CERTAIN TYPES OF SOFT GRAPHS

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In this article, the concepts of soft graphs and vertex-induced soft graphs are presented. Certain types of soft graphs including regular soft graphs, irregular soft graphs, neighbourly irregular soft graphs and highly irregular soft graphs are introduced and investigated.

Keywords: Soft sets, Regular soft graphs, Irregular soft graphs.

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1. Introduction and Preliminaries

Molodtsov [8] initiated the novel concept of soft set theory as a new mathematical tool for dealing with uncertainties. This theory provides a parameterized point of view for uncertainty modelling and soft computing. Let U be the universe of discourse and E be the universe of all possible parameters related to the objects in U. Each parameter is a word or a sentence. In most cases, parameters are considered to be attributes, characteristics or properties of objects in U. The pair (U, E) is also known as a *soft universe*. The power set of U is denoted by $\mathscr{P}(U)$.

Definition 1.1. [8] *A pair (F, A) is called soft set over U, where $A \subseteq E$, F is a set-valued function $F : A \to \mathscr{P}(U)$. In other words, a soft set over U is a parameterized family of subsets of U. For any $\epsilon \in A$, $F(\epsilon)$ may be considered as set of ϵ-approximate elements of soft set (F, A).*

By means of parametrization, a soft set produces a series of approximate descriptions of a complicated object being perceived from various points of view. It is apparent that a soft set $F_A = (F, A)$ over a universe U can be viewed as a parameterized family of subsets of U. For any parameter $\epsilon \in A$, the subset $F(\epsilon) \subseteq U$ may be interpreted as the set of *ϵ-approximate elements*.

In 1975, Rosenfeld [9] first discussed the concept of fuzzy graphs whose basic idea was introduced by Kauffmann [6] in 1973. Rosenfeld also proposed the fuzzy relations between fuzzy sets and developed the structure of fuzzy graphs, obtaining analogs of several graph theoretical concepts. Bhattacharya [3] gave some remarks on fuzzy graphs, and some operations on fuzzy graphs were introduced by Mordeson and Peng [7]. Recently, Akram

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and Nawaz [2] have introduced the novel concepts called fuzzy soft graphs and fuzzy vertex-induced soft graphs. In this paper, we introduce the notion of soft graphs and describe certain types of soft graphs.

2. Certain types of soft graphs

Definition 2.1. *A 4-tuple $G = (G^*, F, K, A)$ is called a soft graph if it satisfies the following conditions:*
(1) *$G^* = (V, E)$ is a simple graph,*
(2) *A is a nonempty set of parameters,*
(3) *(F, A) is a soft set over V,*
(4) *(K, A) is a soft set over E,*
(5) *$(F(a), K(a))$ is a subgraph of G^* for all $a \in A$.*

The subgraph $(F(a), K(a))$ is denoted by $H(a)$ for convenience. A soft graph can also be represented by

$$G = \langle F, K, A\rangle = \{H(x) : x \in A\}.$$

Example 2.1. *Consider a crisp graph $G^* = (V, E)$ as shown in Fig. 2.1.*

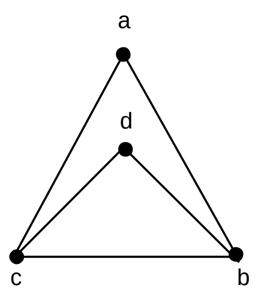

FIGURE 2.1. Simple graph G^*

Let $A = \{a, d\} \subseteq V$ and (F, A) be a soft set with its approximate function $F : A \to \mathscr{P}(V)$ defined by

$$F(x) = \{y \in V : xRy \Leftrightarrow d(x, y) \leq 1\}$$

for all $x \in A$. That is,
$F(a) = \{a, b, c\}$ and $F(d) = \{b, c, d\}$.
Let (K, A) be a soft set over E with its approximate function $K : A \to \mathscr{P}(E)$ defined by

$$K(x) = \{uv \in E : \{u, v\} \subseteq F(x)\}$$

for all $x \in A$. In other words, we have
$K(a) = \{ab, bc, ca\}$ and $K(d) = \{db, bc, cd\}$.
Thus, $H(a) = (F(a), K(a))$ and $H(d) = (F(d), K(d))$ are subgraphs of G^* as shown in Fig. 2.2.

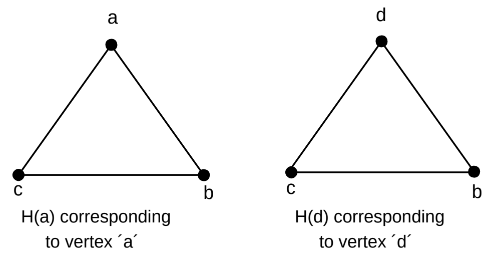

FIGURE 2.2. Subgraphs $H(a)$, $H(d)$

Hence, $G = \{H(a), H(d)\}$ is a soft graph of G^*. It is also called vertex-induced soft graph.

Example 2.2. *Consider a crisp graph $G^* = (V, E)$ such that $V = \{a, b, c, d\}$ and $E = \{ab, bc, cd, ad\}$. Let $A = \{e_1, e_2, e_3\}$ be a nonempty set of parameters. Then the subgraphs of G^* corresponding to parameters e_1, e_2 and e_3 are given below and shown in Fig. 2.3.*

$H(e_1) = (F(e_1), K(e_1)) = (\{a, b\}, \{ab\}),$
$H(e_2) = (F(e_2), K(e_2)) = (\{a, b, c\}, \{ab, bc\}),$
$H(e_3) = (F(e_3), K(e_3)) = (\{a, b, c, d\}, \{ad, , cd, cb\}).$

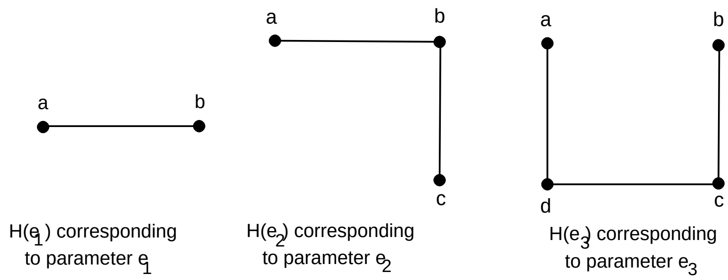

FIGURE 2.3. $H(e_1)$, $H(e_2)$, $H(e_3)$

Hence $G = \{H(e_1), H(e_2), H(e_3)\}$ is a soft graph of G^*. Tabular representation of soft graph G is given in Table. 1

TABLE 1. Tabular representation of a soft graph.

$A\backslash V$	a	b	c	d
e_1	1	1	0	0
e_2	1	1	1	0
e_3	1	1	1	1

$A\backslash E$	ab	bc	cd	da
e_1	1	0	0	0
e_2	1	1	0	0
e_3	0	1	1	1

Definition 2.2. *Let G^* be a simple graph and G be a soft graph of G^*. Then G is said to be regular soft graph if $H(x)$ is a regular graph for all $x \in A$. A soft graph G is called a regular soft graph of degree r if $H(x)$ is a regular graph of degree r for all $x \in A$.*

Example 2.3. *Consider an undirected graph G^* as shown in Fig. 2.4.*

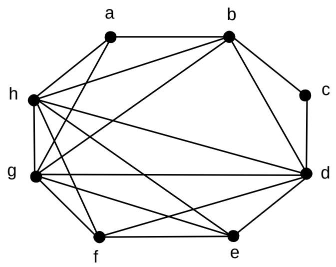

FIGURE 2.4. Undirected graph G^*

Let $A = \{a, c, f\}$. We define an approximate function $F : A \to \mathscr{P}(V)$ by

$$F(x) = \{y \in V : xRy \Leftrightarrow d(x, y) < rad(G^*)\}$$

for all $x \in A$. That is,
$F(a) = \{a, b, h, g\}$, $F(c) = \{b, c, d\}$, $F(f) = \{d, e, f, g, h\}$.
We define an approximate function $K : A \to \mathscr{P}(E)$ by

$$K(x) = \{uv \in E : \{u, v\} \subseteq F(x)\}$$

for all $x \in A$. That is,
$K(a) = \{ab, ah, ag, bh, hg, bg\}$, $K(c) = \{bc, cd, bd\}$ *and* $K(f) = \{de, ef, fg, gh, hd, he, hf, dg, df, ge\}$.
Thus, subgraphs $H(a) = (F(a), K(a))$, $H(c) = (F(c), K(c))$ and $H(f) = (F(f), K(f))$ are regular subgraphs of G^* shown in Fig. 2.5.

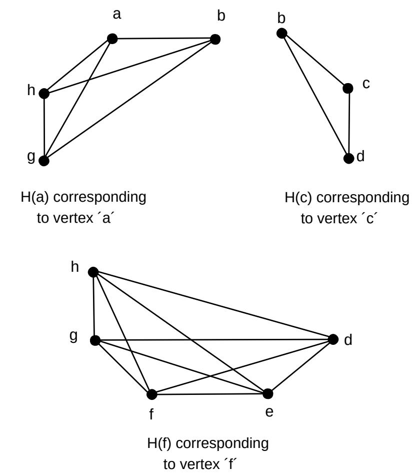

FIGURE 2.5. Subgraphs $H(a)$, $H(c)$ and $H(f)$

Hence G is a regular soft graph of G^*. That is, $G = \{H(a), H(c), H(f)\}$ is a regular soft graph of G^*.

Definition 2.3. *Let G^* be a simple graph and G be a soft graph of G^*. An edge e in G is said to be a soft bridge if its deletion disconnect the subgraph $H(x)$, $x \in A$.*

Example 2.4. *Consider a simple graph G^* as shown in Fig. 2.6.*

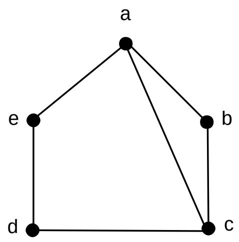

FIGURE 2.6. Simple graph G^*

Let $A = \{b, c\} \subseteq V$ and (F, A) be a soft set over V with approximate function $F : A \to \mathscr{P}(V)$ by

$$F(x) = \{y \in V : xRy \Leftrightarrow d(x, y) \leq rad(G^*)\}$$

for all $x \in A$. That is, $F(b) = \{a, b, c\}$ and $F(c) = \{a, b, c, d\}$.
Let (K, A) be a soft set over E with approximate function $K : A \to \mathscr{P}(E)$ by

$$K(x) = \{uv \in E : \{u, v\} \subseteq F(x)\}$$

for all $x \in A$. That is,
$K(b) = \{ab, bc, ca\}$ and $K(c) = \{ab, bc, ca, cd\}$.
The subgraphs $H(b)$ and $H(c)$ are shown in Fig.2.7.

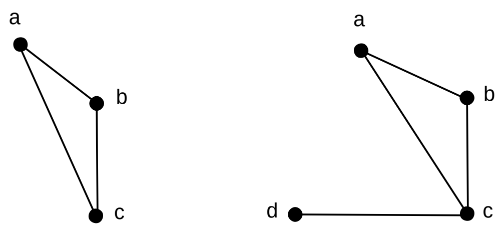

FIGURE 2.7. Subgraphs $H(b)$ and $H(c)$

Then G is a soft graph of G^*. Here, deletion of edge (c, d) in subgraph $H(c)$ disconnect the subgraph $H(c)$. Therefore, (c, d) is a soft bridge.

Definition 2.4. *Let G^* be a simple graph and G be a soft graph of G^*. A vertex v in G is said to be a soft cutvertex if its deletion disconnect the subgraph $H(x)$, $x \in A$.*

Example 2.5. *Consider an undirected graph G^* as shown in Fig. 2.8.*

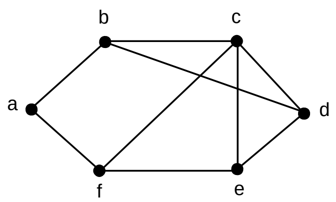

FIGURE 2.8. Undirected graph G^*

Let $A = \{b, f\}$ and (F, A) be a soft set over V with approximate function $F : A \to \mathscr{P}(V)$ by

$$F(x) = \{y \in V : xR\ y \Leftrightarrow d(x, y) < dia(G^*)\}$$

for all $x \in A$. Then $F(b) = \{a, b, c, d\}$ and $F(f) = \{a, c, e, f\}$.
Let (K, A) be a soft set over E with approximate function $K : A \to \mathscr{P}(E)$ by

$$K(x) = \{uv \in E : \{u, v\} \subseteq F(x)\}$$

for all $x \in A$. That is,
$K(b) = \{ab, bc, cd, bd\}$, $K(f) = \{af, fe, ce, fc\}$.
The subgraphs $H(b)$ and $H(f)$ are shown in Fig.2.9.

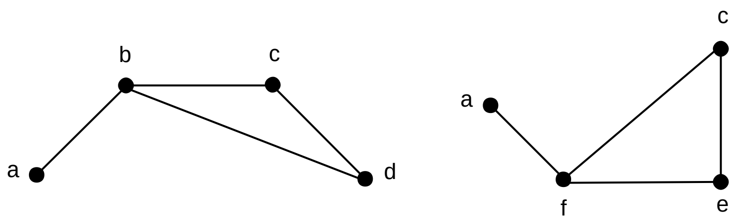

FIGURE 2.9. Subgraphs $H(b)$ and $H(f)$

Clearly, subgraphs $H(b)$ and $H(f)$ are subgraphs of G^*. Thus G is a soft graph of G^*. Here, the deletion of vertex b in subgraph $H(b)$ and f in subgraph $H(f)$ disconnect the subgraphs. Therefore, b, f are soft cutvertices.

Theorem 2.1. A regular soft graph with cardinality of $F(x)$, $|F(x)| \geq 3$ for all $x \in A$ does not have a soft bridge. Hence it does not have a soft cutvertex.

Proof. Suppose that G is a regular soft graph. Then $H(x)$ is a regular graph for all $x \in A$. Since $|F(x)| \geq 3$ for all $x \in A$, the removal of any edge of $H(x)$ does not disconnect the subgraph. Therefore, G has no soft bridge. Hence it does not have a soft cutvertex. □

Proposition 2.1. A regular soft graph of degree k where $k > 0$ with $|F(x)| \geq 3$ for all $x \in A$ does not have an end vertex (a vertex of degree 1).

Definition 2.5. *Let G^* be a simple graph and G be a soft graph of G^*. Then G is said to be soft tree if $H(x)$ is a tree for all $x \in A$. Let G^* be a simple graph and G be a soft graph of G^*. Then G is a soft cycle if $H(x)$ is cycle for all $x \in A$.*

Example 2.6. *Consider a simple graph G^* as shown in Fig. 2.10.*

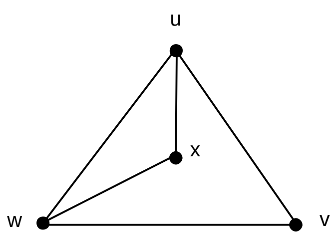

FIGURE 2.10. Simple graph G^*

Let $A = \{v, x\} \subseteq V$ *and* (F, A) *be a soft set over* V *with approximate function* $F : A \to \mathscr{P}(V)$ *by*

$$F(x) = \{y \in V : xRy \Leftrightarrow d(x, y) \leq 1\}$$

for all $x \in A$.
That is, $F(v) = \{u, v, w\}$ *and* $F(x) = \{u, w, x\}$.
Let (K, A) *be a soft set over* E *with approximate function* $K : A \to \mathscr{P}(E)$ *by*

$$K(x) = \{uv \in E : \{u, v\} \subseteq F(x)\}$$

for all $x \in A$. *That is,*
$K(v) = \{uv, vw, wu\}$, $K(x) = \{ux, xw, wu\}$.
Subgraphs $H(v)$ *and* $H(x)$ *are shown in Fig. 2.11.*

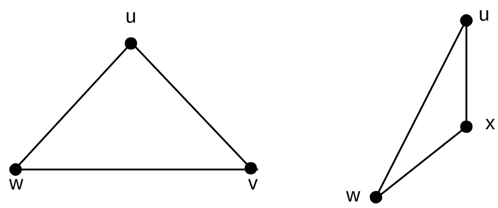

FIGURE 2.11. Subgraphs $H(v)$ and $H(x)$

Then G *is a soft graph of* G^*. *Here,* $H(x)$ *is cycle for all* $x \in A$. *Hence* G *is a soft cycle.*

Theorem 2.2. If G is a soft cycle then G is not soft tree.

Proof. Let G be a soft cycle. Then $H(x)$ is cycle for all $x \in A$. Since a tree contains no cycle so $H(x)$ is not a tree for all $x \in A$. Therefore, G is not a soft tree. □

Remark 2.1. *The converse of above theorem is not true in general i.e., if* G *is not a soft tree then* G *need not be a soft cycle.*

The following example illustrate it.

Example 2.7. *Consider a graph* G^* *as shown below in Fig. 2.6.*
Let $A = \{b, d\} \subseteq V$ *and* (F, A) *be a soft set over* V *with approximate function* $F : A \to \mathscr{P}(V)$ *by*

$$F(x) = \{y \in V : xRy \Leftrightarrow d(x, y) \leq 1\}$$

for all $x \in A$. *That is,* $F(b) = \{a, b, c\}$ *and* $F(d) = \{c, d, e\}$.
Let (K, A) *be a soft set over* E *with approximate function* $K : A \to \mathscr{P}(E)$ *by*

$$K(x) = \{uv \in E : \{u, v\} \subseteq F(x)\}$$

for all $x \in A$. *That is,*
$K(b) = \{ab, bc, ca\}$ *and* $K(d) = \{ed, dc\}$.

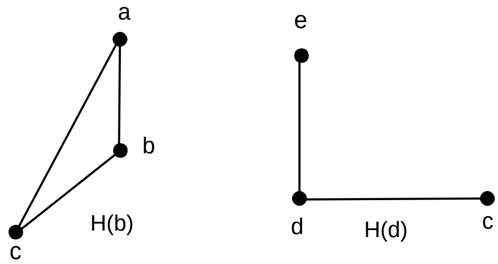

FIGURE 2.12. Subgraphs $H(b)$ and $H(d)$

Then G is a soft graph of G^*. Here, $H(b)$ is not a tree. Therefore, G is not a soft tree. But G is not a soft cycle.

Proposition 2.2. If G is a soft cycle then G is a regular soft graph of degree 2.
OR every soft cycle is a regular soft graph.

Proof. Suppose that G is a soft cycle. Then $H(x)$ is a cycle graph for all $x \in A$. Since a cycle graph is a closed path and each vertex has degree 2 therefore, it is a regular graph of degree 2. So $H(x)$ is a regular graph of degree 2 for all $x \in A$. Hence G is a regular soft graph of degree 2. □

Proposition 2.3. Let G be a regular soft graph and $H(x)$ is a cycle for all $x \in A$. Then G is a soft cycle.

Proposition 2.4. Let graph G^* be a complete graph. Then every soft graph of G^* is a regular soft graph of G^*.

Proof. Let G be a soft graph of G^*. Then $H(x)$ is a complete subgraph of G^* for all $x \in A$ as every induced subgraph of a complete graph is complete and every complete graph is regular. Therefore, G is a regular soft graph of G^*. □

Remark 2.2. *The converse of the above proposition is not true in general. That is, if G is regular soft graph of G^* then G^* need not be a complete graph.*

Example 2.8. *Consider a simple graph G^* as shown in Fig.2.13.*

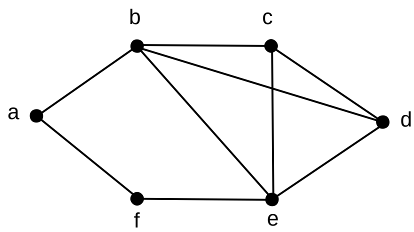

FIGURE 2.13. Simple Graph

Let $A = \{c, d\} \subseteq V$ and (F, A) be a soft set over V with approximate function $F : A \to \mathscr{P}(V)$ by

$$F(x) = \{y \in V : xRy \Leftrightarrow d(x, y) = 1\}$$

for all $x \in A$. That is, $F(c) = \{b, d, e\}$ and $F(d) = \{b, c, e\}$.
Let (K, A) be a soft set over E with approximate function $K : A \to \mathscr{P}(E)$ by

$$K(x) = \{uv \in E : \{u, v\} \subseteq F(x)\}$$

for all $x \in A$. That is,
$K(c) = \{bd, de, eb\}$ and $K(d) = \{bc, ce, eb\}$. Subgraphs $H(c)$ and $H(d)$ are shown in Fig.2.14.

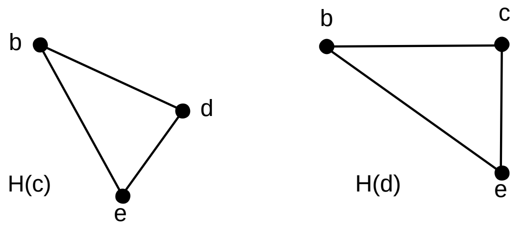

FIGURE 2.14. Subgraphs $H(c)$ and $H(d)$

Then G is a soft graph of G^*. Here, $H(x)$ is a regular graph for all $x \in A$. Therefore, G is a regular soft graph of G^*. But G^* is a not complete graph.

Theorem 2.3. A soft graph G of G^* is a regular soft graph if and only if $H(x)$ is regular graph for all $x \in A$.

Proof. Suppose G is a regular soft graph of G^*. Then clearly $H(x)$ is a regular graph for all $x \in A$.
Conversely, suppose that $H(x)$ is a regular graph of G^* for all $x \in A$. Then G is a regular soft graph of G^*. □

Definition 2.6. *Let G^* be a simple graph and G be a soft graph of G^*. Then G is said to be soft complete graph if $H(x)$ is a complete graph for all $x \in A$.*

Proposition 2.5. Every complete soft graph G of G^* is regular soft graph.

Proof. Let G be a soft complete graph of G^*. Then $H(x)$ is a complete graph for all $x \in A$. Since every complete graph is regular. So $H(x)$ is a regular graph for all $x \in A$. Therefore, G is a regular soft graph. □

Proposition 2.6. Let the graph G^* be a regular. Then every soft graph of G^* may not be a regular soft graph.

Example 2.9. *Consider the regular graph $G^* = (V, E)$ such that $V = \{a, b, c, d\}$ and $E = \{ab, bc, cd, ad\}$. Let $A = \{a, c\} \subseteq V$ and (F, A) be a soft set over V with approximate function $F : A \to \mathscr{P}(V)$ by*

$$F(x) = \{y \in V : xRy \Leftrightarrow d(x, y) < dia(G^*)\}$$

for all $x \in A$. Then $F(a) = \{a, b, d\}$ and $F(c) = \{b, c, d\}$.
Let (K, A) be a soft set over E with approximate function $K : A \to \mathscr{P}(E)$ by

$$K(x) = \{uv \in E : \{u, v\} \subseteq F(x)\}$$

for all $x \in A$. That is,
$K(a) = \{ab, ad\}$ and $K(c) = \{bc, cd\}$. Subgraphs $H(a)$ and $H(c)$ are shown in Fig.2.15.

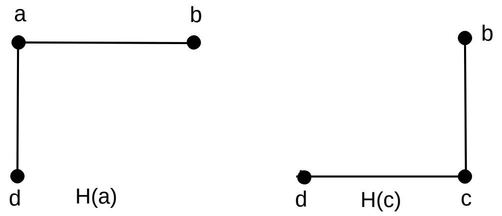

FIGURE 2.15. Subgraphs $H(a)$ and $H(c)$

Then $G = \{H(a), H(c)\}$ is a soft graph of G^*. Here, $H(x)$ is not a regular graph for all $x \in A$. Therefore, G is not a regular soft graph of G^*. But G is a regular graph.

Proposition 2.7. If G is a regular soft graph of G^* then G^c is a regular soft graph of G^*.

Proof. Suppose that G is a regular soft graph of G^*. Then $H(x)$ is a regular graph for all $x \in A$. Since the complement of a regular graph is regular. So the complement of subgraph $H(x)$, $H^c(x)$ is a regular graph for all $x \in A$. Therefore, G^c is a regular soft graph of G^*. □

Theorem 2.4. If $G = (G^*, F, K, A)$ is a complete soft graph of G^* then every soft subgraph $G_1 = (G^*, F_1, K_1, B)$ of complete soft graph is regular soft graph.

Proof. Suppose that G_1 is a soft subgraph of G. Then by definition of soft subgraph, $B \subseteq A$ and $H_1(x)$ is a subgraph of $H(x)$ for all $x \in B$. Since G is a soft complete graph then subgraph $H(x)$ is a complete subgraph for all $x \in A$. Since $H_1(x)$ is a subgraph of $H(x)$ so $H_1(x)$ is a regular graph for all $x \in B$, as each subgraph of a complete graph is complete and every complete graph is regular. Thus, G_1 is a regular soft subgraph of G. □

Definition 2.7. *Let G^* be a simple graph and G be a soft graph of G^*. Then G is said to be irregular soft graph if $H(x)$ is an irregular graph for all $x \in A$.*

Example 2.10. *Consider a simple graph G^* as shown in Fig. 2.16.*

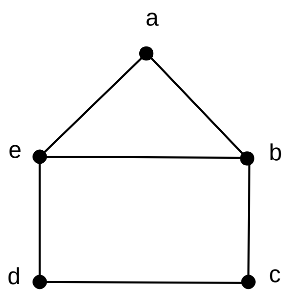

FIGURE 2.16. Simple graph G^*

Let $A = \{c, e\}$. We define an approximate function $F : A \to \mathscr{P}(V)$ by

$$F(x) = \{y \in V : xRy \Leftrightarrow d(x, y) < rad(G^*)\}$$

for all $x \in A$. That is, $F(c) = \{b, c, d\}$ and $F(e) = \{a, b, d, e\}$.
We define an approximate function $K : A \to \mathscr{P}(E)$ by

$$K(x) = \{uv \in E : \{u, v\} \subseteq F(x)\}$$

for all $x \in A$. *That is,*
$K(c) = \{bc, cd\}$ *and* $K(e) = \{ab, ae, eb, ed\}$. *Subgraphs* $H(c)$ *and* $H(e)$ *are shown in Fig.2.17.*

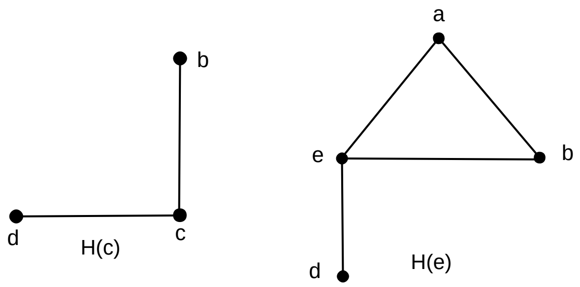

FIGURE 2.17. Subgraphs $H(c)$ and $H(e)$

Then $G = \{H(c), H(e)\}$ *is a soft graph of* G^*. *Here,* $H(x)$ *is an irregular graph for all* $x \in A$. *Therefore,* G *is an irregular soft graph.*

Definition 2.8. *Let* G^* *be an undirected graph and* G *be a soft graph of* G^*. *Then* G *is said to be neighbourly irregular soft graph if* $H(x)$ *is a neighbourly irregular graph for all* $x \in A$.

Example 2.11. *Consider an undirected graph* G^* *as shown in Fig. 2.18.*

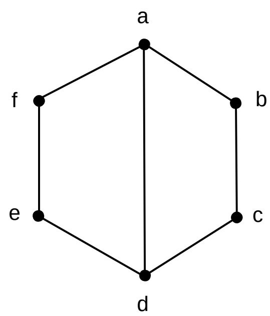

FIGURE 2.18. undirected graph G^*

Let $A = \{a, c, d\}$. *We define an approximate function* $F : A \to \mathscr{P}(V)$ *by*

$$F(x) = \{y \in V : xRy \Leftrightarrow d(x, y) < rad(G^*)\}$$

for all $x \in A$. *That is,* $F(a) = \{a, b, d, f\}$, $F(c) = \{b, c, d\}$ *and* $F(d) = \{a, c, d, e\}$.
We define an approximate function $K : A \to \mathscr{P}(E)$ *by*

$$K(x) = \{uv \in E : \{u, v\} \subseteq F(x)\}$$

for all $x \in A$. *That is,*
$K(a) = \{ab, ad, af\}$, $K(c) = \{bc, cd\}$ *and* $K(d) = \{db, da, df\}$.
Subgraphs $H(a)$, $H(c)$ *and* $H(d)$ *are shown in Fig. 2.19.*

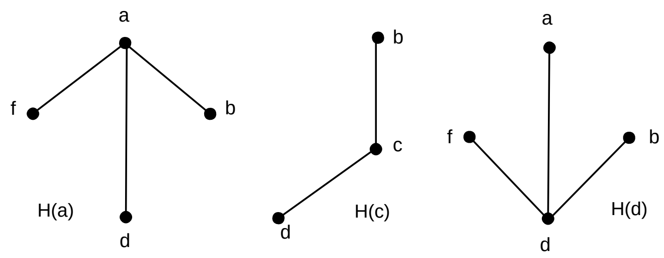

FIGURE 2.19. Subgraphs $H(a)$, $H(c)$ and $H(d)$

Then G is a soft graph of G^*. Here, $H(x)$ is neighbourly irregular graph for all $x \in A$. Therefore, $G = \{H(a), H(c), H(d)\}$ is a neighbourly irregular soft graph of G^*.

Definition 2.9. *Let G^* be a graph and G be a soft graph of G^*. Then G is said to be highly irregular soft graph if $H(x)$ is a highly irregular graph for all $x \in A$.*

Example 2.12. *Consider a nontrivial graph G^* as shown in Fig .2.20.*

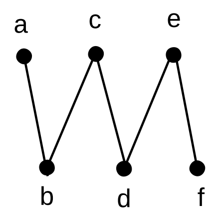

FIGURE 2.20. Nontrivial graph G^*

Let $A = \{b, e\}$. We define an approximate function $F : A \to \mathscr{P}(V)$ by $F(x) = \{y \in V : xRy \Leftrightarrow d(x, y) < rad(G^*)\}$ for all $x \in A$. That is, $F(b) = \{a, b, c, d\}$ and $F(e) = \{c, d, e, f\}$. We define an approximate function $K : A \to \mathscr{P}(E)$ by $K(x) = \{uv \in E : \{u, v\} \subseteq F(x)\}$ for all $x \in A$. That is,
$K(b) = \{ab, bc, cd\}$ and $K(e) = \{cd, de, ef\}$.
Subgraphs $H(b)$ and $H(e)$ are shown in Fig 2.21.

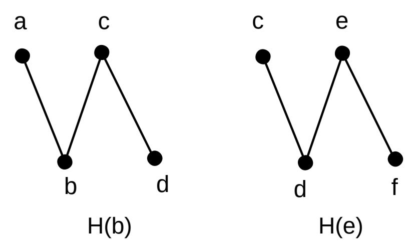

FIGURE 2.21. Subgraphs $H(b)$ and $H(e)$

Then G is a soft graph of G^*. Here, $H(x)$ is highly irregular graph for all $x \in A$. Therefore, $G = \{H(b), H(e)\}$ is a highly irregular soft graph of G^*.

Remark 2.3. *A highly irregular soft graph may not be a neighbourly irregular soft graph.*

In above example 2.12 the adjacent vertices b and c in subgraph $H(b)$ and the adjacent vertices d and e in subgraph $H(e)$ have same degrees.Therefore, $H(x)$ is not neighbourly irregular graph for all $x \in A$. Hence G is not neighbourly irregular soft graph. But it is highly irregular soft graph.

Remark 2.4. *A neighbourly irregular soft graph may not be a highly irregular soft graph.*

Example 2.13. *Consider a simple graph G^* as shown in Fig.2.22.*

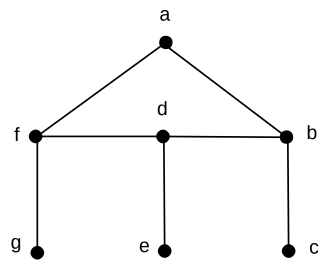

FIGURE 2.22. Simple graph G^*

Let $A = \{a, f\}$. We define an approximate function $F : A \to \mathscr{P}(V)$ by $F(x) = \{y \in V : xRy \Leftrightarrow d(x, y) \leq 1\}$. Then $F(a) = \{a, b, f\}$, $F(f) = \{a, d, g, f\}$. We define an approximate function $K : A \to \mathscr{P}(E)$ by $K(x) = \{uv \in E : \{u, v\} \subseteq F(x)\}$ for all $x \in A$. That is, $K(a) = \{ab, af\}$, $K(f) = \{fa, fd, fg\}$. Subgraphs $H(a)$ and $H(f)$ are shown in Fig. 2.23.

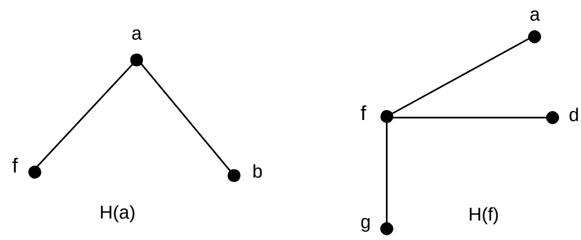

FIGURE 2.23. Subgraphs $H(a)$ and $H(f)$

Here, the adjacent vertices in subgraphs $H(a)$ and $H(f)$ have distinct degree so $H(x)$ is neighbourly irregular for all $x \in A$ but consider a vertex a in subgraph $H(a)$ which is adjacent to the vertices b and f with same degree and a vertex f in subgraph $H(f)$ which is adjacent to the vertices a, d, g with same degree. Therefore, $H(x)$ is not highly irregular for all $x \in A$. Hence G is a neighbourly irregular soft graph but it is not highly irregular soft graph.

We have seen from the above examples that there is no relation between highly irregular soft graphs and neighbourly irregular soft graphs. However, a necessary and sufficient condition of a soft graph G to be both highly irregular and neighbourly irregular is provided in the following theorem.

Theorem 2.5. Let G^* be a simple graph and G be a soft graph of G^*. Then G is both highly irregular soft graph and neighbourly irregular soft graph if and only if the degrees of all the vertices are distinct.

Proof. Let G be a soft graph of G^*. Suppose that G is highly irregular and neighbourly irregular soft graph. Then each vertex in $H(x)$ is adjacent to the vertices with distinct degree for all $x \in A$ since G is highly irregular soft graph. Also in $H(x)$ no two adjacent vertices have same degree for all $x \in A$ since G is neighbourly irregular soft graph. Hence the degrees of all vertices of G are distinct.
Conversely, suppose that all the vertices of G have distinct degrees. This means that every two adjacent vertices have distinct degrees and to every vertex the adjacent vertices have distinct degrees. Hence G is highly irregular soft graph and neighbourly irregular soft graph. □

Proposition 2.8. Let soft graph G_1 of G^* be neighbourly irregular , the soft subgraph G_2 of G_1 may or may not be neighbourly irregular soft graph.

Example 2.14. *Consider an undirected graph G^* as shown in Fig.2.24.*

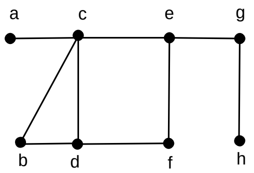

FIGURE 2.24. undirected graph G^*

Let $A = \{c, g, h\}$ and $B = \{g, c\}$. We define an approximate function $F_1 : A \to \mathscr{P}(V)$ by

$$F_1(x) = \{y \in V : xRy \Leftrightarrow d(x, y) < rad(G^*)\}$$

for all $x \in A$. Then $F_1(c) = \{a, b, c, d, e, f, g\}$, $F_1(g) = \{c, e, f, g, h\}$, $F_1(h) = \{e, g, h\}$. We define an approximate function $K_1 : A \to \mathscr{P}(E)$ by

$$K_1(x) = \{uv \in E : \{u, v\} \subseteq F_1(x)\}$$

for all $x \in B$. Then $K_1(c) = \{ac, cb, cd, bd, ce, df, eg, ef\}$, $K_1(g) = \{ce, eg, ef, gh\}$ and $K_1(h) = \{eg, gh\}$. Subgraphs $H_1(c)$, $H_1(g)$ and $H_1(h)$ are shown in Fig. 2.25.

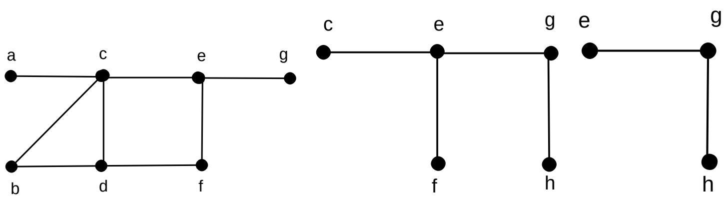

FIGURE 2.25. Subgraphs $H_1(c)$, $H_1(g)$ and $H_1(h)$

Therefore, $G_1 \in \mathcal{SG}(G^*)$. Here $H_1(x)$ is neighbourly irregular for all $x \in A$. Hence G_1 is neighbourly irregular soft graph.

We define an approximate function $F_2 : B \to \mathscr{P}(V)$ *by* $F_2(x) = \{y \in V : xRy \Leftrightarrow d(x, y) < 2\}$ *for all* $x \in B$*. Then* $F_2(c) = \{a, b, c, d, e\}$, $F_2(g) = \{e, g, h\}$*. We define an approximate function* $K_2 : A \to \mathscr{P}(E)$ *by* $K_2(x) = \{uv \in E : \{u, v\} \subseteq F_2(x)\}$ *for all* $x \in B$*. That is,* $K_2(c) = \{ac, cb, cd, bd, ce\}$, $K_2(g) = \{eg, gh\}$*. Subgraphs* $H_2(c) = (F_2(c), K_2(c))$ *and* $H_2(g) = (F_2(g), K_2(g))$ *are shown in Fig. 2.26.*

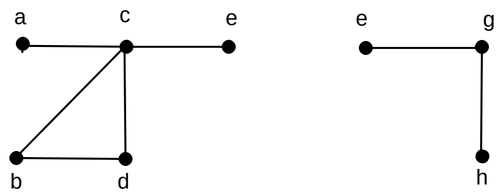

FIGURE 2.26. Subgraphs $H_2(c)$ and $H_2(g)$

Therefore, $G_2 \in \mathcal{SG}(G^*)$*. Here,* $B \subseteq A$ *and* $H_2(x)$ *is a subgraph of* $H_1(x)$ *for all* $x \in B$*. Therefore,* G_2 *is a soft subgraph of* G_1*. Here,* $H_2(c)$ *is not neighbourly irregular,* $c \in A$*. So* G_2 *is not neighbourly irregular soft subgraph.*

Proposition 2.9. If soft graph G of G^* is neither neighbourly irregular nor highly irregular then G is a soft cycle.

Example 2.15. *Consider a simple graph* G^* *as shown in Fig.2.27.*

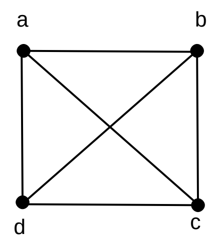

FIGURE 2.27. Simple graph G^*

Let $A = \{a, c\}$*. We define an approximate function* $F : A \to \mathscr{P}(V)$ *by*

$$F(x) = \{y \in V : xRy \Leftrightarrow d(x, y) = 1\}$$

for all $x \in A$*. That is,* $F(a) = \{b, c, d\}$ *and* $F(c) = \{a, b, d\}$*.*
We define an approximate function $K : A \to \mathscr{P}(E)$ *by*

$$K(x) = \{uv \in E : \{u, v\} \subseteq F(x)\}$$

for all $x \in A$*. That is,* $K(a) = \{bc, cd, db\}$ *and* $K(c) = \{ab, bd, da\}$*.*
Subgraphs $H(a)$ *and* $H(c)$ *are shown in Fig. 2.28.*

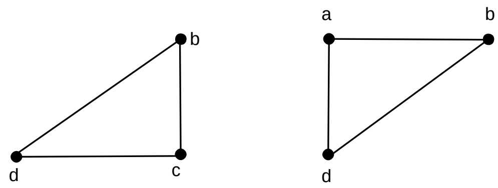

FIGURE 2.28. Subgraphs $H(a)$ and $H(c)$

Then G is a soft graph. Here, $H(x)$ is neither neighbourly irregular nor highly irregular, but it is a cycle for all $x \in A$. Hence G is neither neighbourly irregular nor highly irregular soft graph but it is a soft cycle.

Proposition 2.10. A complete soft graph G is not neighbourly irregular soft graph.

As in Example 2.15 G is a soft complete graph since $H(x)$ is a complete graph for all $x \in A$ but it is not neighbourly irregular soft graph as $H(x)$ is not neighbourly irregular for all $x \in A$.

REFERENCES

[1] *M. Akram*, Bipolar fuzzy graphs with applications, Knowledge Based System, **39**(2-13), $1-8$.

[2] *M. Akram and S. Nawaz*, On fuzzy soft graphs, Itanlian Journal of Pure and Applied Mathematics, **34**(2015), $463-480$.

[3] *P. Bhattacharya*, Some remarks on fuzzy graphs, Pattern Recognition Letter, **6**(1987), $297-302$.

[4] *K. R. Bhutani and A. Battou*, On M-strong fuzzy graphs, Information Sciences, **155**(12)(2003), $103-109$.

[5] *A.N. Gani and S.R. Latha*, On irregular fuzzy graphs, Applied Mathematical Sciences, **6**(2012), $517-523$.

[6] *A. Kauffman*, Introduction a la Theorie des Sous-emsembles Flous, Masson et Cie, **1** (1973).

[7] *J. Mordeson and P.S. Nair*, Fuzzy graphs and fuzzy hypergraphs, Physica Verlag, Heidelberg 1998, Second Edition, 2001.

[8] *D.A. Molodtsov*, Soft set theory-first results, Computers and Mathematics with Applications, **37** (1999), $19-31$.

[9] *A. Rosenfeld*, Fuzzy graphs, in: L.A. Zadeh, K.S. Fu and M. Shimura (Eds.), *Fuzzy Sets and Their Applications*, Academic Press, New York,(1975), $77-95$.

[10] *L.A. Zadeh*, Fuzzy sets, Information and Control, **8** (1965), $378-352$.