A CLASSIFICATION OF CUBIC EDGE-TRANSITIVE GRAPHS OF ORDER 18*p*

Mehdi ALAEIYAN[1], M.K. HOSSEINIPOOR[2]

A graph is called edge-transitive if its automorphism group acts transitively on the set of its edge. In this paper, we classify all connected cubic edge-transitive graphs of order 18p for each prime p.

Keywords: Edge-transitive graphs, Semisymmetric graphs, Symmetric graphs, s-Regular graphs, Regular coverings.

1. Introduction

Throughout this paper graphs are assumed to be finite, simple, undirected and connected. For group theoretic concepts and notation not defined here, we refer the reader to [13, 24]. Given a positive integer n, we shall use the symbol $\mathbb{Z}_n$ to denote the ring of residues modulo n as well as the cyclic group of order n.

For a graph *X*, we use *V(X), E(X)* and Aut(*X*) to we denote its vertex set, edge set and automorphism group, respectively. For $u, v \in V(X), uv$ is the edge incident to u and v in X and $N_X(u)$ is the set of vertices adjacent to u in X. For a subgroup N of Aut(X), denote by X_N the *quotient graph* of X corresponding to the orbits of N, that is, the graph having the orbits of N as vertices with two orbits adjacent in X_N whenever there is an edge between those orbits in X.

A graph $\widetilde{X}$ is called a *covering* of a graph *X* with projection $\varphi:\widetilde{X} \to X$ if there is a surjection $\varphi : \widetilde{X} \to X$ such that $\wp|N_{\widetilde{X}}(\widetilde{v}) : N_{\widetilde{X}}(\widetilde{v}) \to N_X(v)$ is a bijection for any vertex $v \in V(X)$ and $\widetilde{v} \in \varphi^{-1}(v)$. A covering $\widetilde{X}$ of *X* with a projection φ is said to be *regular* (or *k-covering*) if there is a semiregular subgroup *K* of the automorphism group Aut($\widetilde{X}$) such that graph *X* is isomorphic to the quotient graph $\widetilde{X}_K$, say by *h*, and the quotient map $\widetilde{X} \to \widetilde{X}_K$ is the composition φh of φ and *h*. If $\widetilde{X}$ is connected *K* becomes the covering transformation group. The *fibre* of an edge or vertex is its preimage under φ. An automorphism of $\widetilde{X}$ is said to be *fibre-preserving* if it maps a fibre to a fibre, while every covering transformation maps a fibre onto itself. All of fibre-preserving automorphism from a group called the *fibre-preserving group*.

Let *G* be a finite group and *S* a subset of *G* such that 1∉S and $S = S^{-1} = \{s^{-1}|s \in S\}$. The *Cayley graph Cay (G, S)* on G with respect to S is defined to have

---

[1] Prof., Department of Mathematics, Faculty of Science, South Tehran branch, Islamic Azad University, Tehran, Iran, e-mail: m_alaeiyan@azad.ac.ir.
[2] Department of Mathematics, Iran University of Science and Technology, Iran

vertex set G and edge set $\{gh|\ g,h \in G, gh^{-1} \in S\}$. A Cayley graph Cay $(G,\ S)$ is connected if and only if S generates G. It is well known that Aut (Cay $(G,\ S)$) contains the *right regular representation R(G)* of G, the acting group of G by right multiplication, which is regular on vertices. A Cayley graph Cay $(G,\ S)$ is said to be *normal* if $R(G)$ is normal in Aut(Cay(G,S)). A graph X is isomorphic to a Cayley graph on G if and only if Aut(X) has a subgroup isomorphic to G, acting regularly on vertices (see [4, Lemma 16.3]).

Let s be a positive integer. An *s-arc* in a graph X is an ordered $(s+1)$-tuple $(v_0, v_1, \ldots, v_{s-1}, v_s)$ of vertices of X such that v_{i-1} is adjacent to v_i for $1 \leq i \leq s$ and $v_{i-1} \neq v_{i+1}$ for $1 \leq i \leq s-1$. For a graph X and a subgroup G of Aut(X), X is said to be *G-vertex-transitive*, *G-edge-transitive* and *G-s-arc-transitive* if G acts transitively on the sets of vertices, edges and *s-arcs* of X respectively. It is easily seen that a graph X which is *G*-edge but not *G*-vertex-transitive is necessarily bipartite, with the two parts of bipartition coinciding with the orbits of G. In particular, if X is a regular graph, then these two parts have equal cardinalities, and such a graph is then referred to as being *G-semisymmetric*. In the case where $G =$Aut(X) the symbol G may be omitted from the definitions above, so that a graph X is called *vertex-transitive*, *edge-transitive*, *s-arc-transitive* and *semisymmetric* if it is Aut(X)-vertex-transitive, Aut(X)-edge-transitive, Aut(X)-*s*-arc-transitive and Aut(X)-semisymmetric, respectively. In particular 1-arc-transitive means *arc-transitive* or *symmetric*. A symmetric graph X is said to be *s-regular* if Aut(X) acts regularly on the set of *s-arcs* in X. Tutte [21, 23] showed that every cubic symmetric graph is *s*-regular for some $1 \leq s \leq 5$.

Tutte [22], proved that a vertex- and edge-transitive graph with odd valency must be symmetric. Thus a cubic edge-transitive graph is either symmetric or semisymmetric. The classification of cubic symmetric or semisymmetric graphs of different order is given in many papers. For example, the cubic symmetric graphs of order $4p,\ 4p^2, 6p, 6p^2, 10p, 10p^2, 16p$ and $16p^2$ were classified in [12,11,19,3] and the cubic semisymmetric graphs of order $2p^3, 6p^2, 6p^3, 8p$ and $8p^2$ were classified in [18,16,9,2,1]. In this paper, we obtain a classification of cubic edge-transitive graphs of order $18p$.

In order to state the main Theorem 1.1 we first introduce a family of cubic graphs. Let p be a prime such that $p \equiv 1 \pmod 3$, and let k be an element of order 3 in $\mathbb{Z}_{3p}^*$. Set $V(K_{3,3}) = \{a,b,c,x,y,z\}$ to be the vertex set of the complete bipartite graph $K_{3,3}$ with partite sets $\{a,b,c\}$ and $\{x,y,z\}$. The graph CF_{18p} is defined to have vertex set $V(CF_{18p}) = V(K_{3,3}) \times \mathbb{Z}_{3p}$ and edge set

$$E(CF_{18p}) = \{(a,\ i)(x,\ i),\ (a,\ i)(y,\ i),\ (a,\ i)(z,\ i),\ (b,\ i)(y,\ i),\ (b,\ i)(x,\ i+k+1),$$
$$(b,\ i)(z,\ i+1),\ (c,\ i)(x,\ i-1),\ (c,\ i)(y,\ i-k-1),\ (c,\ i)(z,\ i)|i \in \mathbb{Z}3p\}.$$

It will be shown in the Lemma 2.7 that the graph CF_{18p} is independent of the choice of k and hence unique for given order. The following is the main result of this paper.

Theorem 1.1 *Let X be a connected cubic edge-transitive graph of order 18p, where p is a prime. Then X is either semisymmetric or s-regular for some s = 1, 2 or 5.*
Furthermore,

(1) *X is semisymmetric if and only if X is isomorphic to one of the graphs S54 and S126;*

(2) *X is 1-regular if and only if X is isomorphic to the graph* CF_{18p}, *where* $p \equiv 1$ (mod 3);

(3) *X is 2-regular if and only if X is isomorphic to the graph* F_{54};

(4) *X is 5-regular if and only if X is isomorphic to one of the graphs* F_{90} *and* F_{234B}.

2. Preliminaries

Proposition 2.1 [18, Proposition 2.6] *Let X be a G-semisymmetric graph for some subgroup G of Aut(X). Then either X ≡ K_{3,3} or G acts faithfully on each of bipartition sets of X.*

Proposition 2.2 [18, Proposition 2.3] *Let X be a connected bipartite graph admitting an abelian subgroup G ≤ Aut(X) acting regularly on each of the bipartition sets. Then X is vertex-transitive.*

The next Proposition is a special case of [16, Lemma 3.2].

Proposition 2.3 *Let X be a connected G-semisymmetric cubic graph with bipartition sets L(X) and R(X) and let N be a normal subgroup of G. If N is intransitive on bipartition sets, then N acts semiregularly on both L(X) and R(X), and X is an N-covering of a G/N-semisymmetric graph.*

Proposition 2.4 [15. Theorem 9] *Let X be a connected symmetric graph of prime valency and G an s-arc-transitive subgroup of Aut(X) for some s ≥ 1. If a normal subgroup N of G has more than two orbits, then it is semiregular and G/N is an s-arc-transitive subgroup of Aut(X_N), where X_N is the quotient graph of X corresponding to the orbits of N. Furthermore, X is a regular covering of X_N with the covering transformation group N.*

Let $X = Cay\ (G, S)$ be a Cayley graph on a group G with respect to S. Set $A := \mathrm{Aut}(X)$ and $\mathrm{Aut}(G,S) := \{\alpha \in \mathrm{Aut}(G) \mid S^\alpha = S\}$. Then we have:

Proposition 2.5 [25, Proposition 1.5] *The Cayley graph X = Cay (G,S) is normal if and only if* $A_1 = Aut(G,S)$, *where* A_1 *is the stabilizer of the vertex* $1 \in V(X) = G$ *in A.*

Let p be a prime. It is easy to see that the equation

$$x^2 + x + 1 = 0 \tag{1}$$

has no solution in the ring $\mathbb{Z}_{3p}$ for $p = 3$. The following result determines the solutions of *Eq.* (1) in $\mathbb{Z}_{3p}$ for $p \neq 3$.

Lemma 2.6 *Let $p \neq 3$ be a prime. Then there exists $k \in \mathbb{Z}_{3p}$ solving Eq. (1) if and only if k is an element of order* 3 *in $\mathbb{Z}_{3p}^*$.*

Proof. Suppose first that $k \in \mathbb{Z}3_p$ such that $k^2 + k + 1 = 0$. Then $k \neq 1$ and since $k^3 - 1 = (k - 1)(k^2 + k + 1) = 0$, it follows that k is an element of order 3 in $\mathbb{Z}_{3p}^*$.

Conversely, suppose that k is an element of order 3 in $\mathbb{Z}_{3p}^*$. Then $k \neq 1$ and $k^3 = 1$. It follows that $(k - 1)(k^2 + k + 1) = 0$. If $k - 1$ is divisible by 3, then $k^2 + k + 1$ is also divisible by 3. Thus, in order to prove $k^2 + k + 1 = 0$, it suffices to show $k - 1$ is coprime with p. Assume that $(k - 1, p) \neq 1$. Then $k \equiv 1 \pmod{p}$. Let $k = tp + 1$. Then $k^3 = t^3 p^3 + 1$. Since $k^3 = 1$ and $p = 36$, we have $t \equiv 0 \pmod{3}$. Hence $k = 1$, a contradiction. This completed the proof of lemma.

Let p be a prime such that $p \equiv 1 \pmod 3$. Since $\mathbb{Z}_{3p}^* \cong \mathbb{Z}_2 \times \mathbb{Z}_{p-1}$, by Lemma 2.6, there are exactly two elements of order 3, say k and k^2 in $\mathbb{Z}_{3p^2}^*$ solving Eq. (1). Denote by $V(K_{3,3}) = \{a,b,c,x,y,z\}$ the vertex set of $K_{3,3}$ as before. The graphs

CF_{18p} and $\overline{CF}_{18p}$ are defined to have the same vertex set $V(CF_{18p}) = V(\overline{CF}_{18p}) = V(K_{3,3}) \times \mathbb{Z}3_p$ and edge sets

$E(CF_{18p}) = \{(\mathbf{a},i)(\mathbf{x},i), (\mathbf{a},i)(\mathbf{y},i), (\mathbf{a},i)(\mathbf{z},i), (\mathbf{b},i)(\mathbf{y},i), (\mathbf{b},i)(\mathbf{x},i + k + 1), (\mathbf{b},i)(\mathbf{z},i+1),$
$\qquad\qquad\qquad (\mathbf{c},i)((\mathbf{x},i-1), (\mathbf{c},i)(\mathbf{y},i-k-1), (\mathbf{c},i)(\mathbf{z},i)|i \in \mathbb{Z}3_p\},$
$E(CF_{18p}) = \{(\mathbf{a},i)(\mathbf{x},i), (\mathbf{a},i)(\mathbf{y},i), (\mathbf{a},i)(\mathbf{z},i), (\mathbf{b},i)(\mathbf{y},i), (\mathbf{b},i)(\mathbf{x},i + k^2 + 1),$
$\qquad\qquad (\mathbf{b},i)(\mathbf{z},i+1), (\mathbf{c},i)((\mathbf{x},i-1), (\mathbf{c},i)(\mathbf{y},i-k^2-1), (\mathbf{c},i)(\mathbf{z},i)|i \in \mathbb{Z}3_p\},$

respectively. The graph $\overline{CF}_{18p}$ is obtained by replacing k with k^2 in each edge of CF_{18p}. It is easy to see that CF_{18p} and $\overline{CF}_{18p}$ are cubic and bipartite.

Lemma 2.7 The graphs CF_{18p} and $\overline{CF}_{18p}$ are isomorphic.

Proof. Let p be a prime such that $p \equiv 1 \pmod 3$ and k an element of order 3 in $\mathbb{Z}_{3p}^*$. To show $CF_{18p} \cong \overline{CF}_{18p}$ we define a map α from $V(CF_{18p})$ to $V(\overline{CF}_{18p})$ by

$(\mathbf{a},i) \to (\mathbf{a},ki), \qquad (\mathbf{b},i) \to (\mathbf{c},ki), \qquad (\mathbf{c},i) \to (\mathbf{b},ki),$
$(\mathbf{x},i) \to (\mathbf{x},ki), \qquad (\mathbf{y},i) \to (\mathbf{z},ki), \qquad (\mathbf{z},i) \to (\mathbf{y},ki),$

where $i \in \mathbb{Z}_{3p}$. Clearly,

$$N_{CF18p}\big((\mathbf{b},i)\big) = \{(y,i),(x,i+k+1),(z,i+1)\},$$
$$N_{\overline{CF}18p}((\mathbf{b},i)^\alpha) = N_{\overline{CF}18p}\big((c,ki)\big)$$
$$= \{(x,ki-1),(y,ki-k^2-1),(z,ki)\}$$

By Lemma 2.6, $k^2 + k + 1 = 0$. With use of this property, one can show that
$[N_{CF18p}((\mathbf{b},i))]^\alpha = N_{\overline{CF}_{18p}}((\mathbf{b},i)^\alpha),$
Similarly,
$[N_{CF18p}((\mathbf{u},i))]^\alpha = N_{\overline{CF}_{18p}}((\mathbf{u},i)^\alpha),$

for u = a, c. It follows that α is an isomorphism from CF_{18} to $\overline{CF}_{18p}$, because the graphs are bipartite.

In view of [17, Corollary 2.2], [10, Theorem 1.1], Lemma 2.6 and Lemma 2.7 imply the following.

Lemma 2.8 *Let X be a connected cubic $\mathbb{Z}_{3p}$-covering of $K_{3,3}$ whose fibre-preserving group acts edge-transitively on X. Then $p \equiv 1(mod\ 3)$ and X is isomorphic to the 1-regular graph CF_{18p}.*

3. Proof of Theorem 1.1

Lemma 3.1 *Let p be a prime. Then, with the exception of the graphs S54 (the Gray graph) and S126, every connected cubic edge-transitive graph of order 18p is symmetric.*

Proof. By [7], there are two cubic semisymmetric graphs of order 18*p* for $p \leq 41$ which are the graphs S54 and S126. To prove the lemma, we only need to show that no connected cubic semisymmetric graph of order 18*p* exists for $p \geq 43$.

Suppose that there exists a connected cubic semisymmetric graph *X* of order 18*p* with $p \geq 43$. Then *X* is bipartite. Denote by $L(X)$ and $R(X)$ the bipartition sets of *X*. Clearly, $|L(X)| = |R(X)| = 9p$. Set $A:=Aut(X)$. By Proposition 2.1, *A* is faithful on each bipartition sets of *X* and by [18, Proposition 2.4], the vertex stabilizer A_v of $v \in V(X)$ has order $2^r.3$, where $r \geq 0$. Thus $|A| = 2^r.3^3.p$. By [14, pp. 12-14], there is no simple $\{2, 3, p\}$-group for $p \geq 43$. Hence *A* is solvable.

For any prime divisor *q* of $|A|$, denote by $O_q(A)$ the largest normal *q*-subgroup of *A*. Then by Proposition 2.3, $O_q(A)$ is semiregular on $L(X)$ and $R(X)$ and the quotient graph $X_{O_q(A)}$ is $A/O_q(A)$-semisymmetric. Thus $O_2(A) = 1$ and $|O_3(A)|$ is a divisor of 9. By the solvability of *A*, either $O_3(A) \neq 1$ or $O_p(A) \neq 1$. If $O_3(A) \neq 1$ then $|O_3(A)| = 3$ or 9. Let $T/O_3(A)$ be a minimal normal subgroup of $A/O_3(A)$. Since *A* is solvable, $A/O_3(A)$ is also solvable, implying $T/O_3(A)$ is elementary abelian. The quotient graph $X_{O_3(A)}$ is $A/O_3(A)$-semisymmetric. Thus by Proposition 2.3, $O_2(A/O_3(A)) = 1$ and since $O_3(A/O_3(A)) = 1$, we conclude that $T/O_3(A)$ is a *p*-group. Now $|T| = 3p$ or 9*p* and since $p \geq 43$, by Sylow Theorem, *T* has a normal Sylow *p*-subgroup, which is characteristic in *T* and hence normal in *A* because $T \trianglelefteq A$. Therefore $O_p(A) \neq 1$.

Set $P = O_p(A)$. Since $O_p(A) \neq 1$, we have $P \cong \mathbb{Z}p$. Note that the quotient graph X_P is A/P-semisymmetric. Thus X_P is bipartite. Denote by $L(X_P)$ and $R(X_P)$ the bipartition sets of X_P. Then $|L(X_P)| = |R(X_P)| = 9$. Set $C = C_A(P)$. Then $P \leq C$. Suppose $C = P$. Then by [20, Theorem 1.6.13], A/P is isomorphic to a subgroup of $Aut(P) \cong \mathbb{Z}p-1$, which implies that A/P is abelian. Since X_P is A/P semisymmetric, it follows by Proposition 2.1, and [24, Proposition 4.4] that A/P is regular on $L(X_P)$ (and also $R(X_P)$). This implies that $|A| = 9p$, a contradiction. Thus $P < C$, that is, *P* is a proper subgroup of *C*. Let M/P be a minimal normal subgroup of A/P contained in C/P. Then M/P is an elementary abelian 3-group because by Proposition 2.3, $O_2(A/O_3(A)) = 1$. Let *Q* be a Sylow 3-subgroup of *M*. Then $M =$

PQ, implying $M = P \times Q$ because $Q < C$. Hence Q is characteristic in M and since $M \lhd A$, we have $Q \lhd A$. Thus $Q \leq O_3(A)$, it follows that $|Q| = 3$ or 9 and Q is semiregular on $L(X_P)$ and $R(X_P)$. Let $|Q| = 9$. Then M is an abelian group of order $9p$. Since the vertex stabilizer of each vertex of X has order $2^r.3$, it follows by the semiregularity of Q on $L(X)$ and $R(X)$ that M is regular on each of the bipartition sets of X. By Proposition 2.2, X is vertex-transitive, a contradiction.

Thus $Q \cong \mathbb{Z}3$, and hence $M \cong \mathbb{Z}3p$. By Proposition 2.3, X is a $\mathbb{Z}3p$-covering of the bipartite graph $K_{3,3}$ and since $M \lhd A$, the fibre-preserving group is the automorphism group A of X, so it is edge-transitive. But by Lemma 2.8, X is symmetric, which is a contradiction.

Lemma 3.2 *Let X be a connected cubic symmetric graph of order $18p$, where p is a prime. Then X is 1-,2-, or 5-regular. Furthermore,*

(1) *X is 1-regular if and only if X is isomorphic to the 1-regular graph CF_{18p}, where $p \equiv 1 \pmod 3$;*

(2) *X is 2-regular if and only if X isomorphic to the graph F_{54};*

(3) *X is 5-regular if and only if X is isomorphic to one of the graphs F_{90} and F_{234B}.*

Proof. Let X be a cubic graph satisfying the assumptions and let A =Aut(X). Since X is symmetric by Tutte [23], X is s-regular for some $s \leq 5$. Thus $|A| = 2^s.3^3.p$. For each prime $p = 7,19,31$ or 37, by [6] and Lemma 2.8, there is only one connected cubic symmetric graph of order $18p$, which is the 1-regular graph CF_{18p} and for each prime $p = 2,11,17,23,29$ or 41, there is no connected cubic symmetric graph of order $18p$. Similarly, for $p = 3$ or 5, there is only one connected cubic symmetric graph of order $18p$, that is, the 2-regular graph F_{54} and the 5-regular graph F_{90}, and for $p = 13$ there are two connected cubic symmetric graph of order 18×13 which are the 1-regular graph CF_{234} and the 5-regular graph F_{234B}. Thus we may assume that $p \geq 43$. By [14, pp. 12-14], A is solvable.

Let q be a prime divisor of $|A|$. Then by Proposition 2.4, $O_q(A)$ is semiregular on $V(X)$ and the quotient graph $X_{Oq(A)}$ is a cubic symmetric graph. The semiregularity of $O_q(A)$ implies that $|O_q(A)| \mid 18p^2$. If $O_2(A) = 16$, then $O_2(A) \cong \mathbb{Z}2$ and hence $X_{O2(A)}$ has odd order and valency 3, a contradiction. Thus $O_2(A) = 1$ and by the solvability of A, either $O_3(A) \neq 1$ or $O_p(A) \neq 1$. If $O_3(A) \neq 1$, then $|O_3(A)| = 3$ or 9. Let $T/O_3(A)$ be a minimal normal subgroup of $A/O_3(A)$. Then $T/O_3(A)$ is elementary abelian because A is solvable. Since $O_3(A/O_3(A)) = 1$, $T/O_3(A)$ is a 2- or p-group. For the former by Proposition 2.4, the quotient graph X_T would have be a cubic graph of odd order, a contradiction. Thus $T/O_3(A)$ is a p-group.
Now $|T| = 3p$ or $9p$ and since $p \geq 43$, by Sylow Theorem, M has a normal Sylow p-subgroup, which is characteristic in T and hence normal in A. Therefore $O_p(A) \neq 1$.

Set $P = O_p(A)$. Since $O_p(A) \neq 1$, we have $P \cong \mathbb{Z}p$. By Proposition 2.4, the quotient graph X_P is a cubic symmetric graph and A/P is an arc-transitive subgroup of Aut(X_P). Set $C = C_A(P)$. Clearly $P \leq C$. Suppose that $P = C$. Then by [20, Theorem 1.6.13], A/P is isomorphic to a subgroup of Aut(P) $\cong \mathbb{Z}p_{-1}$, which implies that A/P is abelian. Since A/P is transitive on $V(X_P)$, it follows by [24, Proposition 4.4] that A/P is regular on $V(X_P)$. This implies that $|A| = 18p$, a contradiction. Thus $P < C$. Let M/P be a minimal normal subgroup of A/P contained in C/P. Then M/P is an elementary abelian 2- or 3-group. If M/P is a 2-group then by Proposition 2.4, the quotient graph X_M is a cubic symmetric graph of odd order, a contradiction. Thus M/P is a 3-group. Let Q be a Sylow 3-subgroup of M. Then $M = PQ$, implying $M = P \times Q$ because $Q < C$. Hence Q is characteristic in M and since $M \lhd A$, we have $Q \lhd A$. Thus $Q \leq O_3(A)$ and hence $|Q| = 3$ or 9. Note that Q is isomorphic to the elementary abelian group M/P. Then $Q \cong \mathbb{Z}_3$ or $\mathbb{Z}_3^2$.

Let $Q \cong \mathbb{Z}_3$. Then the quotient graph X_Q is a cubic symmetric graph of order $2p$ and A/Q is an arc-transitive subgroup of Aut(X_Q). Since $p \geq 43$, by [5] and [8, Lemma 3.4], X_Q is a normal cubic 1-regular Cayley graph on dihedral group D_{2p}. Thus $A/Q =$Aut(X_Q) and A has a normal subgroup G such that G/Q acts regularly on $V(X_Q)$. Consequently, G is regular on $V(X)$ and hence X is a normal cubic 1-regular Cayley graph on G. Let $X = Cay(G, S)$. Since X has valency 3, S contains at least one involution. By Proposition 2.5, Aut(G,S) is transitive on S, which implies that S consists of three involutions and by the connectivity of X, G can be generated by three involutions. Note that M/Q is a normal Sylow p-subgroup of A/Q. Then $M/Q \lhd G/Q$, implying $M \lhd G$. Since $M \cong \mathbb{Z}_3^2 \times \mathbb{Z}_p$. and $G/Q \cong D_{2p}$, one can conclude that $G \cong \mathbb{Z}_3^2 \times D_{2p}$ or $\mathbb{Z}_3 \times D_{6p}$, which is impossible because in each case G cannot be generated by involutions.

Thus $Q \cong \mathbb{Z}3$, and so $M \cong \mathbb{Z}3p$. By Proposition 2.4, X is a $\mathbb{Z}3p$-covering of the bipartite graph $K_{3,3}$ and since $M \lhd A$, the symmetry of X means that the fibre-preserving group is arc-transitive and hence is edge-transitive. By Lemma 2.8, X is isomorphic to CF_{18p}, where $p \equiv 1 \pmod 3$.

Proof of Theorem 1.1 It follows by Lemmas 3.1 and 3.2.

REFERENCES

[1]. *M. Alaeiyan, M. Ghasemi*, "Cubic edge-transitive graphs of order 8p2", in Bull. Austral. Math., **vol. 77**, 2008, pp. 315-323

[2]. *M. Alaeiyan, M.K. Hosseinipoor*, "Classifying cubic symmetric graphs of order 8p", in Proc. Indian Acad. Sci. (Math. Sci.), **vol. 119**, no. 5, 2011, pp. 647-653

[3]. *M. Alaeiyan, B.N. Onagh, M.K. Hosseinipoor*, "A classification of cubic symmetric graphs of order 16p2", in Proc. Indian Acad. Sci. (Math. Sci.), **vol. 121**, no. 3, 2011, pp. 249-257

[4]. *N. Biggs*, "Algebraic Graph Theory, second ed.", in Cambridge University Press, Cambridge, 1993.

[5]. *Y. Cheng, J. Oxley*, "On weakly symmetric graphs of order twice a prime", in J. Combin. Theory Ser. **vol. 42,** 1987, pp. 196-211.

[6]. *M.D. E. Conder, P. Dobcsˊanyi*, "Trivalent symmetric graphs on up to 768 vertices", in J. Combin. Math. Combin. Comput. **vol. 40**, 2002, pp. 41-63

[7]. *M.D. E. Conder, A. Malniˇc, D. Maruˇsiˇc, P. Potocnik*, "A census of semisymmetric cubic graphs on up to 768 vertices", in J. Algebraic Combin. **vol. 23,** 2006, pp. 255-294.

[8]. *S.F. Du, Y.-Q. Feng, J.H. Kwak,, M.Y. Xu*, "Cubic Cayley graphs on dihedral groups", in Mathematical Analysis and Applications, Narosa Publishing House, New Delhi, 2004, pp. 224-235.

[9]. *Y. Q. Feng, M. Ghasemi and C.Q. Wang*, "Cubic semisymmetric graphs of order 6p3", in Discrete Math., **vol. 310**, 2010, pp. 2342-2355.

[10]. *Y. Q. Feng, J.H. Kwak*, "s-Regular cyclic coverings of the complete bipartite graph K3,3", in J. Graph Theory, **vol. 45**, 2004, pp. 101-112.

[11]. *Y. Q. Feng, J.H. Kwak*, "Classifying cubic symmetric graphs of order 10p or 10p2", in Sci. China Ser. **vol . A 49** , 2006, pp. 300-319.

[12]. *Y. Q. Feng, J.H. Kwak*, "Cubic symmetric graphs of order a small number times a prime or a prime square", in J. Combin. Theory Ser. **vol B 97**, 2007, pp. 627-646.

[13]. *D. Gorenstein*, "Finite Groups", in Harper and Row, New York, 1968.

[14]. *D. Gorenstein*, "Finite Simple Groups", in Plenum Press, New York, 1982.

[15]. *P. Lorimer*, "Vertex-transitive graphs: Symmetric graphs of prime valency",in J. Graph Theory, **vol. 8**, 1984, pp. 55-68.

[16]. *Z. Lu, M.Y. Xu, C.Q. Wang*, "On semisymmetric cubic graphs of order 6p2", in Sci. China Ser., **vol. A 47**, 2004, pp. 1-17.

[17]. *A. Malniˇc, D. Maruˇsiˇc, P. Potocnik, C.Q. Wang*, "An infinite family of cubic edge-but not vertex-transitive graphs", in Discrete Math., **vol. 280,** 2004, pp. 133-148.

[18]. *A. Malniˇc, D. Maruˇsiˇc, C.Q. Wang*, "Cubic edge-transitive graphs of order 2p3", in Discrete Math. **vol. 274,** 2004, pp. 187-198.

[19]. *J.M. Oh*, "A classification of cubic s-regular graphs of order 16p", in Discrete Math., **vol. 309,** 2009, pp. 3150-3155.

[20]. *D.J. Robinson*, "A Course in the Theory of Groups", in Springer-Verlag, New York, 1982.

[21]. *W.T. Tutte*, "A family of cubical graphs", in Proc. Camb. Phil. Soc., **vol. 43**, 1947, pp.459-474.

[22]. *W.T. Tutte*, "Connectivity in Graphs", in University of Toronto Press, Toronto, 1966.

[23]. *W.T. Tutte*, "On the symmetry of cubic graphs", in Canada. J. Math., **vol. 11**, 1959, pp. 621-624.

[24]. *H. Wielandt*, "Finite Permutation Groups", in Academic Press, New York, 1964.

[25]. *M.Y. Xu*, "Automorphism groups and isomorphisms of Cayley digraphs", in Discrete Math. 182 (1998) 309-319.