OPTIMIZATION OF COIL INDUCTANCE EQUATIONS USED IN WIRELESS POWER TRANSFER FOR DRONE CHARGING

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This paper presents the self and mutual inductances of spiral and circular coil determination based on several equations types from the literature which will be chosen for comparison. The calculation equations of the self and mutual inductances will be solved using software; the theoretical results of the self-inductances will be compared with each other but also with the results of accurate measurements made with an RLC meter. At the same time, the exact equations determining the self-inductance results very close to reality will be established. Also, the limits of all self-inductance equations related to the coil dimensions will be drawn.

Keywords: self-inductance, mutual inductance, loop coil, spiral coil, wireless.

1. Introduction

Circular and square coils are the basic components of wireless energy transfer, although, the magnetic flux distribution of the square coil does not have a cylindrical symmetry as circular coils do. This non-symmetry creates a disadvantage in terms of charging the drones, as landing and charging them on the platform must satisfy the condition of perpendicularity of the transmit and receive coil corners.

Thus the study of the self and mutual inductances of the loop and flat spiral coils was chosen. Since the field of wireless energy transfer is similar to the radio field, most of the literature uses radio/telecommunication equations to calculate the inductances used in wireless energy transfer. In the following, programs for calculating self and mutual inductances for circular (loop and planar) coils will be developed, based on equations from the literature; the results will be

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compared with different equations from the literature or specially developed, and with real measurements.

2. Mathematical models

Note that in the case of mutual inductance from equation (1) the conductor radius (r) is neglected compared to coil radius R since $\frac{r}{\mathrm{R}} \ll 1$, a valid condition for equations (1)-(7) and (14).

The calculation hypothesis of the mutual inductance between two loop or flat spiral coils at a distance h from each other, both with concentric circular turns, starts from a basic equation (1) which calculates the mutual inductance of two coils with one turn each, of radius R_i and R_j at a distance h [1]:

$$M(R_i, R_j, h) = \mu_0\sqrt{R_i R_j}\left[\left(\frac{2}{s} - s\right)K(s) - \frac{2}{s}E(s)\right] \tag{1}$$

$$s = \sqrt{\left(\frac{4R_i R_j}{(R_i + R_j)^2 + h^2}\right)} \tag{2}$$

$$K(s) = \int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{1 - s^2 sin^2\theta}} = \int_0^1 \frac{dt}{\sqrt{(1 - t^2)(1 - s^2t^2)}} \tag{3}$$

$$E(s) = \int_0^{\frac{\pi}{2}} \sqrt{1 - s^2 sin^2\theta}\, d\theta = \int_0^1 \sqrt{\frac{1 - s^2t^2}{1 - t^2}}\, dt \tag{4}$$

$$-1 \le s \le 1 \tag{5}$$

where: M - mutual inductance of two coils formed by one turn each, $K(s)/E(s)$ - complete elliptic integrals of degree I and II, R_i - radius of the transmit coil of order i, R_j - radius of receive coil of order j, h - distance between coils, s - variable depending on R_i, R_j, h.

In the case of mutual inductances between two coils containing more than one turn, the following equation is used:

$$M_{i,j} = \sum_{i=1}^{N_i} * \sum_{j=1}^{N_j} M(R_i, R_j, h) \rightarrow \tag{6}$$

where: $M_{i,j}$ - mutual inductance of multi-turn coils, where i=1,2,...N_i and j=1,2,...N_j, $i \neq j$.

The calculation hypothesis of the self-inductance for loop or flat spiral coils, both with concentric circular turns, starts from a basic equation that calculates the self-inductance of a single spiral coil of radius R [1],[2]:

$$L(R, r) = \mu_0 R(\ln\left(\frac{8\mathrm{R}}{r}\right) - 2) \tag{7}$$

where: R - transmit or receive coil radius, r - transmit or receive coil conductor radius.

The material permeability μ, used in the induction equations will have the value of the magnetic permeability of vacuum $\mu_0 = 4\pi 10^{-7}$, since the coils will have no magnetic support (air coil inductors have $\mu_r = \mu_{air} = 1$) in the first construction variant, following the equation:

$$\mu_r = \frac{\mu}{\mu_0} \tag{8}$$

where: μ, μ_0, μ_r - total, vacuum, relative (material) permeability.

The ratio between the inductance of the same coil with magnetic core and without, gives the relative effective permeability which indicates the value of the core permeability:

$$\mu_{ef} = \frac{L_\mu}{L_0} \tag{9}$$

where: μ_{ef} - effective permeability, L_μ - cored coil inductance, L_0 - coreless coil inductance.

In the following, for the self and mutual inductances calculation (using different values of the variable parameters), calculation programs were used in Matlab environment, and these were checked with approximate equations, simplified or designed from various articles and studies.

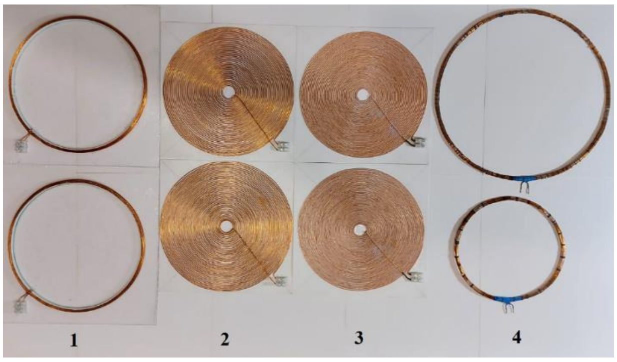

Fig. 1. Coils used in the wireless transfer study: 1,4 - planar loop coil, 2 - planar spiral coil with solid conductor, 3 - planar spiral coil with Litz conductor

3. Material and Methods

3.1 Planar loop coil

The self-inductance for a planar loop coil with N turns, satisfying the condition $\frac{R}{r} \ll 1$, is calculated by multiplying equation (7) by the square number of turns, is defined as follows [3], [4]:

$$L(r,w) = \mu_0 N^2 R\left(\ln\left(\frac{8R}{r}\right) - 2\right) \tag{10}$$

As a calculation technique, the average radius of the turns was used, since the loop coil turns diameters are very similar, the turns being concentrated in a single loop.

The planar loop coils used for testing have the following characteristics:

- coil A (solid conductor): ri = 0.0006m, Ri = 0.125m, N = 10,
- coil B (solid conductor): ri = 0.0006m, Ri = 0.072m, N = 11,
- coil C (solid conductor): ri = 0.001m, Ri = 0.101m, N = 18.

The following inductances results were obtained from calculations (equation (10)): coil A: 85.115µH; coil B: 53.282 µH; coil C: 193.05 µH.

Another equation for self-inductance determining for a mono spiral coil (expressed in µH) inspired by [5] is:

$$\boldsymbol{L(R,r) = 2\pi\mu_0 D(\ln\left(\frac{D}{r}\right) - 0.33)10^{-1}} \tag{11}$$

adapting this equation to multi-turn planar loop coils is done by multiplying the equation (11) by the number of turns N, not by their square N^2 as was done in equation 10, so the equation becomes:

$$\boldsymbol{L(R,r) = N2\pi\mu_0 D(\ln\left(\frac{D}{r}\right) - 0.33)10^{-1}} \tag{12}$$

where: D - coil diameter.

The following inductance results were obtained from the calculations (according to equation (12)): coil A: 111.79µH; coil B: 69.27 µH; coil C: 164.53µH. To certify the accuracy of the calculation equations (10) and (12) using an RLC Meter, the inductance of the three planar loop coils was measured as shown in Fig. 2. The following self-inductance were obtained from the measurements fig. 2: coil A: 69.1µH; coil B: 45.1µH; coil C: 165.8µH.

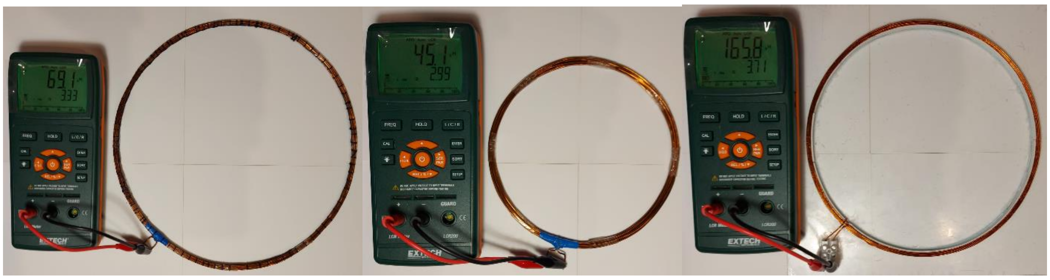

Fig. 2. Checking self-inductors of planar loop coils

The difference between the calculated inductance with equation (10) and the measured one:

- coil A, 85.11µH-69.1µH=15.71 µH, this resulted in an increase of measured value by 23.17%,
- coil B, 53.28µH-45.1µH=8.18µH, this resulted in an increase of measured value by 18.14%,
- coil C, 193.05µH-165.8µH=27.25µH, this resulted in an increase of measured value by 16.43%.

The difference between the calculated inductance with equation (12) and the measured one:

- coil A, 111.79μH-69.1μH=42.69μH, this resulted in an increase of measured value by 69.1%,
- coil B, 69.27 μH-45.1 μH=24.17μH, this resulted in an increase of measured value by 53.59%,
- coil C, 164.53μH-165.8μH=-1.17μH, this resulted in an decreased of measured value by 0.76%.

The mutual inductance of loop coils with N turns with radii R_i(i=1....N_i and R_j(j=1.....N_j), having the coil A construction characteristics with h = 0.01m; was calculated, resulting a mutual inductance of 4.6493e-05H≈46.493μH.

By analogy with the self-inductance calculation with equation (10), the mutual inductance will be decreased by 23.17%, the theoretical mutual inductance will be ≈ 35.72μH.

3.2 Planar spiral coil

Before calculating the planar spiral coil inductances, as shown in fig. 3, a formula for the radii calculation of the turns was defined:

$$R_{N_1} = r_i; R_{N_2} = r_i + (w + p) \rightarrow$$
$$\boldsymbol{R_{N_n} = r_i + (w + p)(N - 1)} \tag{13}$$

where: r_i - inner coil radius, R_N - N turn radius, w - conductor diameter, p - space between turns.

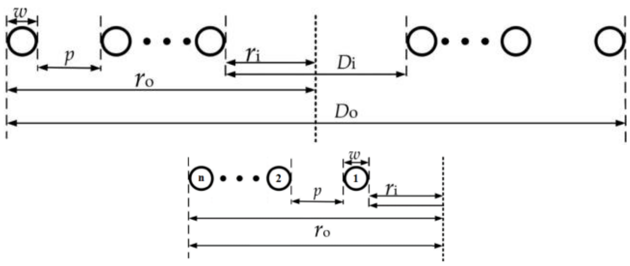

Fig. 3. Turns radii calculation of a planar spiral coil

The planar spiral coil self-inductance, consisting of a sequence of N concentric turns of different radii R_i (i=1,2,....N) and conductor radius r, is defined as follows [1]:

$$L_{tot} = \sum_{i=1}^{N} L(R_i, r_i) + \sum_{i=1}^{N_i} * \sum_{\substack{j=1 \\ i \neq j}}^{N_j} M(R_i, R_j) \rightarrow$$
$$L_{tot} = \sum_{i=1}^{N} \mu_0 r_i (\ln\left(\frac{8R_i}{r_i}\right) - \frac{7}{8}) + \sum_{i=1}^{N_i} * \sum_{i \neq j}^{N_j} \mu_0 \sqrt{R_i R_j}\left[\left(\frac{2}{s} - s\right) K(s) - \frac{2}{s} E(s)\right] \tag{14}$$

where: L_{tot} - coil self-inductance for $i \neq j$.

When calculating this self-inductance it must be remembered that the i and j turns are of the same coil, and the definition of the mutual inductance in equation (14) is made by calculating the mutual couplings between the turns of the same coil defined as i and j, in order to make combinations of i, j mutual inductances.

Calculations were performed for the following coils (equation (14)):

- coil D (solid conductor): ri = 0.0095, w = 0.001, p = 0.0011, D_o = 0.2038, N1=44.
- coil E (Litz conductor 12x0.2mm): ri = 0.0095, w = 0.0012, p = 0.0008, D_o = 0.195, N1= 44.

The following inductance results were obtained from the calculations (according to equation (14)): coil D: 202.7µH, coil E: 195.3 µH.

Another equation for the self-inductance calculation (in µH) for a planar spiral coil (dimensions in inches) is:

$$L = \frac{A^2 N^2}{30A - 11D_i} \qquad A = \frac{D_i + N(w+p)}{2} \tag{15}$$

where: D_i - inside diameter of the coil.

The following self-inductance results were obtained from the calculations (according to equation (15)): coil D: 161.74 µH, coil E: 156.27µH.

To certify the accuracy of the calculation equations (14) and (15) using an RLC Meter, the inductance of the two planar spiral coils was measured as shown in the figure.

Fig. 3. Checking self-inductance planar spiral coil

The following self-inductances were obtained from the measurements: coil D: 165.6µH; coil E: 157.3µH. The difference between the calculated inductance with equation (14) and the measured one:

- coil D: 202.7µH-165.6µH=37.1 µH, this resulted in an increase of measured value by 22.4%,
- coil E: 195.3µH-157.3µH =38µH, this resulted in an increase of measured value by 24.16%.

The difference between the calculated inductance with equation (15) and the measured one:

- coil D: 161.74 µH-165.6µH=-3.86µH, this resulted in an decreased of measured value by 2.33%
- coil E: 156.27µH -157.3µH =-1.03µH, this resulted in an decresed of measured value by 0.65% of the measured value.

Mutual inductance of planar spiral coils made of a sequence of N turns of radii R_i(i=1....N_i și R_j(j=1.....N_j), each coil having different radii for each turn, with coil D characteristics and h=0.01m, was calculated, resulting a mutual inductance of 8.8919e-05H≈88.919µH.

By analogy with the self-inductance calculation with equation (14), the mutual inductance will be decreased by 22.4%, the theoretical mutual inductance will be ≈ 68.99µH.

3.3 Coil quality factor

Six coils (pairs of two with near/identical dimensions) will be compared, whose dimensions are shown in table 1. An RLC Meter will be used to measure inductance, DC resistance and quality factor for different frequencies (1kHz, 10kHz, 100kHz), the results are shown in table 2.

Table 1

Coil dimensions used in the quality coefficient study

No.	Turns no.	Di (m)	Do (m)	W (m)	P (m)	Conductor length (m)	Coil type	
1	44	0.019	0.203	0.001	0.0011	15.398	Solid conductor	Planar spiral coil
2	44	0.019	0.209	0.001	0.00116	15.763		
3	44	0.019	0.197	0.0012	0.0082	14.912	Litz Conductor	
4	44	0.019	0.195	0.0012	0.008	14.79		
5	18	0.202	-	0.001	-	11.42	Solid conductor	Planar loop coil
6	18	0.202	-	0.001	-	11.42		

Table 2

Coil parameters for power supply at frequencies: 1kHz, 10kHz, 100kHz

No.	Planar spiral coil (solid conductor)		No.	Planar spiral coil (Litz type conductor)		No.	Planar loop coil (solid conductor)	
1	**L 165.6µH**	**Rdc 0.38 Ω**	**3**	**L 165.3µH**	**Rdc 0.38 Ω**	**5**	**L 162.5µH**	**Rdc 0.29 Ω**
	f (kHz)	Q		f (kHz)	Q		f (kHz)	Q
	1	2.84		1	2.9		1	3.7
	10	26.2		10	27.3		10	25.5
	100	126		100	257		100	54.3
2	**L 174.1µH**	**Rdc 0.4 Ω**	**4**	**L 157.3µH**	**Rdc 0.36 Ω**	**6**	**L 165.8µH**	**Rdc 0.3 Ω**
	f (kHz)	Q		f (kHz)	Q		f (kHz)	Q
	1	2.87		1	2.82		1	3.75
	10	26.5		10	25.3		10	27.3
	100	126		100	235		100	53.2

All the coils in the experiment will be used for this study, namely: planar loop coils with solid conductor and planar spiral coils with both solid conductor and Litz conductor. The comparison will be made for coils with very similar inductance values, but of different designs, to determine the applicability of each coil type in terms of quality coefficient for a given supply frequency.

The instrument (RLC Meter - type: LCR200 EXTECH) used for the measurement of inductance, resistance, and capacitance has the metrological error ±(0.5%rdg + 5digits) and quality factor range 0.000-999**;** therefore the actual measurement results are also subjected to this measurement error mentioned by the manufacturer.

4. Conclusions

According to the experiments we can conclude the following.

Although the self-inductance equations (10) and (14) are mentioned in numerous papers of literature [1], [6], [7], [8], [9], [10], (from the field of wireless energy transfer) they are not suitable for wireless energy transfer, as the coils do not satisfy the $\frac{r}{R} \ll 1$ condition, the equations being imported from the field of radio engineering where the condition $\frac{r}{R} \ll 1$ is fulfilled.

To theoretically demonstrate the applicability dependence of equations (10) and (14) on the condition $\frac{r}{R} \ll 1$, for a planar spiral coil, the self-inductance was calculated for different values of the ratio $\frac{r}{R}$ with equation (14) and separately with equation (15) which gives precise values, the results are expressed in table 3.

Acording to table 3, it can be seen that as the $\frac{r}{R}$ ratio decreases, equation (14) approaches in results values to equation (15) which give precise values.

Table 3

The inductance equations with reference to r/R

Inner radius R (m)	Conductor radius r (m)	(r/R)	L1 (μH) equation (14)	L2 (μH) equation (15)	L2/ L1
0.0095	0.0005	0.052	202.7	156.27	0.77
0.015	0.0005	0.033	231.7	184.25	0.79
0.095	0.0005	0.005	767.8	708	0.92
0.1	0.0005	0.005	806.3	745.52	0.92
0.124	0.0005	0.004	1000	930.46	0.93

For the self-inductances calculation, equations (12) for planar loop coils and (15) for planar spiral coils gave accurate results, and these equations can be used as a reference for the self-inductances calculation.

From the experiments carried out in the paper, the condition for the use of equations (12) and (15) is to maintain a minimum number of turns:

- planar loop coil: 18,

- planar spiral coil: 44;

this condition is demonstrated by applying the equation (12) for the three types of coils (A,B,C) self-inductance calculation, determining significant deviations from the real values in the case of coil A (10 turns) of 69.1%, smaller deviations in the case of coil B (11 turns) of 53.59%, and insignificant deviations in the case of coil C (18 turns) of -0.76%. It should be highlighted that this minimum number may be lower, as the conclusions are strictly experimental. Equation (15) determined a self-inductance for coils D and E with a small deviation from the real measured inductance of only 1.2%.

When either calculating using equation (15) or measuring the self-inductances for planar spiral coils D and E which differ in the nature of the conductor, 1mm diameter solid conductor with 1.1mm turn spacing for coil D and Litz type conductor with 1.2mm total diameter and 0.8mm turn spacing (consisting of 25 conductors of 0.2mm diameter) for coil E, identical results were obtained.

The total cross-section of the Litz-type conductor of coil E is the same as the cross-section of the conductor of coil D, the space assigned to a turn as well as the geometry of the coils (inner diameter and outer diameter) being approximately identical between the two coils D and E, which shows that the nature of the conductor does not influence the inductance, as long as the geometrical parameters are respected. Coil D having a larger space between turns also determined a larger outer diameter and proportionally higher inductance than coil E. In terms of the mutual inductance (for planar loop and planar spiral coils) determined by calculation programs according to equations (1), (6), if identical transmit and receive coils are used, the error found in the self-inductances calculated by equations (10) and (14) with respect to the equivalent equations giving exact results (12) and (15) is subtracted from the mutual inductance for correction.

From the coil quality factor study we can conclude the following: at low frequencies up to 10kHz the quality factor Q is almost identical for all 6 types of coils, so the type of conductor (Litz or solid) or the type of coil (spiral or loop) has no significant influence. With the increase of the supply frequency, as expected the planar loop coil with Litz type conductor obtained a much higher quality factor than the other coil types mentioned above of about $Q \approx 240$, which recommends this type of coil for use in wireless systems operating at high frequencies above 100kHz.

In comparison (for frequencies above 100kHz), the planar spiral coils type but with a solid conductor obtained a 50% lower Q, whereas the planar loop coils type with a solid conductor obtained by far the lowest Q values of 80% lower than planar spiral type coils with Litz type conductor.

Therefore, in terms of quality coefficient and inductance, for low frequencies around 10kHz planar loop coils with solid conductor are recommended, as they have a robust construction, are simple to build (minimum cost), use a much smaller amount of linear conductor (≈11m as opposed to the other coils with ≈15m), for almost identical inductance values.

R E F E R E N C E S

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