EFFECT OF MECHANICAL AND GEOMETRICAL PROPERTIES ON DYNAMIC BEHAVIOR OF ASYMMETRICAL COMPOSITE SANDWICH BEAM WITH VISCOELASTIC CORE

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This paper presents an approach to the analysis of free and forced vibrations of strengthened sandwich beam with viscoelastic core and composite coats by considering geometrical asymmetry. A higher-orde theory is used considering longitudinal and rotational inertias as well as the asymmetry of sandwich beam. The formulation of the equation of motion is carried out by Hamilton principle. A comparative study to validate the proposed numerical approach is performed comparing the obtained results with other findings. Moreover, a parametric analysis is carried out with different configurations of the sandwich beams in order to analyze the influence of different parameters on the dynamic behavior. The analysis highlighted from the study of fiber orientation influence on dynamic behavior, that the structure damping can be improved by adopting a better composite configuration. However, the obtained results from the thickness ratio effect showed that the sandwich structure has more dissipative capacity for low values of viscoelastic thickness and it is more efficient for asymmetrical sandwich beam.

Keywords: vibration; sandwich; viscoelastic material; composite; loss factor; finite element.

1. Introduction

In recent decades, viscoelastic materials have undergone a great evolution in several fields such as civil engineering, aeronautics, and in the automobile industry, because of their specific mechanical properties. Viscoelastic materials attenuate structural vibrations generated from various dynamic loadings, the damping is provided to the structure through of its property of passing from a slight rigid state (rubbery state) into a rigid state (glassy state). The first studies on sandwich structures with viscoelastic core have been carried out by Kerwin [1] and Ungar et al [2], in these studies an analytical expression of the loss factor as a function of the structure characteristics was employed. Other analytical models were proposed by Ungar [3], Yu [4] to characterize damping properties of viscoelastic sandwich beams based on previous studies [1-2]. Then, DiTaranto [5] defined an equation describing the damping properties (damped pulsation, loss factor) taking into account different boundary conditions. However, vibration

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analysis of sandwich beam with a constant modulus of viscoelastic core has been widely investigated. Rao [6] used Hamilton energetic principle to formulate the governing equation of vibration motion. In addition, several numerical approaches based on finite element method were assumed for composite structures considering more complex geometries. The authors used several kinematic models describing the displacement field of three layers [7-9]. Recently, kinematic models consist of describing the layer-by-layer displacement field in order to improve accuracy, leading to zigzag-type model that combine at the same time the kinematic models of Rao [6] and Reddy [10]. Arikoglu and Ozkol [11] studied the dynamic behavior of three-layer composite beam with a viscoelastic core using the differential transformation method (DTM) to solve governing equation of motion obtained by Hamilton principle. Irazu and Elejabarrieta [12] analyzed the design parameters influence on the dynamic properties of thin sandwich beams with different types of viscoelastic layers and metallic skins using bandwidth method. Daya and Potier-Ferry [13] employed the asymptotic numerical method for the eigenvalue problem characterizing the free vibration of viscoelastic sandwich beams taking into account the frequency dependence of the viscoelastic core. Daya et al [14] applied a nonlinear theory to study the dynamic responses of sandwich beams with viscoelastic core. Bilasse et al [15] have used the Diamant approach to solve the eigenvalue problem in order to analyze the linear and nonlinear vibration of viscoelastic sandwich beams. However, Other researchers have used a constraining layer in viscoelastic sandwich beams to improve the structure damping. This type of structure named Passive Constrained Layer Damping “PCLD” is studied by Cai et al [16], in this work, an analytical approach is proposed to examine the dynamic response using the Lagrange energy method. The model of Mead and Markus [17] was used to describe the kinematic relationships between the three layers. In the same line, an analytical approach have proposed by Cai et al [18] to analysis the vibratory responses for a composite beam with a viscoelastic core layer using an active treatment Active Constrained Layer Damping (ACLD), which the elastic constraining layer in the PCLD principle is replaced by a piezoelectric layer in order to improve the energy dissipation. Arvin et al [19] presented a higher order theory of sandwich with composite faces and viscoelastic core, transverse displacements are considered independent for both face layers and a linear variation along the viscoelastic layer depth.

However, very few researchers have focused on the impact of mechanical and geometrical properties on the dynamic behavior of passive damped structures, when in fact this is very important in the conception of structures with viscoelastic core. The present paper is focused on dynamic behavior of asymmetrical viscoelastic sandwich beams with different mechanical and geometrical configurations under a dynamic point load. A higher theory is used for the asymmetrical sandwich composite beam with viscoelastic core and composite face, where the longitudinal and rotational inertias are considered. In addition, the theory of Euler-Bernoulli is applied to the faces and Timoshenko theory to the viscoelastic core [20].

2. Mathematical Formulation

The dynamic behavior of asymmetric PCLD sandwich beam (figure1) is carried out in the context of small deformations. The assumptions considered by Karmi et al [21] and Tekili et al[22] are modified to take into account the effect of longitudinal and rotational inertia as well as the asymmetry of the sandwich. The displacement field beam is given by Rao's zigzag model [9] based on the first-order deformation theory, where Euler-Bernoulli's theory is applied to the composite sandwich faces and Timoshenko's theory to the viscoelastic core.

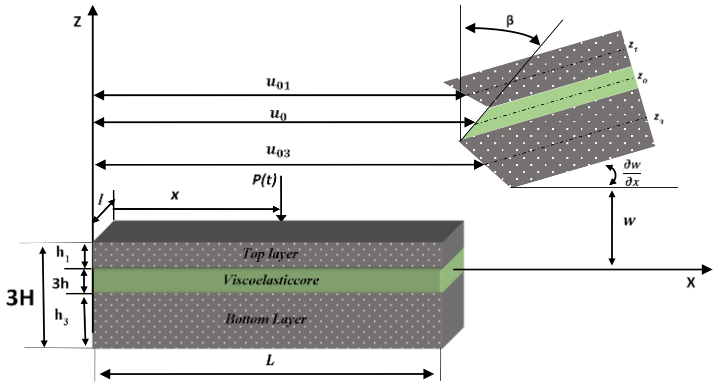

Fig.1. Asymmetric sandwich beam configuration and deformation

where u_{0i} is the longitudinal displacement at the middle plane of ith layer. h_1, h_2 and h_3 represent the thickness of the upper, central and lower layers, respectively. w is the transverse displacement of sandwich, β is the rotation of the normal of the central layer. On the basis of the researches of references [21-22] and considering the new assumptions of the asymmetrical sandwich beam, the application of the variational formulation to the Hamilton's principle yields the governing equation of motion:

$$\int_0^T \delta\Pi dt = \int_0^T \left(\delta U - \delta K - \delta W\right) dt =$$

$$\int_0^T\int_0^L \begin{bmatrix} N_1\delta u_{1,x} + N_2\delta u_{2,x} + N_3\delta u_{3,x} + M_1\delta w_{,xx} + M_2\delta\beta_{,x} + M_3\delta w_{,xx} + T(\delta w_{,x} + \delta\beta) \\ + \rho_1 S_1 \ddot{u}_1\delta u_1 + \rho_3 S_3 \ddot{u}_3 \delta u_3 + \rho_2 S_2 \ddot{u}_2 \delta u_2 + \left(\rho_1 S_1 + \rho_2 S_2 + \rho_3 S_3\right)\ddot{w}\delta w - P\left(x,t\right)\delta w \end{bmatrix} dxdt = 0 \quad (1)$$

where S_i refers the cross-section area of the ith layer, N_i and M_i correspond to the normal force and the bending moment in the ith layer and T is the shear force in the viscoelastic layer, which are given by:

$$N_i = E_i S_i u_{1,x}; M_i = E_i I_i w_{,xx} \quad \text{for i} = 1,3$$

$$N_2 = E_2^*(\omega) S_2 u_{2,x};\ M_2 = E_2^*(\omega) I_2 \beta_{,x};\ T = \frac{S_2}{2(1+\upsilon_2)} E_2^*(\omega)(\beta + w_{,x}) \quad (2)$$

where E_i and I_i denote the Young's modulus and the quadratic moment of the cross section of the ith layer respectively, $E_2{}^*(\omega)$ and υ_2 are the frequency dependent Young's modulus and the poisson's ratio of the viscoelastic layer respectively.

Because of the asymmetry of the sandwich beam, the longitudinal displacements at the middle plane of the face layers are different and can be expressed as a function of the displacements at the middle plane of the central layer by :

$$u_{01} = u_0 + \frac{h_2}{2}\beta - \frac{h_3}{2}\frac{\partial w}{\partial x}\ ;\ u_{03} = u_0 - \frac{h_2}{2}\beta + \frac{h_1}{2}\frac{\partial w}{\partial x} \quad (3)$$

3. Finite element discretization

The discretization of the equation of motion Eq.(16) by the finite element method and the expression of displacement field as a function of the nodal displacements U_e make it possible to form the elementary behavior matrices.

$$w = N_w U_e; U = N_u U_e; \beta = N_\beta U_e \text{ with } U_e = \langle U_i\ \beta_i\ w_i\ \theta_i \rangle^T \text{ i=1,2} \quad (4)$$

N_w, N_u and N_β are the interpolation functions. An element with two nodes is used in this study, the number of degrees of freedom is four (the longitudinal displacement u, the transverse displacement w, the rotation of the normal of the central layer β and the rotation $\theta = dw/dx$.

The elementary matrix system that describes the vibratory behavior of the sandwich beam can be obtained by replacing Eqs. (1-4) into Eq.(1), the following expression is obtained:

$$[M]^e \ddot{U}_e + [K]^e U_e = \{F\}^e \quad (5)$$

where $[M]^e$, $[K]^e$ and $\{F\}^e$ are the elementary mass matrix, stiffness matrix and nodal force vector expressed by:

$$\begin{aligned}
[M]^e &= (\rho_3 S_3 + \rho_1 S_1)\int_0^{L^e}\left([N_u]^T[N_u] + \frac{h_2^2}{4}[N_\beta]^T[N_\beta]\right)dx + h_2(\rho_3 S_3 - \rho_2 S_2)\int_0^{L^e}\left([N_\beta]^T[N_u]\right)dx \\
&+ (\rho_2 S_2 h_2 - \rho_3 S_3 h_3)\int_0^{L^e}\left([N_{w,x}]^T[N_u]\right)dx - \frac{h_c}{2}(\rho_3 S_3 h_3 + \rho_2 S_2 h_2)\int_0^{L^e}\left([N_{w,x}]^T[N_\beta]\right)dx \\
&+ \frac{1}{4}(\rho_3 S_3 h_3^2 + \rho_2 S_2 h_2^2)\int_0^{L^e}\left([N_{w,x}]^T[N_{w,x}]\right)dx + (\rho_2 S_2)\int_0^{L^e}\left([N_u]^T[N_u]\right)dx \\
&+ (E_3 S_3 + E_2 S_2 + E_1 S_1)\int_0^{L^e}\left([N_{w,xx}]^T[N_{w,xx}]\right)dx
\end{aligned} \quad (6)$$

$$[K]^e = (E_3S_3 + E_1S_1)\int_0^{L^e}\left([N_{u,x}]^T[N_{u,x}] + \frac{h_c^2}{4}[N_{\beta,x}]^T[N_{\beta,x}]\right)dx + (E_2I_2)\int_0^{L^e}\left([N_{\beta,x}]^T[N_{\beta,x}]\right)dx$$
$$+h_2(E_3S_3 - E_1S_1)\int_0^{L^e}\left([N_{\beta,x}]^T[N_{u,x}]\right)dx + (E_1S_1h_1 - E_3S_3h_3)\int_0^{L^e}\left([N_{w,xx}]^T[N_{u,x}]\right)dx$$
$$+(E_2S_2)\int_0^{L^e}\left([N_{u,x}]^T[N_{u,x}]\right)dx - \frac{h_c}{2}(E_3S_3h_3 + E_1S_1h_1)\int_0^{L^e}\left([N_{w,xx}]^T[N_{\beta,x}]\right)dx$$
$$+(E_3I_3 + E_1I_1)\int_0^{L^e}\left([N_{w,xx}]^T[N_{w,xx}]\right)dx + \frac{1}{4}\left(E_3S_3h_3^2 + E_1S_1h_1^2\right)\int_0^{L^e}\left([N_{w,xx}]^T[N_{w,xx}]\right)dx$$
$$+\left(\frac{S_2}{2(1+\upsilon_c)}\right)\int_0^{L^e}\left([N_\beta]^T[N_\beta] + 2[N_\beta]^T[N_{w,x}] + [N_{w,x}]^T[N_{w,x}]\right)dx \quad (7)$$

$$\{F\}^e = \int_0^{L^e} P(x,t)[N_w]^T \quad (8)$$

with L^e is the element length, the global matrix system describing the vibratory behavior of the sandwich beam after the assembly of the elementary matrices is written in the form:

$$[M]\ddot{U} + [K]U = \{F\} \quad (9)$$

where *[M]*, *[K]* and *{F}* are respectively the mass matrix, the stiffness matrix and the global nodal force vector. This equation can be solved using harmonic balance method. In order to study the free vibration and establish the modal basis, it is required to solve the eigenvalue problem that can be determined using the QR method combined to the asymptotic numerical method implemented with Matlab code [21].

4. Results and discussions

In the following sections, the dynamic behavior of viscoelastic sandwich beams is studied using several models of configuration presented in figure 1. Firstly, comparative studies are carried out to validate the proposed numerical approach. Next, dynamic responses of the sandwich beams are examined under a harmonic point load in the form:

$$P(x,t) = P_0\delta(x - x_0)e^{i\omega t} \quad (10)$$

with δ is the Dirac distribution, P_0 is the force magnitude and x_0 is the position of the force, where $x_0=\{L, L/2\}$ for cantilever and simply supported beams, respectively. Different beam configurations such as fiber orientation of the face layers and thickness ratio are studied to evaluate their effects on dynamic behavior. The model of viscoelastic behavior is considered with the frequency independent viscoelastic modulus Eq.(11), this model is widely used to study the viscoelastic behavior.

$$E_2 = E_0(1 + i\eta_c) \quad (11)$$

with E_0 is the modulus of delayed elasticity and η_c is the viscoelastic loss factor.

4.1. Results and validation

The obtained results for a cantilever sandwich beam with viscoelastic core placed between two isotropic elastic layers are compared with those obtained by Arvin et al [19]. The mechanical and geometrical properties of the viscoelastic sandwich beam are given in table 1.

Table1

Mechanical and geometrical properties of the viscoelastic sandwich [19]

	Upper face	Viscoelastic core	Lower face
Young's modulus (Pa)	E_1=7.03×10^{10}	E_0=2.097×10^{6}	E_3=7.03×10^{10}
Poisson's ratio υ	υ_1=0.3	υ_2=0.49	υ_3=0.3
Density (Kg/m^3)	P_1=2770	P_2=970	ρ_3=2770
Thickness (mm)	h_1=1.52	h_2=0.127	h_3=1.52
Length (mm); Width (mm)	L=177.8 ×10^{-3}; l=12.7×10^{-3}		

The damping properties corresponding to the first five modes are reported in table 2. By comparison, it can be seen that the obtained results are very close with those obtained by Arvin et al [19]. The precision of the results is illustrated by the residual error $\mathbf{R}(\boldsymbol{U}, \boldsymbol{\lambda})=\|([\mathbf{K}]-\boldsymbol{\omega}^2 [\mathbf{M}])\boldsymbol{U}\|$ where **R**<0.5×10^{-3}, which approves the effectiveness of the proposed approach.

Table 2

Natural frequencies and loss factor for cantilever sandwich

		Proposed formulation			Arvin et al [19]	
η_c	mode	ω (Hz)	η	R (U, λ) ×10^{-3}	f (Hz)	η
0.3	1	65.016	0.0812	0.3315	64.985	0.08181
	2	300.31	0.0706	0.4310	299.47	0.07230
	3	749.78	0.0485	0.3553	749.77	0.04642
	4	1408.3	0.0275	0.4875	1404.1	0.02681
	5	2295.5	0.0125	0.4309	2276.5	0.01725

In this section, the dynamic responses of sandwich beams with Passive Constrained Layer Damping “PCLD” under a harmonic point load are investigated, where the properties of the sandwich are presented in table 3.

Table 3

Mechanical and geometrical properties of the PCLD sandwich with viscoelastic core[16]

		Upper face	Viscoelastic core	Lower face
Young's modulus (Pa)		E_1 =49×10^9	E_2=2G(1+υ_2)	E_3=70×10^9
Shear Modulus (MPa)	Soft	/	G=0.895 + 1.3067i	/
	Hard		G=9.89 + 14.4394i	
Poisson's ratio υ		υ_1=0.3	υ_2=0.49	υ_3=0.3
Density (Kg/m^3)		ρ_1 =7500	ρ_2=1000	ρ_3=2110
Thickness (mm)		h_1=2	h_2=1	h_3=4
Length (m); Width (m)		L=0.4; l=0.03		

The frequency responses of displacement obtained by solving Eq. (9) of the considered sandwich beam are compared with the responses obtained by Cai et al [16], the responses are compared for both soft and hard viscoelastic core models and for a cantilever beam. The obtained responses by the analytical approach of reference [16] and with the proposed finite element approach at the tip of the beam are illustrated in Figs. 2.

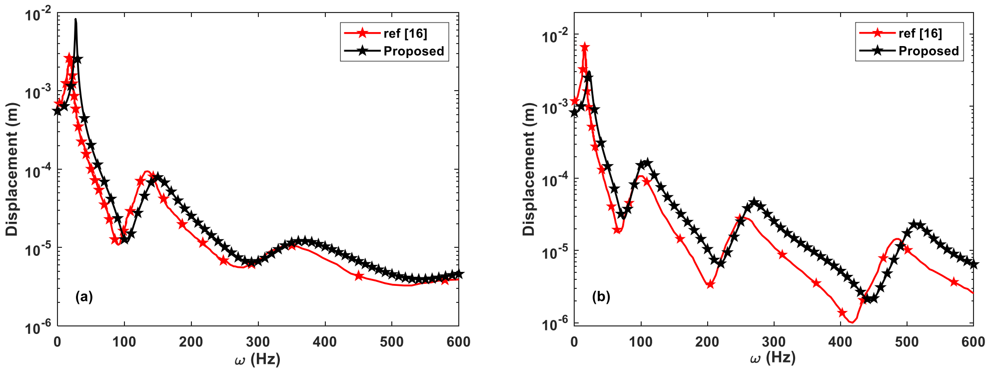

Fig.2. Comparison of frequency responses of the sandwich beam with viscoelastic core between present approach and the analytical approach of the reference [16] "a-hard core; b-soft core"

It can be seen that the natural frequencies obtained by Cai and al [16] are underestimated compared to the proposed approach results obtained for two models of the hard and soft viscoelastic core. In addition, the amplitudes of the frequency responses at resonant frequencies of reference [16] for the hard-core are lower than the corresponding results of the present study even at low frequencies. On the other hand, the amplitudes obtained by the proposed approach for the second soft-core model are lower than those obtained by Cai and al [16].

4.2. Parametric study

The effects of different beam configuration parameters such as the fiber orientation of the face layers θ, the thickness ratio h/H and the asymmetry of sandwich beam on dynamic behavior are analyzed considered in this parametric study. The mechanical and geometrical properties of the considered sandwich beam strengthened by composite coats are given in table 4 and figure 1.

Table 4

Mechanical and geometrical properties of the strengthened sandwich with viscoelastic core

	Upper composite / Lower Face	Viscoelastic core
Young's modulus (Pa)	$E_{11}=98\times10^9$; $E_{22}=7.998\times10^9$; $G_{12}=5.69\times10^9$	$E_0=7.037\times10^5\times(2(1+\upsilon_2))$
Poisson's ratio υ	$\upsilon_3=\upsilon_1=0.28$	$\upsilon_2=0.49$
Density (Kg/m^3)	$\rho_1=\rho_3=1520$	$\rho_2=970$
Thickness(m)	h_1=H-h; $h_3=2h_1$; $H=12\times10^{-3}$	h_2=3h
Length(m); Width (m)	L=0.6; l=0.02	

4.2.1. Effect of fiber orientation

The natural frequencies ω(Hz) and structural loss factor η of the cantilever sandwich beam corresponding to the first three modes with η_c=0.6 and h/H = 0.1 and for different fiber orientation are illustrated in figure 3. It can be observed that the natural frequencies reach maximum values for 0°, 50° and 60° orientations and they reach low values for the other orientation. Conversely, the loss factor values of the structure reach maximum values for orientations that are different at θ = 0 °, 50 ° and 60°.These results show the benefits provided by viscoelastic materials even for low frequency values.

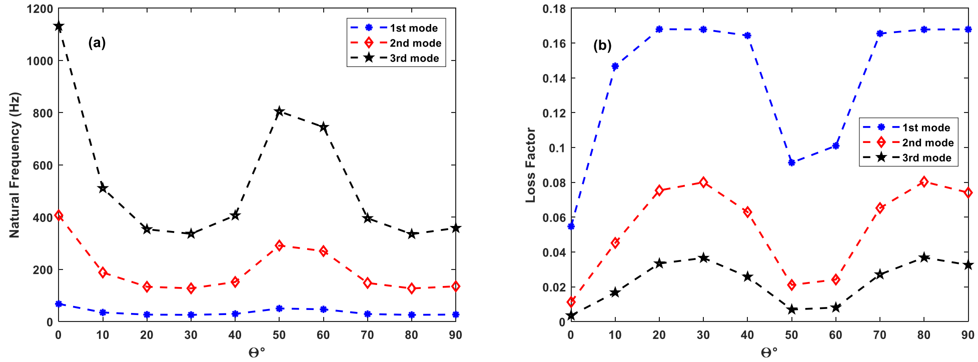

Fig.3. Variations of natural frequencies and loss factors for the first three modes of the simply supported sandwich beam obtained for different values of fiber orientation ((a) natural frequency; (b) loss factor)

The frequency responses of the transverse displacement at the middle and the tip of simple supported and cantilever beams respectively are shown in figure 4. The obtained results show that the frequency ranges for the 30° and 90° configurations are less dispersed compared to the 0 ° and 60 ° configurations. However, the amplitude peaks for configurations with θ =30 ° and θ =90 ° are higher compared to those obtained for θ =0° and θ =60° in particular the first peak. The same conclusions for the cantilever beam have been drawn.

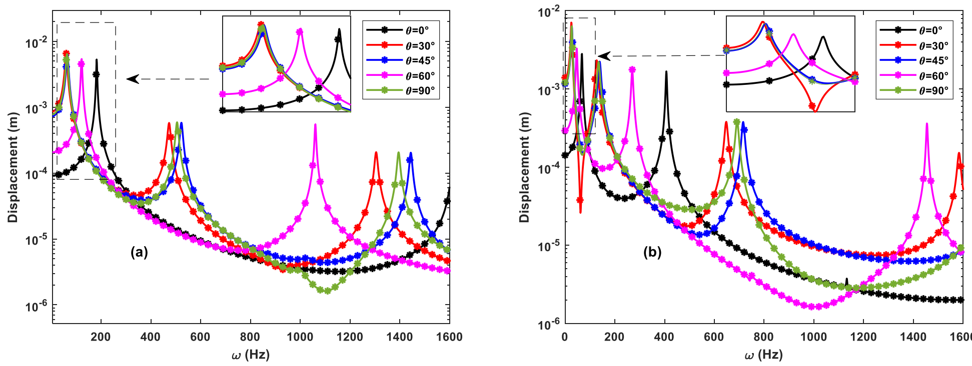

Fig.4. Comparison of frequency responses of the sandwich beam obtained for different values of fiber orientation θ ((a) simply supported; (b) cantilever beam)

4.2.2. Effect of thickness ratio

The variations of natural frequencies and loss factor as a function of the thickness ratio *h/H* corresponding to the first three modes with η_c=0.6 and *0*=0°

are illustrated in figure 5. These results illustrate that the frequencies are inversely proportional to the thickness ratios *h/H*, which means that the natural frequencies decrease when the thickness ratio increases. However, the loss factor variation is proportional to the thickness ratio, which the loss factors reach large values implying an increase in the structural damping with low frequencies.

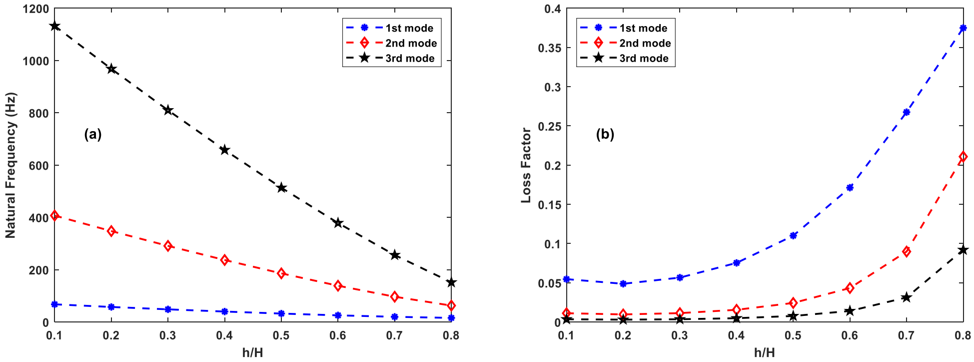

Fig.5.Variations of natural frequencies and loss factors for the first three modes of the simply supported sandwich beam obtained for different values of thickness ratio *h/H* ((a) natural frequency; (b) loss factor)

The frequency responses of the sandwich beam with viscoelastic core considered in table 4 are shown in figure 6 for both conditions simply supported and cantilever beams. The obtained results show that the amplitudes of the frequency response for *h/H* = 0.1 are much smaller. Moreover, it is noticed that the natural frequencies obtained for *h/H*=0.6 and *h/H*=0.8 corresponding to the first three eigenmodes are less dispersed by comparing the results with those obtained for *h/H*=0.1 and *h/H* =0.3. This means that the frequencies decrease with the increase in the thickness of the viscoelastic layer inducing high amplitudes.

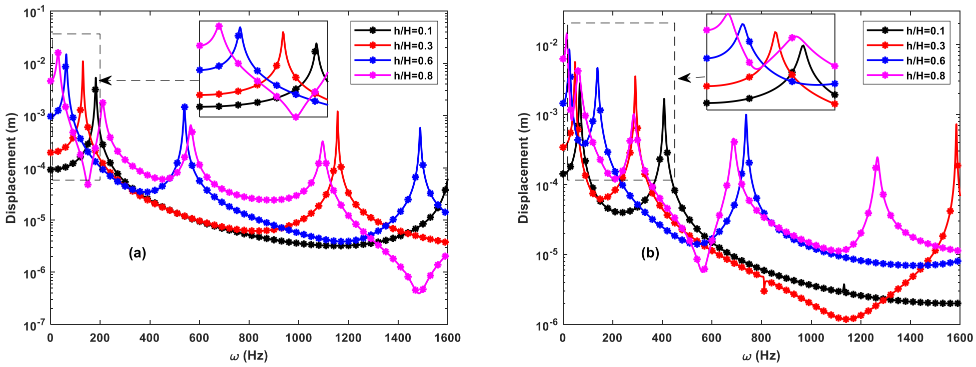

Fig.6. Comparison of frequency responses of the sandwich beam obtained for different values of *h/H* ((a) simply supported; (b) cantilever beam)

4.2.3. Effect of sandwich asymmetry

In this section, the effect of the asymmetry of sandwich beam with viscoelastic core is studied by varying the thickness of the bottom layer with respect to the top layer by keeping the overall thickness of the sandwich beam *3H* constant (figure (1)). The natural frequency and loss factor variations for the first three modes and for a cantilever beam are shown in figure 7. It is very clear that natural frequencies are inversely proportional to the thickness ratio for $h_3/h_1<1$ whereas they become proportional to the thickness ratio for $h_3/h_1>1$. This means that the natural frequencies increase when the thickness of the bottom layer is strictly different from that of the top layer, the highest natural frequencies are obtained for $h_3/h_1=0.1$ and $h_3/h_1=9$ and the lowest value is obtained for $h_3/h_1=0.1$. Reciprocally, the loss factor is proportional to the variation of the thickness ratio for $h_3/h_1<1$ and inversely proportional for $h_3/h_1>1$, where highest value is obtained for $h_3/h_1=1$.

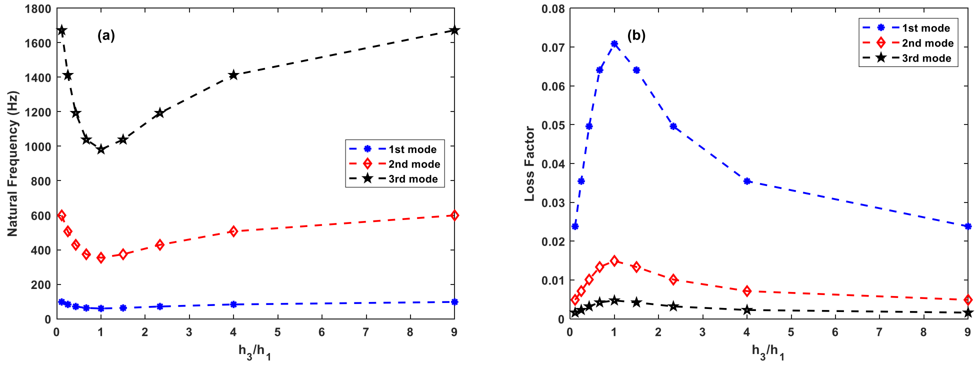

Fig.7. Variations of natural frequencies and loss factors for the first three modes of the cantilever sandwich beam obtained for different values of thickness ratio h_3/h_1 ((a) natural frequency; (b) loss factor)

In order to evaluate the effect of asymmetry on the dynamic behavior of the sandwich beam under dynamic load, the different frequency responses are obtained and presented in figure 8. It can be seen that the increase of thickness ratio h_3/h_1 caused a shift of the amplitude peaks of different responses due to the variation of natural frequencies. The largest shift of amplitude peaks for the simply supported beam is obtained with the configuration $h_3/h_1=0.25$ and $h_3/h_1=3$ corresponding to the natural frequencies $\omega=227$ and $\omega=208$, respectively, while the smallest shift is obtained for $h_3/h_1=1$ corresponding to the lowest natural frequency $\omega=159$. It can also be observed that the peak amplitudes are very close for different thickness values because of the interaction between reduced stiffness and improved damping. The same remarks have been noticed for the cantilever sandwich beam. Given that the lowest frequencies are the most critical for the structure, it is evident that the structure has better performance when it becomes asymmetrical.

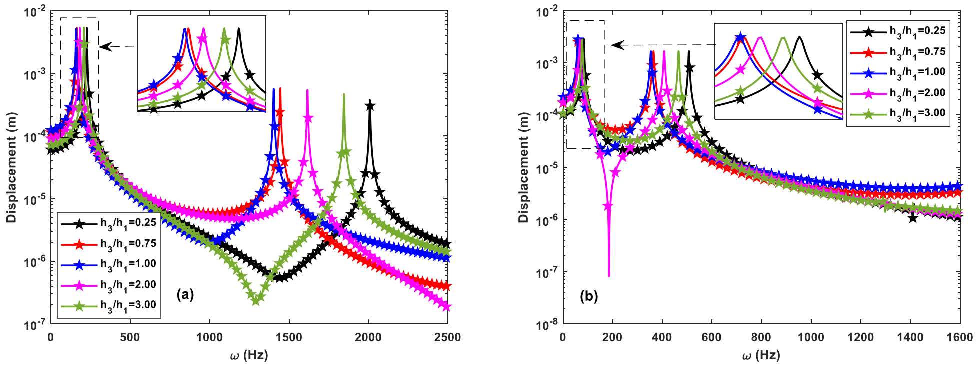

Fig.8. Comparison of frequency responses of the sandwich beam obtained for different values of thickness ratio h_3/h_1 ((a) simply supported; (b) cantilever beam)

5. Conclusions

In this work, a higher order theory was used to study frequency responses of asymmetric sandwich beams with viscoelastic core by considering the longitudinal and rotational inertias. An evaluation of the damping of sandwich beams with viscoelastic materials strengthened by composite coats has been carried out using an improved numerical approach based on the finite element method, which has been validated by comparison with other research results. In the face of the lack of research investigating the optimization of the configuration of passive damping treatment by viscoelastic layer, in this research, the different mechanical and geometrical properties as well as the asymmetry of the sandwich beam that affect the dynamic behavior have been properly examined in order to find an optimal configuration providing a high damping ability.

From the obtained results, the following conclusions can be drawn:

- The natural frequencies reach high values for θ = 0°, θ = 50° and θ = 60° while they reach low frequency values for configurations with θ = 90° and θ = 30°. However, the amplitude peaks of the frequency responses for 30° and 90° configurations are higher compared to those obtained for 0° and 60°, in particular for the first peaks.
- The natural frequencies are inversely proportional to the thickness ratio. Therefore, the amplitudes of the peaks are proportional to this ratio.
- The natural frequencies increase when the sandwich beam becomes asymmetrical, which caused a shift of the amplitude peaks of different responses.

This analysis shows that the loss of stiffness due to the fiber orientation of the face layers, which is possibly the main cause of the increase of amplitudes of dynamic responses. In addition, the obtained results reflect the high damping proprieties of the structure when the thickness of the viscoelastic core layer becomes thinner. However, the structure is more efficient and resistant to dynamic load when the thickness of the bottom layer is different from that of the top layer.

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