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# INTERVAL VALUED $(\alpha, \beta)$-FUZZY HYPERIDEALS OF SEMIHYPERRINGS

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*The concept of quasi-coincidence of interval valued fuzzy point with an interval valued fuzzy subset has considered. By using this idea, the notion of interval valued $(\alpha, \beta)$-fuzzy hyperideal in a semihyperring introduced and consequently, a generalization of interval valued fuzzy hyperideals and interval valued fuzzy hyperideals have defined. In this paper, we study the related properties of the interval valued $(\alpha, \beta)$-fuzzy hyperideals and in particular, the interval valued $(\in, \in \vee q)$-fuzzy hyperideals in semihyperrings will be investigated. Moreover, we also consider the concept of implication-based interval valued fuzzy hyperideals in a semihyperring.*

**Keywords:** Semihyperring, fuzzy hyperideal, interval valued $(\alpha, \beta)$-fuzzy hyperideal

**MSC2010:** 16Y99, 16D25, 03E72

## 1. Introduction

Hyperstructure theory was born in 1934 when Marty [19] defined hyper groups, began to analysis their properties and applied them to groups, rational algebraic functions. Now they are widely studied from theoretical point of view and for their applications to many subjects of pure and applied properties. Recently Abdullah et al defined prime bi-Γ-hyperideals, M-hypersystems and N-hypersystems in Γ-semihypergroups [1, 2]. Abdullah studied on topological structure on semihypergroups [3].

The notion of a fuzzy set was introduced by Zadeh [5]. Fuzzy set theory has been shown to be a useful tool to describe situations in which the data are imprecise or vague. Fuzzy sets handle such situations by attributing a degree to which a certain object belongs to a set. The fuzzy algebraic structures play a prominent role in mathematics with wide applications in many other branches such as theoretical physics, computer sciences, control engineering, information sciences, coding theory, topological spaces, logic, set theory, group theory, real analysis, measure theory etc. Rosenfeld studied fuzzy subgroups of a group [6]. The study of fuzzy semigroups was studied by Kuroki in his classical papers [7]. Recently Jun and Kang [8] considered fuzzification of generalized tarasaki filter in tarasaki algebra. Recently, fuzzy set theory has been well developed in the context of hyperalgebraic

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structure theory. Davvaz introduced the concept of fuzzy hyperideals in a semihypergroup. Recently in [18], Davvaz and Leoreanu-Fotea studied the structure of fuzzy Γ-hyperideals in Γ-semihypergroups. Ameri and Noari [9] initiated fuzzy hyperalgebras and introduced some important results. Davvaz et.al., proposed fuzzy $H_v$-ideals in Γ-$H_v$-rings [10] and fuzzy Γ-hypernearrings [11]. Recently Aslam et al introduced fuzzy m-hypersystems and rough M-hypersystems in Γ-semihypergroups [4]. Davvaz [12], initiated fuzzy krasner $(m, n)$-hyperrings. Sun et.al deeply studied fuzzy hypergraphs on fuzzy relations in [13]. Using the notions "*belong to*" relation ($\in$) introduced by Pu and Lia [14]. In [15], Morali proposed the concept of a fuzzy point belonging to a fuzzy subset under natural equivalence on fuzzy subset. Bhakat and Das introduced the concepts of $(\alpha, \beta)$-fuzzy subgroups by using the "*belong to*" relation ($\in$) and "*quasi-coincident with*" relation ($q$) between a fuzzy point and a fuzzy subgroup, and defined an $(\in, \in \vee q)$-fuzzy subgroup of a group [16]. In [17], Tariq et. al. introduced the concept of $(\alpha, \beta)$-fuzzy hyperideals and $(\in, \in \vee q)$-fuzzy hyperideals in semihypergroups.

## 2. **Preliminaries**

By a semihyperring, we mean an algebraic hyper system $(R,+,.)$ consisting of a non-empty set $R$ together with two binary operations on $R$, " addition and multiplication", such that $(R,+)$ and $(R,.)$ are semihypergroups and for all $x, y, z \in R$, we have $x(y+z) = xy + xz$, which is called distributivity. A subsemihyperring of $R$, we mean a non-empty subset $S$ of $R$ such that for all $x, y \in S$, we have $x.y \subseteq S$ and $x + y \subseteq S$. A subsemihyperring $I$ of $R$ is called a left (right) hyperideal of $R$ such that for all $r \in R$ and $x \in I$, we have $r.x \subseteq I$ $(x.r \subseteq I)$.

A subsemihyperring of $R$ which is both a left and a right hyperideal of $R$ is said to be a hyperideal of $R$.

**Definition 2.1.** *A fuzzy subset $\widetilde{\mu}$ of a hyperring $R$ is called a fuzzy hyperideal of $R$ if the following conditions hold:*

(i) $\min\{\widetilde{\mu}(x), \widetilde{\mu}(y)\} \leq \inf_{\, z \in x+y}\widetilde{\mu}(z)$, for all $x, y \in R$;
(ii) $\widetilde{\mu}(rx) \geq \widetilde{\mu}(x)$ and $\widetilde{\mu}(xr) \geq \widetilde{\mu}(x)$.

**Theorem 2.1.** *Let $\widetilde{\mu}$ be a fuzzy set in a semihyperring $R$. Then, $\widetilde{\mu}$ is a fuzzy hyperideal of $R$ if and only if for every $t \in (0, 1]$, the level subset $\mu_t (\neq \Phi)$ is a hyperideal of $R$, where $\mu(\widetilde{\mu}; t) = \{x \in R|\ \widetilde{\mu}(x) \geq t\}$, $t \in [0, 1]$.*

*Proof.* The proof is straightforward by considering the definition. □

**Lemma 2.1.** *Let $\widetilde{\mu}$ be a fuzzy set in a semihyperring $R$. If $R$ has zero element, then $\widetilde{\mu}(0) \geq \widetilde{\mu}(x)$ for all $x \in R$.*

*Proof.* The proof is clear. □

It is clear that $(D[0, 1], \leq, \vee, \wedge)$ is a complete lattice with $0 = [0, 0]$ as the least element and $1 = [1, 1]$ as the greatest element. By an interval valued fuzzy set $\widetilde{\mu}$ on $X$, we mean the set,

$$\widetilde{\mu} = \{(x,\ [\mu^-(x), \mu^+(x)]) \mid x \in X\},$$

where $\mu^-_{\widetilde{\mu}}$ and $\mu^+_{\widetilde{\mu}}$ are two fuzzy subsets of $X$ such that $\mu^-(x) \leq \mu^+(x)$ for all $x \in X$. Putting $\widetilde{\mu}(x) = [\mu^-(x), \mu^+(x)]$, then we see that

$$\widetilde{\mu} = \{(x, \widetilde{\mu}(x)) \mid x \in X\},$$

where $\widetilde{\mu} : X \longrightarrow D[0, 1]$.

## 3. Interval valued $(\alpha, \beta)$-fuzzy hyperideals of semihyperrings

An interval valued fuzzy set $\widetilde{\mu}$ of a semihyperring $R$ of the form

$$\widetilde{\mu}(y) = \begin{cases} \tilde{t} \neq [0,0] & \text{if } y = x, \\ [0,0] & \text{if } y \neq x, \end{cases}$$

is said to be a interval valued fuzzy point with support $x$, interval value $\tilde{t}$ and is denoted by $\mu(x; \tilde{t})$. A interval valued fuzzy point $\mu(x; \tilde{t})$ is said to be belong to (resp. quasi-coincident with) an interval valued fuzzy set $\widetilde{\mu}$, written as

$\mu(x; \tilde{t}) \in \widetilde{\mu}(resp. \mu(x; \tilde{t})q\widetilde{\mu})$ if $\widetilde{\mu}(x) \geq \tilde{t}(\widetilde{\mu}(x) + \tilde{t} > [1, 1])$. If $\mu(x; \tilde{t}) \in \widetilde{\mu}$ or (resp. and) $\mu(x; \tilde{t})q\widetilde{\mu}$, then we write $\mu(x; \tilde{t}) \in \vee q\widetilde{\mu}$. If $\mu(x; \tilde{t}) \in \widetilde{\mu}$ and $\mu(x; \tilde{t})q\widetilde{\mu}$, then we write $\mu(x; \tilde{t}) \in \wedge q\widetilde{\mu}$. The symbol $\overline{\in \vee q}$ means neither $\in$ nor $q$ holds. The symbol $\overline{\in \wedge q}$ means $\in$ or $q$ does not hold.

**Definition 3.1.** *An interval valued fuzzy set $\widetilde{\mu}$ in $R$ is called an interval valued $(\alpha, \beta)$-fuzzy hyperideal of $R$, where $\alpha \neq \in \wedge q$, if for all $r, x, y \in R$ and the following conditions hold:*

(i) $\mu(x; \tilde{t})\alpha\widetilde{\mu}$ and $\mu(y; \tilde{t})\alpha\widetilde{\mu}$ imply $u(z; r \min\{\tilde{t}, \tilde{r}\})\beta\widetilde{\mu}$, for all $z \in x + y$,

(ii) $\mu(x; \tilde{t})\alpha\widetilde{\mu}$ implies $\mu(rx; \tilde{t})\beta\widetilde{\mu}$ and $\mu(xr; \tilde{t})\beta\widetilde{\mu}$.

In the next theorem, by an interval valued $(\alpha, \beta)$-fuzzy hyperideal of $R$, we construct an ordinary hyperideal of $R$.

**Theorem 3.1.** *Let $\widetilde{\mu}$ be a non-zero interval valued $(\alpha, \beta)$-fuzzy hyperideal of $R$. Then, the set $supp(\widetilde{\mu}) = \{x \in R | \widetilde{\mu}(x) > [0, 0]\}$ is a hyperideal of $R$.*

*Proof.* Suppose that $x, y \in \text{supp}(\widetilde{\mu})$ and $t, r \in (0, 1]$. Then $\mu(x; \tilde{t}) > [0, 0]$ and $\widetilde{\mu}(y; \tilde{r}) > [0, 0]$. Assume that $\widetilde{\mu}(z) = [0, 0]$ for all $z \in x + y$. If $\alpha \in \{\in, \in \vee q\}$ then $\mu(x; \tilde{t})\alpha\widetilde{\mu}$ and $\mu(y; \tilde{r})\alpha\widetilde{\mu}$. But, for all $z \in x + y$, $\mu(z; r \min\{\tilde{t}, \tilde{r}\})\overline{\beta}\widetilde{\mu}$, for every $\beta \in \{\in, q, \in \vee q, \in \wedge q\}$, which is a contradiction. Note that $\mu(x; \tilde{t})\alpha\widetilde{\mu}$ and $\mu(y; \tilde{r})\alpha\widetilde{\mu}$ but, for all $z \in x + y$,

$$\mu(z; r \min\{[1, 1], [1, 1]\}) = \mu(z; [1, 1])\overline{\beta}\widetilde{\mu},$$

for every $\beta \in \{\in, q, \in \vee q, \in \wedge q\}$, which is a contradiction. Hence, for all $z \in x + y$, $\widetilde{\mu}(z) > [0,\ 0]$, that is, for all $z \in x + y$, $z \in \text{supp}(\widetilde{\mu})$. Also, let there exists $r \in R$ such that $\widetilde{\mu}(xr; \tilde{t}) = [0, 0]$. If $\alpha \in \{\in, \in \vee q\}$, then $\mu(x; \tilde{t})\alpha\widetilde{\mu}$. But $\mu(xr; \tilde{t})\overline{\beta}\widetilde{\mu}$, for every $\beta \in \{\in, q, \in \vee q, \in \wedge q\}$, which is a contradiction. We know that $\mu(x; [1, 1])q\widetilde{\mu}$. But $\mu(xr\ ; \tilde{t})\overline{\beta}\widetilde{\mu}$ for every $\beta \in \{\in, q, \in \vee q, \in \wedge q\}$, which is a contradiction. Hence, $\mu(xr\ ; \tilde{t}) > [0, 0]$ that is $xr \in \text{supp}(\widetilde{\mu})$. Similarly, we can show that $rx \in \text{supp}(\widetilde{\mu})$. Therefore, $\text{supp}(\widetilde{\mu})$ is a hyperideal of $R$. □

In the next theorem, we see that an interval valued $(q, q)$-fuzzy hyperideal is constant under suitable condition.

**Theorem 3.2.** *Let $R$ has zero element and $\widetilde{\mu}$ be a non-zero interval valued $(q, q)$-fuzzy hyperideal of $R$. Then, $\widetilde{\mu}$ is a constant on supp$(\widetilde{\mu})$.*

*Proof.* By lemma 2.5, we know that

$$\mu(\widetilde{\mu};[0,0]) = \vee\{\widetilde{\mu}(x) > [0,0] | x \in R\}.$$

Suppose that there exists $x \in$supp$(\widetilde{\mu})$ such that $\tilde{t}_x = \widetilde{\mu}(x) \neq \tilde{t}_0$,then $\tilde{t}_x < \tilde{t}_0$. Choose $\tilde{t}_1$, $\tilde{t}_2 \in D(0,1]$ such that

$$1 - \tilde{t}_0 < \tilde{t}_1 < 1 - \tilde{t}_x < \tilde{t}_2.$$

Then $\mu([0,0];\tilde{t}_1)q\widetilde{\mu}$ and $\mu(x;\tilde{t}_2)q\widetilde{\mu}$ but for all $z \in 0 + x$,

$$\mu(z; r\min\{\tilde{t}_1,\ \tilde{t}_2\} = \mu(x;\tilde{t}_1)\overline{q}\widetilde{\mu}$$

and for all $z \in x + 0$,

$$\mu(z; r\min\{\tilde{t}_1,\ \tilde{t}_2\}) = \mu(x;\tilde{t}_1)\overline{q}\widetilde{\mu}$$

which is a contradiction. Thus, $\widetilde{\mu}(x) = \widetilde{\mu}(0)$, for all $x \in$ supp$(\widetilde{\mu})$. Therefore, $\widetilde{\mu}$ is constant on supp$(\widetilde{\mu})$. □

In the following theorem, we investigate some conditions that make an interval valued fuzzy set $\widetilde{\mu}$ in $R$ as an interval valued $(q, \in\vee q)$-fuzzy hyperideal.

**Theorem 3.3.** *Let $I$ be a hyperideal of $R$ and $\widetilde{\mu}$ an interval valued fuzzy set in $R$ such that*

(i) $\forall\ x \in R\backslash I$ , $\widetilde{\mu}(x) = [0,0]$,
(ii) $\forall\ x \in I$, $\widetilde{\mu}(x) \geqslant [0.5, 0.5]$,
Then, $\widetilde{\mu}$ is an interval valued $(q, \in\vee q)$-fuzzy hyperideal of $R$.

*Proof.* Suppose that $x, y \in R$ and $\tilde{t}_1$, $\tilde{t}_2 \in D\ (0,1]$ such that $\mu(x;\tilde{t}_1)q\widetilde{\mu}$ and $(y;\tilde{t}_2)q\widetilde{\mu}$. Then $x, y \in I$, and so $z \subseteq I$ for all $z \in x + y$. We can consider the following cases:

(1) In case of $\tilde{t}_1 \wedge \tilde{t}_2 \leq [0.5, 0.5]$, then

$$\widetilde{\mu}(z) \geq [0.5, 0.5] \geq \tilde{t}_1 \wedge \tilde{t}_2,$$

for all $z \in x + y$ and hence $(z; r\min\{\tilde{t}_1,\ \tilde{t}_2\}) \in \widetilde{\mu}$ for all $z \in x + y$.

(2) In case of $\tilde{t}_1 \wedge \tilde{t}_2 > [0.5, 0.5]$,then

$$\begin{aligned} \widetilde{\mu}(z) + \tilde{t}_1 \wedge \tilde{t}_2 \quad > \quad & [0.5, 0.5] + [0.5, 0.5] = [1, 1] \\ & \mu(z; r\min\{\tilde{t}_1, \tilde{t}_2\})q\widetilde{\mu}. \end{aligned}$$

Therefore,

$$\mu(z; r\min\{\tilde{t}_1, \tilde{t}_2\} \in \vee q\widetilde{\mu} \text{ for all } z \in x + y.$$

Now, suppose that $r \in R$ and $\tilde{t} \in D\ (0,1]$ such that $\mu(x;\tilde{t})q\widetilde{\mu}$, then, $x \in I$, and so $rx \subseteq I$. We can see two following cases:

(1) In case of $\tilde{t} \leq [0.5, 0.5]$, then

$$\widetilde{\mu}(rx) \geq [0.5, 0.5] \geq \tilde{t}$$

and hence $\mu(rx;\tilde{t}) \in \widetilde{\mu}$. Similarly, $\mu(xr;\tilde{t}) \in \widetilde{\mu}$.

(2) In case of $\tilde{t} > [0.5, 0.5]$, then

$$\widetilde{\mu}(rx) + [1, 1] > [0.5, 0.5] + [0.5, 0.5] = [1, 1]$$

and so $\mu(rx;\tilde{t})q\widetilde{\mu}$. Similarly, $\mu(xr;\tilde{t})q\widetilde{\mu}$. Therefore, $\mu(rx;\tilde{t}) \in \vee q\widetilde{\mu}$ and $\mu(xr;\tilde{t}) \in \vee q\widetilde{\mu}$.This completes the proof. □

**Theorem 3.4.** *Let $R$ be a semihyperring with zero and $\widetilde{\mu}$ is a interval valued $(q, \in \vee q)$-fuzzy hyperideal of $R$, such that $\widetilde{\mu}$ is not constant on $supp(\widetilde{\mu})$. Then, $\widetilde{\mu}(x) \geq [0.5, 0.5]$ for all $x \in supp(\widetilde{\mu})$.*

*Proof.* Straightforward. □

An interval valued fuzzy set $\widetilde{\mu}$ in $R$ is said to be proper if $\mathrm{Im}(\widetilde{\mu})$ has at least two elements. The two interval valued fuzzy sets are said to be equivalent if they have same family of interval valued level subsets, otherwise, they are said to be non-equivalent. Now, we can discuss on interval valued $(\in, \in)$-fuzzy hyperideal of $R$ which can be expressed as the union of two proper non-equivalent interval valued $(\in, \in)$-fuzzy hyperideals.

**Theorem 3.5.** *Let $R$ have some proper hyperideals. Then a proper interval valued $(\in, \in)$-fuzzy hyperideal $\widetilde{\mu}$ of $R$ such that $3 \leq |Im(\widetilde{\mu})| < \infty$, can be expressed as the union of two proper non-equivalent inteval valued $(\in, \in)$-fuzzy hyperideals of $R$.*

*Proof.* Straightforward □

## 4. **Interval valued $(\in, \in \vee q)$-fuzzy hyperideals**

In this section, we investigate some results and properties of interval valued $(\alpha, \beta)$-fuzzy hyperideals (specifically, ( $\in, \in \vee q$)-fuzzy hyperideals ) of $R$.

**Theorem 4.1.** *Every interval valued $(\in \vee q, \in \vee q)$-fuzzy hyperideal of $R$ is an interval valued $(\in, \in \vee q)$-fuzzy hyperideals of $R$.*

*Proof.* The proof is straight forward. □

**Theorem 4.2.** *Every interval valued $(\in, \in)$-fuzzy hyperideal of $R$ is an interval valued ( $\in, \in \vee q$)-fuzzy hyperideal.*

*Proof.* The proof is straight forward. □

**Proposition 4.1.** *If $I$ is a hyperideal of $R$, then $X_I$ the characteristic function of $I$ is an interval valued $(\in, \in)$-fuzzy hyperideal of $R$.*

*Proof.* We omit the proof. □

**Example 4.1.** *On four element semihyperring $(R, +, .)$ defined by the following two tables:*

| $+$ | $0$ | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| $0$ | $\{0\}$ | $\{a\}$ | $\{b\}$ | $\{c\}$ |
| $a$ | $\{a\}$ | $\{a, b\}$ | $\{b\}$ | $\{c\}$ |
| $b$ | $\{b\}$ | $\{b\}$ | $\{0, b\}$ | $\{c\}$ |
| $c$ | $\{c\}$ | $\{c\}$ | $\{c\}$ | $\{0, c\}$ |

| $.$ | $0$ | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| $0$ | $0$ | $0$ | $0$ | $0$ |
| $a$ | $0$ | $a$ | $a$ | $a$ |
| $b$ | $0$ | $b$ | $b$ | $b$ |
| $c$ | $0$ | $c$ | $c$ | $c$ |

*consider a fuzyy $\widetilde{\mu}$ set as follows:*

$$\tilde{u}_{\widetilde{\mu}}(x) = \begin{cases} [0.8, 0.9] & \text{if } x = 0 \\ [0.6, 0.7] & \text{if } x = a, b \\ [0.2, 0.3] & \text{if } x = c \end{cases}$$

*It is easy to see that $\widetilde{\mu}$ is an interval valued $(\in, \in \vee q)$-fuzzy hyperideal of $(R, +, .)$.*

In the next theorem, we prove an equivalent condition for interval valued $(\in, \in \vee q)$-fuzzy hyperideals.

**Theorem 4.3.** *An interval valued fuzzy set $\widetilde{\mu}$ in $R$ is an interval valued $(\in, \in \vee q)$-fuzzy hyperideal of $R$ if and only if for all $\widetilde{t}, \widetilde{r} \in D(0,1]$ and $x, y \in R$ the following two conditions hold:*

*(i) $r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y), [0.5, 0.5]\} \leq r\inf\{\widetilde{\mu}(z)$ for all $z \in x+y\}$, for all $x, y \in R$.*

*(ii) $\widetilde{\mu}(rx) \geq r\min\{\widetilde{\mu}(x), [0.5, 0.5]\}$ and $\widetilde{\mu}(xr) \geq r\min\{\widetilde{\mu}(x), [0.5, 0.5]\}$,*

*Proof.* Let $\widetilde{\mu}$ be an interval valued $(\in, \in \vee q)$-fuzzy hyperideal of $R$ and $x, y \in R$. We can consider the following cases:

(1) $r\inf\{\widetilde{\mu}(x), \widetilde{\mu}(y)\} < [0.5, 0.5]$. In this case, $r\min\{\{\widetilde{\mu}(z) : \text{for all } z \in x+y\}\} < r\min\{\widetilde{\mu}(x),\ \widetilde{\mu}(y)\}$

which is a contradiction. Thus, for all $z \in x+y$,

$$\widetilde{\mu}(z) \geq r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y)\} = r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y), [0.5, 0.5]\}.$$

(2) $r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y)\} \geq [0.5, 0.5]$. In this case, we have $\widetilde{\mu}_{[0.5,0.5]} \in \widetilde{\mu}$ and $\mu(y; [0.5, 0.5]) \in \widetilde{\mu}$. If for all $z \in x+y$, we have

$$\begin{aligned} ,\widetilde{\mu}(z) &< [0.5, 0.5] \Rightarrow \mu(z; [0.5, 0.5]) \overline{\in} \widetilde{\mu} \\ \widetilde{\mu}(z) + [0.5, 0.5] &< [1, 1] \text{ (or } \mu(z; [0.5, 0.5]) \overline{q} \widetilde{\mu}, \text{ for all } z \in x+y). \end{aligned}$$

Hence, $\mu(z; r\min\{[0.5, 0.5], [0.5, 0.5]\}) \overline{\in \vee q \widetilde{\mu}}$, for all $z \in x+y$, which is a contradiction. Thus, for all $z \in x+y$

$$\widetilde{\mu}(z) \geq [0.5, 0.5] = r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y), [0.5, 0.5]\},$$

Also, if $r, x \in R$ we can consider two following cases:

(1) $\widetilde{\mu}(x) \geq [0.5, 0.5]$. In this case, if $\widetilde{\mu}(xr) < \widetilde{\mu}(x)$, we can choose $\tilde{t} \in (0, 0.5)$ such that $\widetilde{\mu}(rx) < \tilde{t} < \widetilde{\mu}(x)$. Then $\mu(x; \tilde{t}) \in \widetilde{\mu}$, but $\mu(rx; \tilde{t}) \overline{\in \vee q \widetilde{\mu}}$, which is a contradiction. Thus,

$$\widetilde{\mu}(rx) \geq \widetilde{\mu}(x) = r\min\{\widetilde{\mu}(x), [0.5, 0.5]\}.$$

Similarly,

$$\widetilde{\mu}(xr) \geq \widetilde{\mu}(x) = r\min\{\widetilde{\mu}(x), [0.5, 0.5]\}.$$

(2) $\widetilde{\mu}(x) \geq [0.5, 0.5]$. In this case, we have $\widetilde{\mu}_{[0.5,0.5]} \in \widetilde{\mu}$. If $\widetilde{\mu}(rx) < [0.5, 0.5]$ then $\mu(rx; [0.5, 0.5]) \overline{\in} \widetilde{\mu}$

and $\widetilde{\mu}(rx) + [0.5, 0.5] < [1, 1]$ (or $\mu(rx; [0.5, 0.5]) \overline{q} \widetilde{\mu}$).

Hence $\mu(rx; [0.5, 0.5]) \overline{\in \vee q \widetilde{\mu}}$ which is a contradiction. Thus,

$$\widetilde{\mu}(rx) \geq [0.5, 0.5] = r\min\{\widetilde{\mu}(x), [0.5, 0.5]\}.$$

Similarly,

$$\widetilde{\mu}(xr) \geq r\min\{\widetilde{\mu}(x), [0.5, 0.5]\}.$$

Conversely, suppose that $\widetilde{\mu}$ satisfies condition (i) and (ii) Let $x, y \in R$ and $\tilde{t}_1,\ \tilde{t}_2 \in D$ $(0, 1]$ such that

$$\begin{aligned} \mu(x; \tilde{t}_1) &\in \widetilde{\mu} \text{ and } \mu(y; \tilde{t}_2) \in \widetilde{\mu} \\ \widetilde{\mu}(x) &\geq \tilde{t}_1 \text{ and } \widetilde{\mu}(y) \geq \tilde{t}_2 \end{aligned}$$

Suppose that for all $z \in x+y$,

$$\widetilde{\mu}(z) < r\min\{\tilde{t}_1, \tilde{t}_2\}.$$

If

$$r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y) < [0.5, 0.5],$$

then, for all $z \in x + y$,

$$\widetilde{\mu}(z) \geq r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y), [0.5, 0.5]\} = r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y)\} > r\min\{\tilde{t}_1, \tilde{t}_2\},$$

which is a contradiction. So

$$r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y)\} \geq [0.5, 0.5].$$

It follows that, for all $z \in x + y$,

$$\widetilde{\mu}(z) + r\min(\tilde{t}_1, \tilde{t}_2) > 2\widetilde{\mu}(z) \geq 2r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y), [0.5, 0.5]\} = [1, 1].$$

Hence, $\mu(z; r\min\{\tilde{t}_1, \tilde{t}_2\})q\widetilde{\mu}$, for all $z \in x+y$, which implies $\mu(z; r\min\{\tilde{t}_1, \tilde{t}_2\} \in \vee q\widetilde{\mu}$, for all $z \in x + y$. Also, let $r, x \in R$ and $\tilde{t} \in D\ (0, 1]$ such that $\mu(x; \tilde{t}) \in \widetilde{\mu}$, then, $\widetilde{\mu}(x) \geq \tilde{t}$. Suppose that $\widetilde{\mu}(rx) < \tilde{t}$. If $\widetilde{\mu}(x) < [0.5, 0.5]$, then

$$\widetilde{\mu}(rx) \geq r\min\{\widetilde{\mu}(x), [0.5, 0.5]\} = \widetilde{\mu}(x) \geq \tilde{t},$$

which is a contradiction, and so $\widetilde{\mu}(x) \geq [0.5, 0.5]$. It follows that

$$\widetilde{\mu}(rx) + \tilde{t} > 2\widetilde{\mu}(rx) \geq 2r\min\{\widetilde{\mu}(x), [0.5, 0.5]\} = [1, 1].$$

Hence, $\mu(rx; \tilde{t})q\widetilde{\mu}$, which implies $\mu(rx; \tilde{t}) \in \vee q\widetilde{\mu}$. Similarly, $\mu(xr; \tilde{t}) \in \vee q\widetilde{\mu}$. Therefore, $\widetilde{\mu}$ is an inteval valued $(\in, \in \vee q)$-fuzzy hyperideal of $R$. □

In the following theorem, we characterize interval valued $(\in, \in \vee q)$-fuzzy hyperideals based on level subsets.

**Theorem 4.4.** *Let $\widetilde{\mu}$ be an interval valued fuzzy set in $R$. If $\widetilde{\mu}$ is an interval valued $(\in, \in \vee q)$-fuzzy hyperideal of $R$, then for all $[0, 0] < \tilde{t} \leq [0.5, 0.5]$, $\mu(\widetilde{\mu}; \tilde{t}) = \Phi$ or $\mu(\widetilde{\mu}; \tilde{t})$ is an hyperideal of $R$.*

Conversely**,** If $\mu(\widetilde{\mu}; \tilde{t})(\neq \Phi)$ is a hyperideal of $R$ for all $[0, 0] < \tilde{t} \leq [0.5, 0, 5]$ then $\widetilde{\mu}$ is an interval valued $(\in, \in \vee q)$-fuzzy hyperideal of $R$.

*Proof.* Let $\widetilde{\mu}$ be an interval valued $(\in, \in \vee q)$-fuzzy hyperideal of $R$ and $[0, 0] < \tilde{t} \leq [0.5, 0.5]$. If $x, y \in \mu(\widetilde{\mu}; \tilde{t})$, then $\widetilde{\mu}(x) \geq \tilde{t}$ and $\widetilde{\mu}(y) \geq \tilde{t}$. Hence, for all $z \in x + y$,

$$\widetilde{\mu}(z) \geq r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y), [0.5, 0.5]\} \geq r\min\{\tilde{t}, [0.5, 0.5]\} = \tilde{t},$$

which implies that $\widetilde{\mu}(z) \geq \tilde{t}$, for all $z \in x + y$. That is $z \in \mu(\widetilde{\mu}; \tilde{t})$ for all $z \in x + y$. Now, suppose that $x \in \mu(\widetilde{\mu}; \tilde{t})$ and $r \in R$. Then, $\widetilde{\mu}(x) \geq \tilde{t}$, and hence

$$\widetilde{\mu}(rx) \geq r\min\{\widetilde{\mu}(x), [0.5, 0.5]\} \geq r\min\{\tilde{t}, [0.5, 0.5]\} = \tilde{t}.$$

It implies $\widetilde{\mu}(rx) \geq \tilde{t}$, that is $rx \in \mu(\widetilde{\mu}; \tilde{t})$. Similarly, $xr \in \mu(\widetilde{\mu}; \tilde{t})$. Therefore, $\mu(\widetilde{\mu}; \tilde{t})$ is a hyperideal of $R$.

Conversely**,** Let $\widetilde{\mu}$ be an interval valued fuzzy set in $R$ such that $\mu(\widetilde{\mu}; \tilde{t}) \neq \Phi)$ is a hyperideal of $R$ for all $[0, 0] < \tilde{t} \leq [0.5, 0.5]$. If $x, y \in R$, we have

$$\begin{aligned} \widetilde{\mu}(x) &\geq r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y), [0.5, 0.5]\} = \tilde{t}_0 \\ \widetilde{\mu}(y) &\geq r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y), [0.5, 0.5]\} = \tilde{t}_{0,} \end{aligned}$$

then $x, y \in \mu(\widetilde{\mu}; \tilde{t}_0)$, and so $z \in \mu(\widetilde{\mu}; \tilde{t}_0)$ for all $z \in x + y$. Now, we have

$$\widetilde{\mu}(z) \geq \tilde{t}_0 = r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y), [0.5, 0.5]\},$$

for all $z \in x + y$. Hence, condition (i) of the Theorem 4.5 is verified. Now, if $x \in R$, we have

$$\widetilde{\mu}(x) \geq r\min\{\widetilde{\mu}(x), [0.5, 0.5]\} = \tilde{t}'_0.$$

Then, $x \in \mu(\widetilde{\mu}; \tilde{t}_0)$, so $rx \in \mu(\widetilde{\mu}; \tilde{t}_0)$, for all $r \in R$. Hence,

$$\widetilde{\mu}(x) \geq \tilde{t}_0 = r\min\{\widetilde{\mu}(a), [0.5, 0.5]\}.$$

Similarly,

$$\widetilde{\mu}(xr) \geq \tilde{t}_0 = r\min\{\widetilde{\mu}(a), [0.5, 0.5]\}.$$

This shows condition (ii) of the Theorem 4.5 holds. Therefore, $\widetilde{\mu}$ is an interval valued $(\in, \in \vee q)$-fuzzy hyperideal of $R$. □

In Theorem 4.6, we discuss on level subsets in the interval $D(0, 0.5]$. In the next theorem, we see what happen to the subsets in interval $D(0.5, 1]$.

**Theorem 4.5.** *Let $\widetilde{\mu}$ be an interval valued fuzzy set in $R$. Then, $\mu(\widetilde{\mu}; \tilde{t}_0)(\neq \Phi)$ is a hyperideal of $R$ for all $\tilde{t} \in D(0.5, 1]$ if and only if for all $x, y \in R$.*

*(i) $r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y)\} \leq r\max\{\widetilde{\mu}(z, [0.5, 0.5]\}$, for all $z \in x + y$.*

*(ii) $\widetilde{\mu}(x) \leq r\max\{\widetilde{\mu}(rx), [0.5, 0.5]\}$ and $\widetilde{\mu}(x) \leq r\max\{\widetilde{\mu}(xr), [0.5, 0.5]\}$.*

*Proof.* Let $\mu(\widetilde{\mu}; \tilde{t}_0)(\neq \Phi)$ be a hyperideal of $R$ for all $\tilde{t} \in (0.5, 1]$.

If there exists $x, y \in R$ such that, for all $z \in x + y$,

$$r\max\{\widetilde{\mu}(z), [0.5.0.5]\} < r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y)\} = \tilde{t}$$

then $\tilde{t} \in D(0.5, 1]$, $\widetilde{\mu}(z) < \tilde{t}$, $x \in \mu(\widetilde{\mu}; \tilde{t})$ and $y \in \mu(\widetilde{\mu}; \tilde{t})$ for all $z \in x + y$. Hence, $z \in \mu(\widetilde{\mu}; \tilde{t})$, for all $z \in x+y$ and so $\widetilde{\mu}(z) \geq \tilde{t}$, for all $z \in x+y$, which is a contradiction. Therefore, for all $x, y \in R$, we have for all $z \in x + y$,

$$r\max\{\widetilde{\mu}(z), [0.5, 0.5]\} \geq r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y)\},$$

Thus, (1) is proved. Also, if there exist $r, x \in R$ such that

$$r\max\{\widetilde{\mu}(rx), [0.5, 0.5]\} < \widetilde{\mu}(x) = r\max\{\widetilde{\mu}(xr), [0.5, 0.5]\} < \widetilde{\mu}(x) = \tilde{t},$$

then $\tilde{t} \in (0.5, 1], \widetilde{\mu}(rx) < \tilde{t}$ and $x \in \mu(\widetilde{\mu}; \tilde{t})$. Hence, $rx \in \mu(\widetilde{\mu}; \tilde{t})$ and so $\widetilde{\mu}(rx) \geq \tilde{t}$, which is a contradiction. Therefore, for all $r, x \in R$, we have

$$r\max\{\widetilde{\mu}(rx), [0.5, 0.5]\} \geq \widetilde{\mu}(x)$$

and

$$r\max\{\widetilde{\mu}(xr), [0.5, 0.5]\} \geq \widetilde{\mu}(x).$$

Thus, (2) is proved.

Conversely, let (1) and (2) hold. Assume that $\tilde{t} \in (0.5, 1]$ and $x, y \in \mu(\widetilde{\mu}; \tilde{t})$. Then, by (1) we have

$$[0.5, 0.5] < \tilde{t} \leq r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y)\} \leq r\max\{\widetilde{\mu}(z), [0.5, 0.5]\}$$

for all $z \in x + y$. It implies that

$$[0.5, 0.5] < \tilde{t} \leq r\max\{\widetilde{\mu}(z), [0.5, 0.5]\}$$

for all $z \in x + y$. Hence, $\widetilde{\mu}(z) \geq \tilde{t}$, for all $z \in x + y$, which means $z \in \mu(\widetilde{\mu}; \tilde{t})$, for all $z \in x + y$. Also, suppose that $\tilde{t} \in (0.5, 1]$, $x \in \mu(\widetilde{\mu}; \tilde{t})$ and $r \in R$. Then, by (2) we have

$$[0.5, 0.5] < \tilde{t} \leq \widetilde{\mu}(x) \leq r\max\{\widetilde{\mu}(rx), [0.5, 0.5]\}.$$

It implies

$$[0.5, 0.5] < \tilde{t} \leq r\max\{\widetilde{\mu}(rx), [0.5, 0.5]\}.$$

Hence, $\widetilde{\mu}(rx) \geq \tilde{t}$, which means $rx \in \mu(\widetilde{\mu}; \tilde{t})$, similarly, $xr \in \mu(\widetilde{\mu}; \tilde{t})$. Therefore, $\mu(\widetilde{\mu}; \tilde{t})$ is a hyperideal of $R$. Let $\widetilde{\mu}$ be an interval valued fuzzy set in $R$ and $J$ be the set of

$\tilde{t} \in D(0,1]$ such that $\mu(\widetilde{\mu}; \tilde{t}) = \emptyset$ or $\mu(\widetilde{\mu}; \tilde{t})$ is a hyperideal of $R$. If $J = D(0,1]$, then by Theorem 4.6, $\widetilde{\mu}$ is an interval valued $(\in, \in \vee q)$-fuzzy hyperideal of $R$. Naturally, a corresponding result should be considered when $J = (0.5, 1]$. □

**Definition 4.1.** *An interval valued fuzzy set $\widetilde{\mu}$ in $R$ is called interval valued $(\overline{\in}, \overline{\in \wedge q})$-fuzzy hyperideal of $R$ if for all $\tilde{t}_1, \tilde{t}_2 \in D(0,1]$ and $r, x, y \in R$, the following condition hold:*

*(i) $\mu(z; r\min\{\tilde{t}_1, \tilde{t}_2\})\overline{\in}\widetilde{\mu}$, for all $z \in x+y$, implies $\mu(x; \tilde{t}_1)\overline{\in \wedge q}\widetilde{\mu}$ or $\mu(y; \tilde{t}_2)\overline{\in \wedge q}\widetilde{\mu}$.*

*(ii) $\mu(rx; \tilde{t})\overline{\in}\widetilde{\mu}$ or $\mu(xr; \tilde{t})\overline{\in}\widetilde{\mu}$ implies $\mu(x; \tilde{t})\ \overline{\in \wedge q}\widetilde{\mu}$.*

In the next theorem, we prove an equivalent condition for interval valued $(\overline{\in}, \overline{\in \wedge q})$- fuzzy hyperideals.

**Theorem 4.6.** *Let $\widetilde{\mu}$ be an interval valued fuzzy set in $R$. Then, $\widetilde{\mu}$ is an interval valued $(\overline{\in}, \overline{\in \wedge q})$- fuzzy hyperideal of $R$ if and only if for all $r, x, y \in R$, the following conditions hold:*

*(i) $r\max\{\widetilde{\mu}(z), [0.5, 0.5]\} \geq r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y)\}$, for all $z \in x+y$.*

*(ii) $r\max\{\widetilde{\mu}(rx), [0.5, 0.5]\} \geq \widetilde{\mu}(x)$ and $r\max\{\widetilde{\mu}(xr), [0.5, 0.5]\} \geq \widetilde{\mu}(x)$.*

*Proof.* Let $\widetilde{\mu}$ be an interval valued $(\overline{\in}, \overline{\in \wedge q})$- fuzzy hyperideal of $R$. If there exist $x, y \in R$ such that

$$r\max\{\widetilde{\mu}(z), [0.5, 0.5]\} < r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y)\} = \tilde{t}$$

for all $z \in x+y$, then $\tilde{t} \in (0.5, 1]$, $\mu(z; \tilde{t})\overline{\in}\widetilde{\mu}$ for all $z \in x+y$ and $\mu(x; \tilde{t})$, $\mu(y; \tilde{t}) \in \widetilde{\mu}$. By Definition 4.8, it follows that $\mu(x; \tilde{t})\overline{q}\widetilde{\mu}$ or $\mu(y; \tilde{t})\overline{q}\widetilde{\mu}$. Then, $(\widetilde{\mu}(x) \geq \tilde{t}$ and $\widetilde{\mu}(x) + \tilde{t} \leq [1,1])$ or $(\widetilde{\mu}(y) \geq \tilde{t}$ and $\widetilde{\mu}(y) + \tilde{t} \leq [1,1])$. It follows that $\tilde{t} \leq [0.5, 0.5]$ which is a contradiction. Hence, (i) holds. Also, if there exist $r, x \in R$ such that

$$r\max(\widetilde{\mu}(rx), [0.5, 0.5]) < \widetilde{\mu}(x) = \tilde{t},$$

or

$$r\max(\widetilde{\mu}(xr), [0.5, 0.5]) < \widetilde{\mu}(x) = \tilde{t},$$

then $\tilde{t} \in (0.5, 1]$, $\mu(rx; \tilde{t})\overline{\in}\widetilde{\mu}$ and $\mu(x; \tilde{t}) \in \widetilde{\mu}$. By Definition 4.8, it follows that $\mu(x; \tilde{t})\overline{q}\widetilde{\mu}$. Then, $\widetilde{\mu}(x) \geq \tilde{t}$ and $\widetilde{\mu}(x) + \tilde{t} \leq 1$. It concludes that $\tilde{t} \leq [0.5, 0.5]$ which is a contradiction. Thus, (2) holds

Conversely, let conditions (i) and (ii) hold. Also, let $x, y \in R$ such that for all $z \in x+y$

$$\mu(z; r\min\{\tilde{t}_1,\ \tilde{t}_2\})\overline{\in}\widetilde{\mu}$$

$\widetilde{\mu}(z) < r\min\{\tilde{t}_1,\ \tilde{t}_2\}$. Then, we can consider the following cases:

(a) If for all $z \in x+y$

$$\widetilde{\mu}(z) \geq r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y)\},$$

then

$$r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y)\} < r\min\{\tilde{t}_1, \tilde{t}_2\}$$

and so

$$\begin{aligned} \widetilde{\mu}(x) \quad < \quad & \tilde{t}_1 \text{ or } \widetilde{\mu}(y) < \tilde{t}_2 \\ & \mu(x; \tilde{t}_1)\overline{\in}\widetilde{\mu} \text{ or } \mu(y; \tilde{t}_2)\overline{\in}\widetilde{\mu} \\ & \mu(x; \tilde{t}_1)\overline{\in \wedge q}\widetilde{\mu} \text{ or } \mu(x; \tilde{t}_2)\overline{\in \wedge q}\widetilde{\mu}. \end{aligned}$$

(b) If

$$\widetilde{\mu}(z) < r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y)\} \text{ for all } z \in x+y,$$

then by (i), we have

$$[0.5, 0.5] \geq r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y)\}.$$

Hence for all $z \in x+y$,

$$\widetilde{\mu}(z) \vee [0.5, 0.5] \geq \widetilde{\mu}(x) \wedge \widetilde{\mu}(y)$$

Now, if $\mu(x; \tilde{t}_1)$, $\mu(y; \tilde{t}_2) \in \widetilde{\mu}$, then

$$\tilde{t}_1 \leq \widetilde{\mu}(x) \leq [0.5, 0.5]$$

or

$$\tilde{t}_2 \leq \widetilde{\mu}(y) \leq [0.5, 0.5].$$

It follows that $\mu(x; \tilde{t})\overline{q}\widetilde{\mu}$ or $\mu(x; \tilde{t})\overline{q}\widetilde{\mu}$.

Which implies that $\mu(x; \tilde{t}_1)\overline{\in \wedge q\widetilde{\mu}}$ or $\mu(y; \tilde{t}_2)\overline{\in \wedge q\widetilde{\mu}}$. Now, let $r, x, y \in R$ such that $\mu(rx; \tilde{t})\overline{\in}\widetilde{\mu}$ or $\mu(xr; \tilde{t})\overline{\in}\widetilde{\mu}$, then $\widetilde{\mu}(rx)\tilde{t}$ or $\widetilde{\mu}(xr) < \tilde{t}$..We can consider two following cases:

(a) If $\widetilde{\mu}(rx) \geq \widetilde{\mu}(x)$, then $\widetilde{\mu}(x) < \tilde{t}$. It follows that $\mu(x; \tilde{t})\overline{\in}\widetilde{\mu}$, which implies that $\mu(x; \tilde{t}_1)\overline{\in \wedge q\widetilde{\mu}}$.

(b) If $\widetilde{\mu}(rx) \geq \widetilde{\mu}(x)$, then by (ii) we have $[0.5, 0.5] \geq \widetilde{\mu}(x)$. Hence,

$$r\max(\widetilde{\mu}(rx), [0.5, 0.5]) \geq \widetilde{\mu}(x).$$

Now if $\mu(x; \tilde{t}) \in \widetilde{\mu}$, then

$$\tilde{t} \leq \widetilde{\mu}(x) \leq [0.5, 0.5].$$

It follows that $\mu(x; \tilde{t})\overline{q}\widetilde{\mu}$, which implies that $\mu(x; \tilde{t})\overline{\in \wedge q\widetilde{\mu}}$. Therefore, $\widetilde{\mu}$ is an interval valued $(\overline{\in}, \overline{\in \wedge q})$-fuzzy hyperideal of $R$. □

In the following theorem, we characterize interval valued $(\overline{\in}, \overline{\in \wedge q})$-fuzzy hyperideals based on level subsets.

**Theorem 4.7.** *An interval valued fuzzy set $\widetilde{\mu}$ in $R$ is an interval valued $(\overline{\in}, \overline{\in \wedge q})$-fuzzy hyperideal of $R$ if and only if $\mu(x; \tilde{t})(\neq \Phi)$ is a hyperideal of $R$ for all $\tilde{t} \in (0.5, 1]$.*

*Proof.* It follws by Theorem 4.7 and 4.9. For any interval valued fuzzy set $\widetilde{\mu}$ in $R$ and $\tilde{t} \in D(0, 1]$, we put

$$\mu(x; \tilde{t}) = \{x \in R / \mu(x; \tilde{t})q\widetilde{\mu}\},$$

and

$$\overline{|\ \widetilde{\mu}\ |}_{\tilde{t}} = \{x \in R | \mu(x; \tilde{t}) \in \vee q\widetilde{\mu}\}.$$

Clearly $\overline{|\ \widetilde{\mu}\ |}_{\tilde{t}} = \widetilde{\mu}\ _{\tilde{t}} \cup \widetilde{\mu}_{\tilde{t}}$. In fact, $\widetilde{\mu}_{\tilde{t}}$ and $\overline{|\ \widetilde{\mu}\ |}_{\tilde{t}}$ are generalized level subsets. Now, we can characterize interval valued $(\in, \in \vee q)$-fuzzy hyperideals based on generalized level subsets. □

**Theorem 4.8.** *An interval valued fuzzy set $\widetilde{\mu}$ in $R$ is an interval valued $(\in, \in \vee q)$-fuzzy hyperideal of $R$ if and only if $\overline{|\ \widetilde{\mu}\ |}_{\tilde{t}}$is a hyperideal of $R$ for all $\tilde{t} \in (0.5, 1]$.*

*Proof.* Let $\widetilde{\mu}$ be an interval valued $(\in,\in\vee q)$-fuzzy hyperideal of $R$ and $x,y\in\overline{\mid\widetilde{\mu}\mid}_{\tilde{t}}$ for $\tilde{t}\in(0.5,1]$. Then, $\mu(x;\tilde{t})\in\vee q\widetilde{\mu}$ and $\mu(y;\tilde{t})\in\vee q\widetilde{\mu}$, which means $\widetilde{\mu}(x)\geq\tilde{t}$ or $\widetilde{\mu}(x)+\tilde{t}>1$, and $\widetilde{\mu}(y)\geq\tilde{t}$ or $\widetilde{\mu}(y)+\tilde{t}>[1,1]$. On the other hand, by Theorem 4.5, we know, for all $z\in x+y$,

$$\widetilde{\mu}(z)\geq r\min\{\widetilde{\mu}(x),\widetilde{\mu}(y),[0.5.0.5]\}$$

so for all $z\in x+y$,

$$\widetilde{\mu}(z)\geq r\min\{\tilde{t},[0.5,0.5]).$$

Since, if for all $z\in x+y$,

$$\widetilde{\mu}(z)<r\min\{\tilde{t},[0.5,0.5]\}$$

then for all $z\in x+y$,

$$r\min\{\widetilde{\mu}(x),\widetilde{\mu}(y),[0.5,0.5])\leq\widetilde{\mu}(z)<r\min\{\tilde{t},[0.5,0.5]\},$$

which implies,

$$r\min\{\widetilde{\mu}(x),\widetilde{\mu}(y),[0.5,0.5]\}<r\min\{\tilde{t},[0.5,0.5]).$$

Hence, $\widetilde{\mu}(x)<\tilde{t}$ or $\widetilde{\mu}(y)<\tilde{t}$, that is $\mu(x;\tilde{t})\overline{\in}\widetilde{\mu}$ or $\mu(y;\tilde{t})\overline{\in}\widetilde{\mu}$. Thus, $\mu(x;\tilde{t})\overline{\in\vee q\widetilde{\mu}}$ or $\mu(y;\tilde{t})\overline{\in\vee q\widetilde{\mu}}$ which is a contradiction. We know $\tilde{t}\in[0.5,0.5]$ then

$$\widetilde{\mu}(z)\geq r\min\{\tilde{t},[0.5,0.5]\}=[1,1]$$

and so $z\in\widetilde{\mu}_{\tilde{t}}\subseteq\overline{\mid\widetilde{\mu}\mid}_{\tilde{t}}$ for all $z\in x+y$. Also, let $r\in R$ and $x\in\overline{\mid\widetilde{\mu}\mid}_{\tilde{t}}$ for $\tilde{t}\in(0,0.5]$. Then, $\mu(x;\tilde{t})\in\vee q\widetilde{\mu}$ which means $\widetilde{\mu}(x)\geq\tilde{t}$ or $\widetilde{\mu}(x)+\tilde{t}>1$. On the other hand, by Theorem 4.5, we know that

$$\widetilde{\mu}(rx)\geq r\min\{\widetilde{\mu}(x),[0.5,0.5]\}$$

so

$$\widetilde{\mu}(rx)\geq r\min\{\tilde{t},[0.5,0.5]\}$$

Since if,

$$\widetilde{\mu}(rx)<r\min\{\tilde{t},[0.5,0.5]\}$$

then,

$$r\min\{\widetilde{\mu}(x),[0.5,0.5]\}\leq\widetilde{\mu}(rx)<r\min\{\tilde{t},[0.5,0.5]).$$

Hence, $\widetilde{\mu}(x)<\tilde{t}$, that is $\mu(x;\tilde{t})\overline{\in}\widetilde{\mu}$, thus $\mu(x;\tilde{t})\overline{\in\wedge q\widetilde{\mu}}$, which is a contradiction. We know $\tilde{t}\leq[0.5,0.5]$ then

$$\widetilde{\mu}(rx)\geq r\min\{\tilde{t},[0.5,0.5]\}=\tilde{t}$$

and so $rx\in\widetilde{\mu}_{\tilde{t}}\subseteq\overline{\mid\widetilde{\mu}\mid}_{\tilde{t}}$. Similarly, $xr\in\overline{\mid\widetilde{\mu}\mid}_{\tilde{t}}$, therefore, $\overline{\mid\widetilde{\mu}\mid}_{\tilde{t}}$ is a hyperideal of $R$.

Conversely, let $\overline{\mid\widetilde{\mu}\mid}_{\tilde{t}}$ be a hyperideal of $R$ for $\tilde{t}\in(0,0.5]$. Suppose $x,y\in R$ such that, for all $z\in x+y$,

$$\widetilde{\mu}(z)<r\min\{\widetilde{\mu}(x),\widetilde{\mu}(y),[0.5,0.5]\}$$

Then, there exists $\tilde{t}\in(0,0.5)$ such that for all $z\in x+y$

$$\widetilde{\mu}(z)<\tilde{t}<\widetilde{\mu}(x)\wedge\widetilde{\mu}(y)\wedge[0.5,0.5]\}$$

It follows $x,y\in\widetilde{\mu}_{\tilde{t}}\subseteq\overline{\mid\widetilde{\mu}\mid}_{\tilde{t}}$, which implies $z\in\overline{\mid\widetilde{\mu}\mid}_{\tilde{t}}$, for all $z\in x+y$. Hence, $\widetilde{\mu}(z)\geq\tilde{t}$ or $\widetilde{\mu}(z)+\tilde{t}>[1,1]$, for all $z\in x+y$, which is a contradiction. Therefore, for all $z\in x+y$,

$$\widetilde{\mu}(z)\geq r\min\{\widetilde{\mu}(x),\widetilde{\mu}(y),[0.5,0.5]\}$$

Also, suppose $r,\ x \in R$ such that

$$\widetilde{\mu}(rx) < r\min\{\widetilde{\mu}(x), [0.5, 0.5]\}$$

then there exists $\tilde{t} \in (0, 0.5)$ such that

$$\widetilde{\mu}(rx) < \tilde{t} < r\min\{\widetilde{\mu}(x), [0.5, 0.5]\}.$$

It follows $x \in \widetilde{\mu}\ _{\tilde{t}} \subseteq \overline{|\ \widetilde{\mu}\ |}_{\tilde{t}}$, which implies $rx \in \overline{|\ \widetilde{\mu}\ |}_{\tilde{t}}$.

Hence, $\widetilde{\mu}(rx) \geq \tilde{t}$ or $\widetilde{\mu}(rx) + \tilde{t} > [1, 1]$ which is a contradiction. Thus,

$$\widetilde{\mu}(rx) \geq r\min\{\widetilde{\mu}(x), [0.5, 0.5]\}$$

Similarly,

$$\widetilde{\mu}(xr) \geq r\min\{\widetilde{\mu}(x), [0.5, 0.5]\}$$

therefore, the proof is completed. □

In the next theorem, we discuss an interval valued $(\in, \in \vee q)$-fuzzy hyperideal of $R$ which can be expressed as the union of two proper non-equivalent interval valued $(\in, \in \vee q)$- fuzzy hyperideals.

**Theorem 4.9.** *Let $\widetilde{\mu}$ be a proper interval valued $(\in, \in \vee q)$-fuzzy hyperideal of $R$ such that*

$$2 \leq |\ \{\widetilde{\mu}(x) \mid \widetilde{\mu}(x) < [0.5, 0.5]\}\ | < \infty.$$

*Then, there exist two proper non-equivalent interval valued $(\in, \in \vee q)$- fuzzy hyperideal of $R$ such that $\widetilde{\mu}$ can be expressed as the union of them.*

*Proof.* Let

$$\{\widetilde{\mu}(x) \mid \widetilde{\mu}(x) < [0.5, 0.5]\} = \{\tilde{t}_0,\ \tilde{t}_{1,...,}\ \tilde{t}_n\},$$

where $\tilde{t}_1 > \tilde{t}_2 > ... > \tilde{t}_r$ and $r \geq 2$. Then, the chain of interval valued $(\in \vee q)$-level hyperideals of $R$ is

$$\overline{|\ \widetilde{\mu}\ |}_{[0.5,0.5]} \subseteq \overline{|\ \widetilde{\mu}\ |}_{\tilde{t}_1} \subseteq \overline{|\ \widetilde{\mu}\ |}_{\tilde{t}_2} ... \subseteq \overline{|\ \widetilde{\mu}\ |}_{\tilde{t}_r} = R.$$

Let $\bar{\nu}$ and $\bar{\theta}$ be fuzzy sets in $R$ defined by

$$\bar{\nu}(x) = \begin{cases} \tilde{t}_1 & \text{if } x \in \overline{|\ \widetilde{\mu}\ |}_{\tilde{t}_1}, \\ \tilde{t}_2 & \text{if } x \in \overline{|\ \widetilde{\mu}\ |}_{\tilde{t}_2} \backslash \overline{|\ \widetilde{\mu}\ |}_{\tilde{t}_1}, \\ . \\ . \\ . \\ \tilde{t}_r & \text{if } x \in \overline{|\ \widetilde{\mu}\ |}_{\tilde{t}_r} \\ & backslash \overline{|\ \widetilde{\mu}\ |}_{\tilde{t}_r - 1}, \end{cases}$$

and

$$\bar{\theta}\,(x) = \begin{cases} \widetilde{\mu}(x) & \text{if } x \in \overline{|\ \widetilde{\mu}\ |}_{[0.5,0.5]}\ , \\ k & \text{if } x \in \overline{|\ \widetilde{\mu}\ |}_{\tilde{t}_2} \backslash \overline{|\ \widetilde{\mu}\ |}_{[0.5,0.5]}, \\ \tilde{t}_3 & \text{if } x \in \overline{|\ \widetilde{\mu}\ |}_{\tilde{t}_3} \backslash \overline{|\ \widetilde{\mu}\ |}_{\tilde{t}_2}, \\ \tilde{t}_4 & \text{if } x \in \overline{|\ \widetilde{\mu}\ |}_{\tilde{t}_4} \backslash \overline{|\ \widetilde{\mu}\ |}_{\tilde{t}_3}, \\ . \\ . \\ . \\ \tilde{t}_r & \text{if } x \in \overline{|\ \widetilde{\mu}\ |}_{\tilde{t}_r} \backslash \overline{|\ \widetilde{\mu}\ |}_{\tilde{t}_r - 1}, \end{cases}$$

where $\tilde{t}_3 < k < \tilde{t}_2$. The, $\bar{\nu}$ and $\bar{\theta}$ are interval valued $(\in, \in \vee q)$- fuzzy hyperideals of $R$, and $\bar{\nu}, \bar{\theta} \leq \widetilde{\mu}$. The chains of interval valued $(\in \vee q)$-level hyperideals of $\bar{\nu}$ and $\bar{\theta}$ are, respectively, given by

$$\overline{|\,\widetilde{\mu}\,|}_{[0.5,0.5]} \subseteq \overline{|\,\widetilde{\mu}\,|}_{\tilde{t}_1} \subseteq \overline{|\,\widetilde{\mu}\,|}_{\tilde{t}_2} \cdots \subseteq \overline{|\,\widetilde{\mu}\,|}_{\tilde{t}_r}$$

and

$$\overline{|\,\widetilde{\mu}\,|}_{[0.5,0.5]} \subseteq \overline{|\,\widetilde{\mu}\,|}_{\tilde{t}_1} \subseteq \overline{|\,\widetilde{\mu}\,|}_{\tilde{t}_2} \cdots \subseteq \overline{|\,\widetilde{\mu}\,|}_{\tilde{t}_r}.$$

Thus, $\bar{\nu}$ and $\bar{\theta}$ are non-equivalent and clearly $\widetilde{\mu} = \bar{\nu} \vee \bar{\theta}$. Therefore, $\widetilde{\mu}$ be expressed as the union of two proper non-equivalent interval valued $(\in, \in \vee q)$- fuzzy hyperideal of $R$. □

## 5. t-implication-based interval valued fuzzy hyperideals of semirings

In this section, we generalize the notion of ordinary fuzzy hyperideals, interval valued $(\in, \in \vee q)$- fuzzy hyperideals and interval valued $(\overline{\in}, \overline{\in \vee q})$- fuzzy hyperideals. Specially, we characterize fuzzy hyperideals, interval valued $(\in, \in \vee q)$- fuzzy hyperideals and interval valued $(\overline{\in}, \overline{\in \vee q})$-fuzzy hyperideals based on implication operators.

**Definition 5.1.** *Let $\tilde{m}, \tilde{n} \in [0, 1]$, $\tilde{m} < \tilde{n}$ and $\widetilde{\mu}$ be an interval valued fuzzy set in $R$. Then, $\widetilde{\mu}$ is said to be a fuzzy hyperideal with thresholds $(\tilde{m}, \tilde{n})$ of $R$, if for all $r, x, y \in R$, the following conditions hold:*

(i) $r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y), \tilde{n}\} \leq r\max\{\widetilde{\mu}(z), \tilde{m}\}$, for all $z \in x + y$,
(ii) $r\min\{\widetilde{\mu}(x), \tilde{n}\} \leq r\max\{\widetilde{\mu}(rx, \tilde{m}\}$ and

$$r\min\{\widetilde{\mu}(x), \tilde{n}\} \leq r\max\{\widetilde{\mu}(xr), \tilde{m}\}.$$

Clearly, every interval valued fuzzy hyperideal with threshods $(\tilde{m}, \tilde{n})$ of $R$ is an ordinary interval valued fuzzy hyperideal when $\tilde{m} = [0, 0]$ and $\tilde{n} = [1, 1]$ (see definition 1). Also, it is an interval valued $(\in, \in \vee q)$-fuzzy (resp. interval valued $(\overline{\in}, \overline{\in \vee q})$-fuzzy) hyperideals when $\tilde{m} = [0, 0]$ and $\tilde{n} = [0.5, 0.5]$ (resp. $\tilde{m} = [0, 0]$ and $\tilde{n} = [0.5, 0.5]$) (see Theorem 4.7 and 4.9).

**Theorem 5.1.** *An interval valued fuzzy set $\widetilde{\mu}$ in $R$ is a interval valued fuzzy hyperideal with threshold $(\tilde{m}, \tilde{n})$ of $R$ if and only if $\widetilde{\mu}_{\tilde{t}}(\neq \Phi)$ is a hyperideal of $R$ for all $\tilde{t} \in (\tilde{m}, \tilde{n}]$.*

*Proof.* Suppose that $\widetilde{\mu}$ is an interval valued fuzzy hyperideal with thresholds $(\tilde{m}, \tilde{n})$ of $R$ and $\tilde{t} \in (\tilde{m}, \tilde{n}]$. If $x, y \in \widetilde{\mu}_{\tilde{t}}$, then $\widetilde{\mu}(x) \geq \tilde{t}$ and $\widetilde{\mu}(y) \geq \tilde{t}$. We have, for all $z \in x + y$,

$$r\max\{\widetilde{\mu}(z), \tilde{m}\} \geq r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y), \tilde{n}\} \geq r\min\{\tilde{t}, \tilde{n}\} = \tilde{t} > \tilde{m},$$

Hence, $r\max\{\widetilde{\mu}(z), \tilde{m}\} \geq \tilde{t} > \tilde{m}$, for all $z \in x + y$,

which implies $\widetilde{\mu}(z) \geq \tilde{t}$, for all $z \in x + y$, that is $z \in \widetilde{\mu}_{\tilde{t}}$ for all $z \in x + y$. Now, if $x \in \widetilde{\mu}_{\ \tilde{t}}$ and $r \in R$, then $\widetilde{\mu}(x) \geq \tilde{t}$. We have

$$r\max\{\widetilde{\mu}(rx), \tilde{m}\} \geq r\min\{\widetilde{\mu}(x), \tilde{n}\} \geq \ r\min\{\tilde{t}, \tilde{n}\} = \tilde{t} > \tilde{m}.$$

Hence,

$$r\max\{\widetilde{\mu}(rx), \tilde{m}\} \geq \tilde{t} > \tilde{m},$$

which implies $\widetilde{\mu}(rx) \geq \tilde{t}$, that is $rx \in \widetilde{\mu}_{\tilde{t}}$. Similarly, $xr \in \widetilde{\mu}_{\tilde{t}}$. Therefore, $\widetilde{\mu}_{\tilde{t}}$ is a hyperideal of $R$.

**Conversely,** let $\widetilde{\mu}$ be an interval valued fuzzy set in $R$. If there exist $x, y \in R$ such that

$$r\max\{\widetilde{\mu}(z), \tilde{m}\} < r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y), \tilde{n}\} = \tilde{t},$$

for all $z \in x + y$, then $\tilde{t} \in (\tilde{m}, \tilde{n}]$, $\widetilde{\mu}(z) < \tilde{t}$, $x \in \widetilde{\mu}$ and $y \in \widetilde{\mu}_{\tilde{t}}$, for all $z \in x + y$. Since $\widetilde{\mu}_{\tilde{t}}$ is a hyperideal of $R$, we have $z \in \widetilde{\mu}\ _{\tilde{t}}$ for all $z \in x + y$. Thus, $z \subseteq \widetilde{\mu}_{\tilde{t}}$, for all $z \in x + y$. Hence, $\widetilde{\mu}(z) \geq \tilde{t}$ for all $z \in x + y$, which is a contradiction. Therefore, for all $x, y \in R$, we have for all $z \in x + y$

$$r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y)\} \leq r\max\{\widetilde{\mu}(z), \tilde{m}\},$$

Also, if there exist $r, x \in R$ such that

$$r\max\{\widetilde{\mu}(rx), \tilde{m}\} \geq r\min\{\widetilde{\mu}(x), \tilde{n}\} \geq r\min\{\tilde{t}, \tilde{n}\} = \tilde{t},$$

then $\tilde{t} \in (\tilde{m}, \tilde{n}]$, $\widetilde{\mu}(rx) \geq \tilde{t}$, which is a contradiction. Thus, for all $r, x \in R$, we have

$$r\min\{\widetilde{\mu}(x), \tilde{n}\} \leq r\max\{\widetilde{\mu}(rx), \tilde{m}\}.$$

Similarly,

$$r\min\{\widetilde{\mu}(x), \tilde{n}\} \leq r\max\{\widetilde{\mu}(xr), \tilde{m}\},$$

therefore, $\widetilde{\mu}$ is an interval valued fuzzy hyperideal with thresholds $(\tilde{m}, \tilde{n})$ of $R$. □

Set theoretic multivalued logic is a special case of fuzzy logic such that the truth values are linguistic variables (or terms of the linguistic variables truth). By using extension principal some operators like $\wedge, \vee, \neg, \longrightarrow$ can be applied in fuzzy logic. In fuzzy logic, $[P]$ means the truth value of fuzzy proposition $P$. In the following, we show a correspondence between fuzzy logic and set-theoretical notions.

$[x \in \widetilde{\mu}] = \widetilde{\mu}(x)$, $\quad [x \neq \widetilde{\mu}] = [1, 1] - \widetilde{\mu}(x)$,

$[P \wedge Q] = \min\{[P], [Q]\}$, $\quad [P \vee Q] = \max\{[P], [Q]\}$,

$[P \longrightarrow Q] = \min\{[1, 1], [1, 1] - [P] + [Q]\}$,

$[\forall x \in P(x)] = \inf[P(x)]$,

$\models P$ if and only if $[P] = [1, 1]$for all valuations.

We show some of important implication operators, where $\alpha$ denotes the degree of membership of the premise and $\beta$ is the degree of membership of the consequence, and $I$ the resulting degree of truth for the implication.

Early Zadeh $\quad I_m(\bar{\alpha}, \bar{\beta}) = \max\{[1, 1] - \bar{\alpha}, \min\{\bar{\alpha}, \bar{\beta}\}\}$,

Lukasiewicz $\quad I_\alpha(\bar{\alpha}, \bar{\beta}) = \min\{[1, 1] - \bar{\alpha} + \bar{\beta}\}$,

Standard Star (Godel)

$$I_g(\bar{\alpha}, \bar{\beta}) = \begin{cases} [1, 1] & \bar{\alpha} \leq \bar{\beta} \\ \bar{\beta} & \text{otherwise} \end{cases},$$

Contraposition of (Godel)

$$I_{cg}(\bar{\alpha}, \bar{\beta}) = \begin{cases} [1, 1] & \bar{\alpha} \leq \bar{\beta} \\ [1, 1] - \bar{\alpha} & \text{otherwise} \end{cases}$$

Gaines-Rescher

$$I_{gr}(\bar{\alpha}, \bar{\beta}) = \begin{cases} [1, 1] & \bar{\alpha} \leq \bar{\beta} \\ [0, 0] & \text{otherwise} \end{cases}$$

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$$I_b(\bar{\alpha}, \bar{\beta}) = \max\{[1, 1] - \bar{\alpha}, \bar{\beta}\}.$$

**Definition 5.2.** *An interval valued fuzzy set $\widetilde{\mu}$ in $R$ is called fuzzifying hyperideal of $R$, if and only if for all $r, x, y \in R$ it satiesfies:*

(1) $\models [[x \in \widetilde{\mu}] \wedge [y \in \widetilde{\mu}] \longrightarrow [z \in \widetilde{\mu}]]$, for all $z \in x + y$,
(2) $\models [[x \in \widetilde{\mu}] \wedge [rx \in \widetilde{\mu}]$ and $\models [[x \in \widetilde{\mu}] \longrightarrow [xr \in \widetilde{\mu}]]$.

Clearly, Definition 26 is eqivalent to Definition 1. Therefore, a fuzzifying hyperideal is an ordinary fuzzy hyperideal. We have the notion of $\tilde{t}$-tautology. In fact $\models_{\tilde{t}} P$, if and only if $[P] \geq \tilde{t}$ (see [20]).

**Definition 5.3.** *An interval valued fuzzy set $\widetilde{\mu}$ in $R$ is said to be $\tilde{t}$-implication-based fuzzy left (resp. right) interval valued hyperideal of $R$ with respect to the implication $\longrightarrow$ if the following conditions hold for all $r, x, y \in R$ :*

(1) $\models_{\tilde{t}} [[x \in \widetilde{\mu}] \wedge [y \in \widetilde{\mu}] \longrightarrow [z \in \widetilde{\mu}]]$ for all $z \in x + y$,
(2) $\models_{\tilde{t}} [[x \in \widetilde{\mu}] \wedge [rx \in \widetilde{\mu}]$ (resp. $\models_{\tilde{t}} [[x \in \widetilde{\mu}] \longrightarrow [xr \in \widetilde{\mu}]]$.).

An interval valued fuzzy set $\widetilde{\mu}$ in $R$ is said to be $\tilde{t}$-implication-based interval valued fuzzy hyperideal of $R$ with respect to the implication $\longrightarrow$ if $\widetilde{\mu}$ is both $\tilde{t}$-implication-based interval valued fuzzy left and right hyperideal of $R$ with respect to the implication $\longrightarrow$.

**Proposition 5.1.** *An interval valued fuzzy set $\widetilde{\mu}$ of $R$ is a $\tilde{t}$-implication-based interval valued fuzzy hyperideal of $R$ with respect to the implication operator $I$ if and only if for all $r, x, y \in R$.*

(i) $I\,(r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y)\}, \widetilde{\mu}(z)) \geq \tilde{t}$ for all $z \in x + y$,
(ii) $I\,(\widetilde{\mu}(x), \widetilde{\mu}(rx)) \geq \tilde{t}$ and $I(\widetilde{\mu}(x), \widetilde{\mu}(xr)) \geq \tilde{t}$.

*Proof.* The proof is clear by considering the definitions. □

**Theorem 5.2.** *(1) Let $I = I_{gr}$ (Gaines-Rescher). Then, $\widetilde{\mu}$ is an $[0.5, 0.5]$-implication-based interval valued fuzzy hyperideal of $R$ if and only if $\widetilde{\mu}$ is an interval valued fuzzy hyperideal with thresholds $m = [0, 0]$ and $n = [1, 1]$ of $R$ (or equivalent, $\widetilde{\mu}$ is an ordinary interval valued fuzzy hyperideal of $R$).*

*(2) Let $I = I_{gr}$ (Godel). Then, $\widetilde{\mu}$ is an $[0.5, 0.5]$-implication-based interval valued fuzzy hyperideal of $R$ if and only if $\widetilde{\mu}$ is an interval valued fuzzy hyperideal with thresholds $m = [0, 0]$ and $n = [0.5, 0.5]$ of $R$ (or equivalent, $\widetilde{\mu}$ is an interval valued $(\in, \in \vee q)-$fuzzy hyperideal of $R$).*

*(3) Let $I = I_{cg}$ (Contraposition of Godel). Then, $\widetilde{\mu}$ is an $[0.5, 0.5]$-implication-based interval valued fuzzy hyperideal of $R$ if and only if $\widetilde{\mu}$ is an interval valued fuzzy hyperideal with thresholds $m = [0.5, 0.5]$ and $n = [1, 1]$ of $R$ (or equivalent, $\widetilde{\mu}$ is an interval valued $(\overline{\in}, \overline{\in \vee q})$- fuzzy hyperideal of $R$).*

*Proof.* (1) Let $\widetilde{\mu}$ be an $[0.5, 0.5]$-implication-based interval valued fuzzy hyperideal of $R$. Then for all $z \in x + y$

$$I_{gr}(r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y)\}, \widetilde{\mu}(z)) \geq [0.5, 0, 5],$$

Which implies for all $z \in x + y$

$$\widetilde{\mu}(z) \geq r\min\{\widetilde{\mu}(x)), \widetilde{\mu}(y)\},$$

Also,

$$I_{gr}(\widetilde{\mu}(x), \widetilde{\mu}(rx)) \geq [0.5, 0.5]$$

which implication $\widetilde{\mu}(rx) \geq \widetilde{\mu}(x)$. Similarly, $\widetilde{\mu}(xr) \geq \widetilde{\mu}(x)$. Therefore, $\widetilde{\mu}$ is an interval valued fuzzy hyperideal with threshold $m = [0, 0]$ and $n = [1, 1]$ of $R$.

**Conversely,** let $\widetilde{\mu}$ be an interval valued fuzzy hyperideal with threshold $m = [0, 0]$ and $n = [1, 1]$ of $R$. Then,

$$\begin{aligned} \widetilde{\mu}(z) &\geq r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y)\}, \text{ for all } z \in x + y, \\ \widetilde{\mu}(rx) &\geq \widetilde{\mu}(x), \widetilde{\mu}(xr) \geq \widetilde{\mu}(x), \text{ for all } r, x, y \in R. \end{aligned}$$

Hence, for all $z \in x + y$

$$I_{gr}(r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y)\}, \widetilde{\mu}(z)) = [1, 1]$$

$$I_{gr}(\widetilde{\mu}(x), \widetilde{\mu}(xr)) = [1, 1] = I_{gr}(\widetilde{\mu}(x), \widetilde{\mu}(rx)).$$

Thus, for all $z \in x + y$,

$$I_{gr}(r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y)\}, \widetilde{\mu}(z)) \geq [0.5, 0.5],$$

$$I_{gr}(\widetilde{\mu}(x), \widetilde{\mu}(rx)) \geq [0.5, 0.5]$$

and

$$I_{gr}(\widetilde{\mu}(x), \widetilde{\mu}(xr)) \geq [0.5, 0.5].$$

Therefore, is a $[0.5, 0.5]$-implication-based interval valued fuzzy hyperideal of $R$.

(2) Let $\widetilde{\mu}$ be an $[0.5, 0.5]$-implication-based interval valued fuzzy $\widetilde{\mu}$ hyperideal of $R$. Then, for all $r, x, y \in R$, we have, for all $z \in x + y$,

$$I_g(r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y)\}, \widetilde{\mu}(z)) \geq [0.5, 0.5]$$

$$I_g(\widetilde{\mu}(x), \widetilde{\mu}(rx) \geq [0.5, 0.5]$$

and

$$I_g((\widetilde{\mu}(x), \widetilde{\mu}(xr)) \geq [0.5, 0.5].$$

By the definition of $I_g$, we can consider the following cases:

(a) $I_g(r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y)\}, \widetilde{\mu}(z)) = [1, 1]$ for all $z \in x+y$, then $r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y)\} \leq \widetilde{\mu}(z)$, for all $z \in x + y$, which implies, for all $z \in x + y$

$$r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y), [0.5, 0.5]\} \leq \widetilde{\mu}(z),$$

(b) $I_g(r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y)\}, \widetilde{\mu}(x + y)) = [1, 1]$ then $\widetilde{\mu}(z) = \widetilde{\mu}(z)$, for all $z \in x + y$ then $\widetilde{\mu}(z) \geq [0.5, 0.5]$ for all $z \in x + y$. Which implies, for all $z \in x + y$

$$r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y), [0.5, 0.5]\} \leq \widetilde{\mu}(z)$$

Similarly, we can show that

$$r\min\{\widetilde{\mu}(x), [0.5, 0.5]\} \leq \widetilde{\mu}(rx)$$

and

$$r\min\{\widetilde{\mu}(x), [0.5, 0.5]\} \leq \widetilde{\mu}(xr).$$

Therefore, $\widetilde{\mu}$ is an interval valued fuzzy hyperideal with thresholds $m = [0, 0]$ and $n = [0.5, 0.5]$ of $R$.

Conversely, let $\widetilde{\mu}$ is an interval valued fuzzy hyperideal with thresholds $m = [0, 0]$ and $n = [0.5, 0.5]$ of $R$. Then, for all $r, x, y \in R$, by Definition 5.2 for all $z \in x + y$,

$$r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y), [0.5, 0.5]\} \leq \widetilde{\mu}(z),$$

and

$$r\min\{\widetilde{\mu}(x), [0.5, 0.5]\} \leq \widetilde{\mu}(rx)$$

and

$$r\min\{\widetilde{\mu}(x), [0.5, 0.5]\} \leq \widetilde{\mu}(xr).$$

Hence, in each case, for all $z \in x + y$

$$I_g(r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y)\}, \widetilde{\mu}(z)) \geq [0.5, 0.5]$$

$$I_g(\widetilde{\mu}(x), \widetilde{\mu}(rx)) \geq [0.5, 0.5]$$

and

$$I_g(\widetilde{\mu}(x), \widetilde{\mu}(xr)) \geq [0.5, 0.5].$$

Therefore, $\widetilde{\mu}$ is an $[0.5, 0.5]$-implication-based interval valuedfuzzy hyperideal of $R$.

(3) Let $\widetilde{\mu}$ be an $[0.5, 0.5]$-implication-based interval valued fuzzy hyperideal of $R$. Then, for all $r, x, y \in R$, we have for all $z \in x + y$,

$$I_{cg}(r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y)\}, \widetilde{\mu}(z)) \geq [0.5, 0.5],$$

$$I_{cg}(\widetilde{\mu}(x), \widetilde{\mu}(rx)) \geq [0.5, 0.5]$$

and

$$I_{cg}(\widetilde{\mu}(x), \widetilde{\mu}(xr)) \geq [0.5, 0.5].$$

By definition of $I_{cg}$, we can consider the following cases:

(a) If $I_{cg}(r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y)\}, \widetilde{\mu}(z)) = [1, 1]$,for all $z \in x + y$, then for all $z \in x + y$,

$$r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y)\} \leq \widetilde{\mu}(z),$$

which implies that for all $z \in x + y$.

$$r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y)\} \leq r\max\{\widetilde{\mu}(z), [0.5, 0.5]\}$$

(b) If for all $z \in x + y$,

$$I_{cg}(r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y)\}, \widetilde{\mu}(z)) = [1, 1] - (\widetilde{\mu}(x) \wedge \widetilde{\mu}(y)),$$

then

$$[1, 1] - r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y)\} \geq [0.5, 0.5]$$

it implies that

$$r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y)\} \leq [0.5, 0.5]$$

and hence for all $z \in x + y$

$$r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y)\} \leq r\max\{\widetilde{\mu}(z), [0.5, 0.5]\}.$$

Similarly, we can show that

$$\widetilde{\mu}(x) \leq r\max\{\widetilde{\mu}(rx), [0.5, 0.5]\}$$

and

$$\widetilde{\mu}(x) \leq r\max\{\widetilde{\mu}(xr), [0.5, 0, 5]\}.$$

Therefore, $\widetilde{\mu}$ is an interval valued fuzzy hyperideal with threshold $m = [0.5, 0.5]$ and $n = [1, 1]$ of $R$.

Conversely, let $\widetilde{\mu}$ be an interval valued fuzzy hyperideal with threshold $m = [0.5, 0.5]$ and $n = [1, 1]$ of $R$. Then, for all $r, x, y \in R$,

we have, for all $z \in x + y$

$$r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y)\} \leq r\max\{\widetilde{\mu}(z), [0.5, 0.5]\}$$

$$\widetilde{\mu}(x) \leq r\max\{\widetilde{\mu}(rx), [0.5, 0.5]\}$$

and

$$\widetilde{\mu}(x) \leq r\max\{\widetilde{\mu}(xr), [0.5, 0.5]\}$$

Now, we can consider two following cases:

(a) For all $z \in x + y$,

$$r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y)\} \leq \widetilde{\mu}(z),$$

which implies, for all $z \in x + y$,

$$I_{cg}(r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y)\}, \widetilde{\mu}(z)) = [1, 1] \geq [0.5, 0.5]$$

(b) If $r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y)\} > \widetilde{\mu}(z)$, for all $z \in x + y$, which implies $r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y)\} \geq [0.5, 0.5]$. Hence, $[1, 1] - r\min\{\widetilde{\mu}(x), \widetilde{\mu}(y)\} \geq [0.5, 0.5]$.

Thus, for all $z \in x+y$, $I_{cg}(\widetilde{\mu}(x) \wedge \widetilde{\mu}(y), \widetilde{\mu}(z)) = [1, 1] - (\widetilde{\mu}(x) \wedge \widetilde{\mu}(y)) \geq [0.5, 0.5]$. Similarly, we can prove that $I_{cg}(\widetilde{\mu}(x), \widetilde{\mu}(rx)) \geq [0.5, 0.5]$ and $I_{cg}(\widetilde{\mu}(x), \widetilde{\mu}(xr)) \geq [0.5, 0.5]$. Therefore, $\widetilde{\mu}$ is an $[0.5, 0.5]$-implication-based interval valued fuzzy hyperideal of $R$. □

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