EFFECT OF SHAPE FACTOR UPON STRESS CONCENTRATION FACTOR IN ISOTROPIC/ORTHOTROPIC PLATES WITH CENTRAL HOLE SUBJECTED TO TENSION LOAD

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Determining stress concentration factors is of practical importance for many engineering structures because geometric discontinuities are frequently the site of failure. In this paper, the work is carried out to analyze the stress concentration factor, around circular central hole in isotropic and orthotropic rectangular plates, subjected to tension load by using analytical and finite element method. Different plate hole diameter - width ratios (d/W) have been considered to provide global and net stress concentration factors. A comparison between FE method and analytical results obtained with Heywood and Howland formulations was carried out. The effect of length/width ratio upon stress concentration factor was also investigated. Global and net SCF were determined for a group of isotropic and orthotropic plates with various ratios (L/W), while (d/W) ratio was changed from 0.1 to 0.9. For both Isotropic and orthotropic plates, analytical formulations of Howland and Heywood are much more valid for a length-to-width ratio equal to 2.

Keywords: Stress concentration factor, isotropic plates, orthotropic plates, finite element analysis.

1. Introduction

Isotropic and orthotropic plates with circular holes under tensile loading have found widespread applications in various fields of engineering such as aeronautic, marine and automobile industry [1-3]. Stress concentrations around holes have big practical importance because they are the main cause of failure. In addition, crack initiation happens near the stress concentration region. The stress concentration near a geometric discontinuity like a hole is estimated by a parameter called stress concentration factor *(SCF)*.The coefficient of stress concentration (K_t) in a plate is defined as the ratio of the actual maximum stress

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(σ) acting on the zone of discontinuity to the nominal stress (σ_{nom}) applied to the plate extremity.

Several analytical, numerical and experimental researches have been performed on stress concentration. Heywood [4], Pilkey [5] and Peterson [6] studied different forms of stress concentration for isotropic materials and plates with a large range of cutouts. Muskhelishvili [7], Timoshenko and Goodier [8] examined the phenomena of stress concentration around holes for infinite width plates and presented classical solution for bi-dimensional analysis. Howland [9] analyzed the stress concentration in plates subjected to tension and proposed a formulation for calculation of the stress concentration factors of an isotropic plate with circular hole. Hwai Chung and Bin [10] developed an empirical model to calculate the stress concentration factor in isotropic /orthotropic plates with circular holes. Jain and Mittal [11] investigated the effect of hole's diameter to plate width upon stress concentration factor and deflection in isotropic, orthotropic and laminated composite plates under different transverse static loading condition. Troyani *et al.* [12] have determined the in-plane stress concentration factors for short rectangular plates with circular holes subjected to tensile field using finite element method. Toubal *et al.* [13] evaluated stress concentration in a circular hole in composite plate. Mittal and Jain [14] studied the effect of fibre orientation on stress concentration factor in fibrous plate with central circular hole under transverse static loading by using two dimension finite element methods. Hashem et al. [15] performed a numerical analysis on stress concentration factor for randomly oriented discontinuous fiber Laminas with circular/square holes. Mhallah and Bouraoui [16] presented experimental analysis based on digital image correlation to determine the stress concentration factor for orthotropic and isotropic materials. Lekhnitskii [17] *et al.* and Tan [18] proposed various formulations to investigate stress concentration for infinite and finite orthotropic plates. Enayat and David L [19] developed an efficient boundary element method (BEM) for use in the analysis of loaded holes in composite structures. Nicholas and Christoph studied the stress concentration factors for cylindrically orthotropic plates [20].

The main objective of this study is to evaluate the stress concentration factors for isotropic and orthotropic rectangular plates with central hole subjected to axial tension (Figure1). The investigation has been carried out by using analytical and finite element methods. A comparison between the results obtained with Heywood and Howland formulations and finite element method has been done. In addition, the purpose of this work is to study the effect of length/width ratio (shape factor *L/W*) for isotropic/orthotropic plates under stress concentration factor. From the above review, it can be noted that no researches have been conducted studies on this particular subject.

2. Theoretical stress concentration factor for finite width plates with a hole

According to Peterson [6], the stress concentration factor is defined as the ratio of the maximum stress under the actual loads in the zone of singularity (hole, notch) to the nominal stress in the section:

$$K_T = \frac{\sigma_{max}}{\sigma_{nom}} \tag{1}$$

where σ_{max} is evaluated by numerical methods or by analytical approaches in the case of simple geometries. It can be also estimated using experimental methods such as photoelasticity or digital image correlation. On the other hand, σ_{nom} is computable with the aid of strength material formulas.

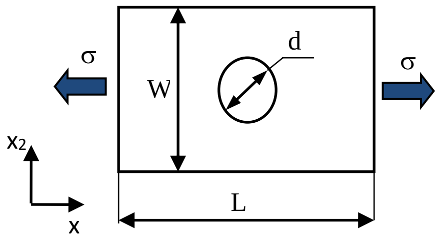

Fig.1. Plate with central hole subjected to uniaxial tension

According to the investigation of Heywood [4] on a finite rectangular orthotropic plate with center hole under action of unidirectional axial load, the *SCF* is given by the following equation:

$$\frac{K_{Tg}^{\alpha}}{K_{Tg}} = \frac{3\left(1-\frac{d}{W}\right)}{2+\left(1-\frac{d}{W}\right)^3} + \frac{1}{2}\left(\frac{d}{W}M\right)^8\left(K_T^{\alpha}-3\right)\left(1-\left(\frac{d}{W}M\right)^3\right) \tag{2}$$

where K_{Tg}^{α} and K_{Tg} are respectively the stress concentration factor in infinite and finite plate. *M*, called the magnification factor, is only a function of the ratio *(d/W)*.

$$M^2 = \sqrt{1-8\left(\frac{3\left(1-\frac{d}{W}\right)}{2+\left(1-\frac{d}{W}\right)^3}-1\right)} \Big/ 2\left(\frac{d}{W}\right)^2 \tag{3}$$

Equation 2 can be used for both orthotropic and isotropic finite width plates with a central circular hole subjected to tension load for *(d/W)* up to 0.9 [18-21].

Leknitskii [22] introduced an expression of the stress concentration factor K_t, for infinite orthotropic plates with circular holes K_T^{α} as the form:

$$K_T^{\alpha} = 1 + \sqrt{2\left(\left(\sqrt{\frac{E_{11}}{E_{22}}}\right) - \nu_{12}\right) + \frac{E_{11}}{G_{12}}} \quad (4)$$

where E_{11} and E_{22} are elasticity moduli on the main directions, G_{12} is the in-plane shear modulus and ν_{12} is the Poisson ratio. Infinite stress concentration factor equal to 3 for an isotropic plate with circular hole simplifies equation 2 to:

$$\frac{K_{Tg}^{\alpha}}{K_{Tg}} = \frac{3\left(1 - \frac{d}{W}\right)}{2 + \left(1 - \frac{d}{W}\right)^3} \quad (5)$$

Another alternative to calculate the stress concentration factor is the application of the average stress across the net section taking into account the presence of the hole.

$$\sigma_{avg} = \frac{\sigma_{nom}}{1 - \frac{d}{W}} \quad (6)$$

In this case, the stress concentration factor is known as the net stress concentration factor K_{Tn} and can be related to the global stress concentration factor K_{Tg} by [18-21]:

$$K_{Tn} = K_{Tg}\left(1 - \frac{d}{W}\right) \quad (7)$$

The stress concentration factor using Heywood formulation will be then:

$$K_{Heywood} = K_{Tg}\left(1 - \frac{d}{W}\right) \quad (8)$$

An expression of the global stress concentration factor, for isotropic rectangular plate with central circular hole under uniaxial tension is given by Howland [9] as:

$$K_{Tg} = 0.284 + \frac{2}{\left(1 - \frac{d}{W}\right)} - 0.6\left(1 - \frac{d}{W}\right) + 1.32\left(1 - \frac{d}{W}\right)^2 \quad (9)$$

And the net stress concentration factor is:

$$K_{Howland} = K_{Tg}\left(1 - \frac{d}{W}\right) \quad (10)$$

3. Description of the problem

To study the stress concentration factor for isotropic and orthotropic plates, we used a rectangular plate with central hole of diameter *d*. For both cases the length, width and thickness of the plate were equal to 200 mm, 100 mm and 1 mm, respectively. The plates are subjected to unidirectional tensile load. The opening diameter to width ratio (d/W) is changed from 0.1 to 0.9.The global and the net stress concentration factor for isotropic and orthotropic plates are calculated with Howland and Heywood formulations and compared with FE results. In the second step of this investigation, we examined the effect of the shape factor on *SCF* for different values of (d/W) ratio varying from 0.1 to 0.9.The dimensionless variable ratios employed in the analyses covered $L/W = 1$, 1.3 ,1.5 ,1.7, 2, 3 and 5; totaling 144 simulation cases.

4. Finite element analysis

The Finite element method (FEM) is a powerful computational technique widely used for numerical simulation and optimization of structural geometry, especially when dealing with stress raisers or concentrators [23-24]. In this investigation, the geometric and FE model is carried out using the ABAQUS software [25]. The element 'S8R', defined by eight nodes was employed, because quadratic elements are more effective in capturing stress concentrations. The mesh is refined near the hole in order to have a steady value of the maximum stress. Figure 2 provides the example of the meshed model for $d/W = 0.2$ .The plate is fixed at one end and a tensile load of 100 MPa is applied on the other one. The plate geometry and the boundary conditions used are shown in Figure 3.

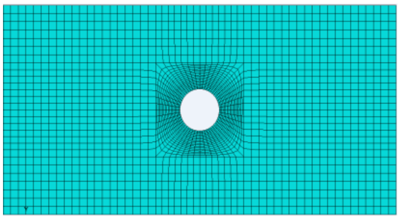

Fig. 2.Typical example of finite element mesh for $d/W = 0.2$

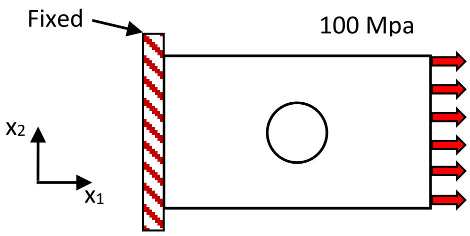

Fig.3. Boundary conditions and loads

5. Results and discussion

5.1 Isotropic plate

Since the *SCF* is independent of mechanical characteristics of the plate, the material chosen for simulation is the ordinary steel. The plate has elastic modulus $E = 210000$ MPa, Poisson's ratio $\nu = 0.3$ and mass density $\rho = 7800$ kg/m^3. For various (d/W) ratios, global and net stress concentration factors for isotropic plate with central hole are calculated using Heywood and Howland formulations (Equations 2, 8, 9 and 10) and compared with FE results. Table 1 show global and net stress concentration factors (K_{Tg} and K_{Tn}) obtained by FE and analytical method. The relative error in this table is estimated by the relation:

$$\varepsilon = \frac{SCF_{the} - SCF_{FE}}{SCF_{the}} \tag{11}$$

where SCF_{th} and SCF_{FE} are, respectively, the stress concentration factor calculated with analytical and FE method. Note that infinite stress concentration factor equals to 3 for isotropic materials with circular hole. In each computation step, global and net stress concentration factors are obtained by dividing maximum stress concentration extracted from ABAQUS software by the nominal and average stress. Maximum stress distribution and stress concentration around the hole with $d/W = 0.2$ are shown in Figure 4. As it can be seen from Table 1, the agreement between results obtained with Heywood-Howland formulations and the finite element method, for isotropic plate is very good. Figures 5 and 6, show respectively, the graphical representation of the numerical (from ABAQUS) and theoretical values of global and net stress concentration factors (Heywood and Howland formulations).

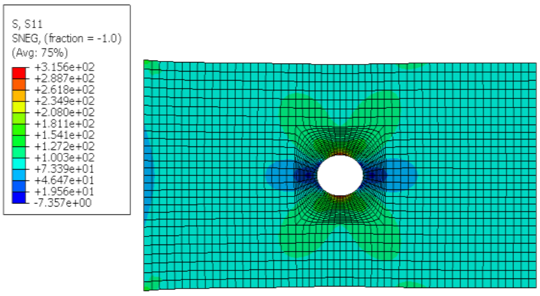

Fig. 4.Stress distribution for isotropic plate with circular hole $d/W = 0.2$

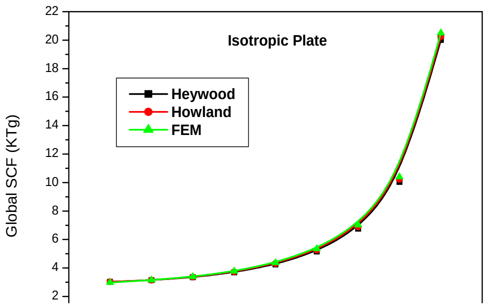

Fig.5.Global stress concentration factor for isotropic plate with central circular hole using different methods

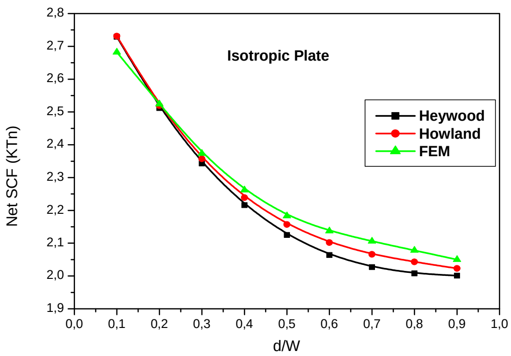

Fig. 6.Net stress concentration factor for isotropic plate with central circular hole using different methods

Table 1

Global and net stress concentration factor for isotropic plate with circular hole using Heywood, Howland and FEM

	Heywood		Howland		FEM				Error	
d/w	K_{Tg}	K_{Hey}	K_{Tg}	K_{How}	σ_{avg}	σ_{max}	K_{Tg}	K_{FEM}	$\varepsilon_{Hey/FE}$	$\varepsilon_{How/FE}$
0.1	3.032	2.729	3.035	2.731	268.29	298.1	2.981	2.682	0.016	0.017
0.2	3.140	2.512	3.148	2.519	252.48	315.6	3.156	2.524	0.005	0.002
0.3	3.347	2.343	3.367	2.357	237.37	339.1	3.391	2.373	0.013	0.006
0.4	3.693	2.216	3.732	2.239	226.26	377.1	3.771	2.262	0.021	0.010
0.5	4.250	2.125	4.314	2.157	218.3	436.6	4.366	2.183	0.027	0.012
0.6	5.160	2.064	5.255	2.102	213.76	534.4	5.344	2.137	0.035	0.016
0.7	6.756	2.027	6.889	2.066	210.6	702	7.020	2.106	0.038	0.018
0.8	10.040	2.008	10.216	2.043	207.8	1039	10.39	2.078	0.034	0.016
0.9	20.010	2.001	20.237	2.023	205	2050	20.5	2.050	0.024	0.012

5.2 Orthotropic plate

This analysis was performed for a plate with one orthotropic ply. The mechanical properties of modeled lamina are given in Table 2. Note that E_{11} and E_{22} are the longitudinal and transversal moduli, respectively, G_{12}, G_{13} and G_{23} shear moduli, υ_{12} Poisson's ratio. For orthotropic plate global and net stress concentration factors are also calculated using analytical method (Heywood formulation) according to Equations 2 and 8. In Table 3, obtained results are compared with FE values. The relative error presented in this table is also computed by Equation 11. Infinite stress concentration factor for orthotropic plate with circular hole is evaluated by using Equation 4. Furthermore, maximum stress has been extracted from ABAQUS software, in order to compute global and net *SCF*, using Equations 1 and 6. Similar to isotropic plate, maximum stress distribution for orthotropic plate with $d/W = 0.3$ is shown in Fig.re 7. In this case, we can also see the very good agreement between results obtained with Heywood formulation and the finite element method. The curves of numerical and analytical values of global and net stress concentration versus *(d/W)* ratios are presented in Figures 8 and 9.

For both isotropic and orthotropic cases, global stress concentration factor (K_{Tg}) values increase as the *(d/W)* ratio increases. This indicates an increase in the stress concentration factor for a reduction in the diameter (Figs. 5 and 8). In the other hand, the net stress concentration factor (K_{Tn}) for both isotropic and orthotropic plates decreases as the *(d/W)* ratio increases and this indicates a decrease in *SCF* for a reduction in diameter.

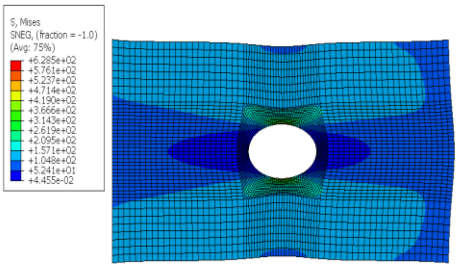

Fig.7. Stress distribution for orthotropic plate with central circular hole *d/W* = 0.3

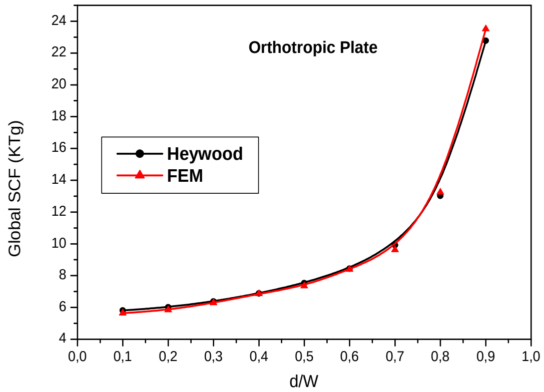

Fig.8. Global stress concentration factor for orthotropic plate with central circular hole using different methods

Table 2

The orthotropic material parameters

E_{11}(MPa)	E_{22}(MPa)	υ_{12}	G_{12}(MPa)	G_{13}(MPa)	G_{23}(MPa)
50000	14500	0.33	2560	2560	2240

Table 3

Global and net stress concentration factor for orthotropic plate with circular hole using Heywood and FEM

		Heywood		FEM				Error
d/w	$K_{T\infty}$	K_{Tg}	$K_{Heywood}$	σ_{avg}	σ_{max}	K_{Tg}	K_{FEM}	$\varepsilon_{Hey/FE}$
0.1	5.752	5.814	5.232	507.06	563.4	5.634	5.070	0,030
0.2	5.752	6.016	4.813	467.12	583.9	5.839	4.671	0,029
0.3	5.752	6.373	4.461	439.95	628.5	6.285	4.399	0,013
0.4	5.752	6.878	4.127	411.9	686.5	6.865	4.119	0,001
0.5	5.752	7.533	3.766	367.3	734.6	7.346	3.673	0,024
0.6	5.752	8.434	3.373	335.64	839.1	8.391	3.356	0,005
0.7	5.752	9.917	2.975	288.51	961.7	9.617	2.885	0,030
0.8	5.752	13.018	2.603	264.8	1324	13.240	2.648	0,017
0.9	5.7523	22.7755	2.2775	235.1	2351	23.510	2.3510	0,0322

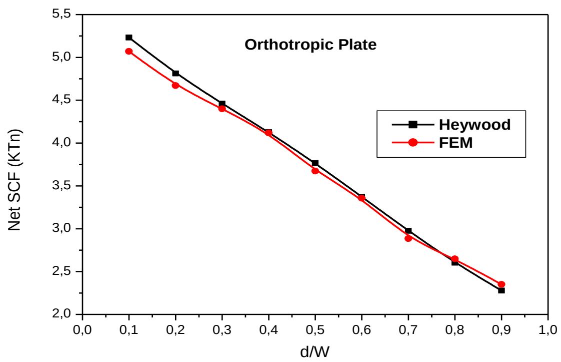

Fig.9. Net stress concentration factor for orthotropic plate with central circular hole using different methods

5.3 Effect of shape factor upon stress concentration factor

In the previous section, stress concentration factor was determined for isotropic and orthotropic finite rectangular plates with central circular hole, subjected to a tensile load by using analytical and FE methods. In this section the effect of length/width ratio on the stress concentration factor is investigated. The length/width ratio denoted (*L/W*) is the ratio between the length of the plate and its width. The global and the net *SCF* are determined for a group of isotropic and orthotropic plates with various ratios *L/W* = 1, 1.3, 1.5, 1.7, 2, 3 and 5, while the *(d/W)* ratio is changed from 0.1 to 0.9. For each ratio, stress concentration factors are calculated analytically by using Heywood and Howland formulations for isotropic case (Equations 2, 8, 9 and 10) and Heywood formulations (Equations 2 and 8) for orthotropic case. The stress concentration factors are also computed according to (Equations 1 and 6 using FE data extracted from ABAQUS software. The global (K_{Tg}) and net (K_{Tn}) stress concentration factors which were calculated with analytical and FE methods for isotropic and orthotropic plates. Figures 10-13 illustrates the relationship between various ratios *(L/W)* and the global and net concentration factors for various *(d/W)* ratios for a group of plates. For isotropic plates, according to Figures 10 and 11 there is a very good agreement between analytical and finite element results of global and net stress concentration factors as long as *(d/W)* ratio is less than 0.4 for all *(L/W)* ratios. When *(d/W)* ratio is more than 0.4 we can see a difference between the numerical and analytical values of *SCF* for *(L/W)* ratios equaling 1, 1.1 and 1.3 because of the proximity of the hole. For *(L/W)* ratios superior than 1.5, we can see an excellent agreement between all curves. Note that for *(L/W)* equaling 2 there is a very good accuracy between Heywood, Howland formulations and FE results. For orthotropic plates, according to Figures 12 and 13 there is a good agreement between analytical and finite element results of global and net *SCF* when *(d/W)* ratio is less than 0.5 for all *(L/W)* ratios. When *(d/W)* ratio is more than 0.5 we can see a difference between numerical and analytical curves for *(L/W)* ratios equaling 1,1.1,1.3,1.5 and 1.7 because of the proximity of the hole and the effect of the material orthotropy. For *(L/W)* ratios superior than 1.7, we can see a good agreement between all curves. Similar to isotropic plates, it can be seen that for *(L/W)* equal to 2 there is a very good accuracy between Heywood formulation and FE results.

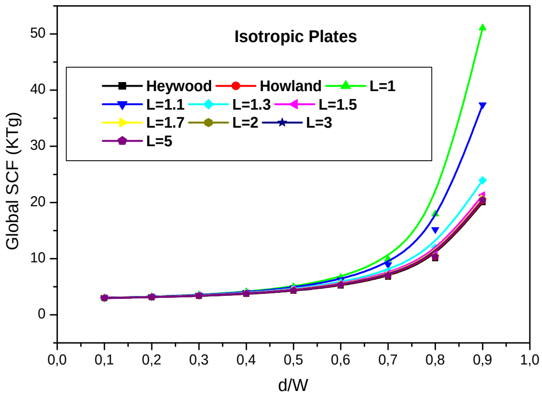

Fig.10. Comparison between (KTg) values obtained from FE results and Heywood's and Howland formulations at different length/width ratio for isotropic plates

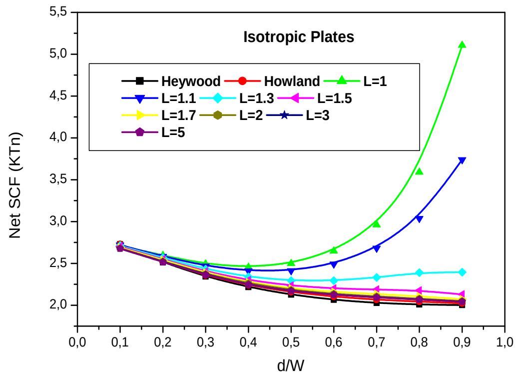

Fig.11. Comparison between (KTn) values Obtained from FE results and Heywood's and Howland formulations at different length/width ratio for isotropic plates

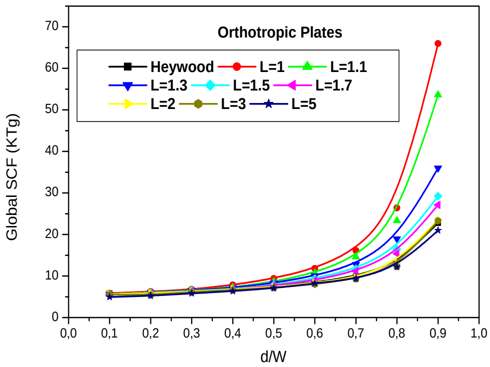

Fig.12. Comparison between (KTg) values obtained from FE results and Heywood's formulation at different length/width

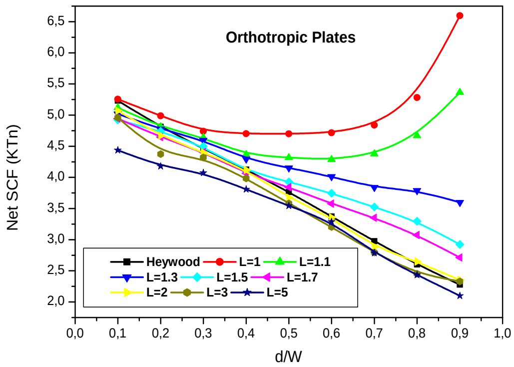

Fig.13. Comparison between (KTn) values obtained from FE results and Heywood's formulation at different length/width

6. Conclusions

The present study provides a detailed discussion of stress concentration factors in isotropic/ orthotropic rectangular plates with central hole subjected to a tensile load field. Analytical analyses based on Heywood and Howland formulations were carried out to evaluate global and net stress concentration factors. The obtained results were compared with those computed with ABAQUS software. The second part of this work covered the study of the effect of length/width ratio upon stress concentration factor. From the analytical and numerical results the following conclusions can be stated:

1. For isotropic plate, the stress concentration factors (*KTg* and *KTn*) values obtained from FE method results are in a very good agreement with the analytical values, obtained with Heywood and Howland formulations with a maximum error of 0.038. Also, for orthotropic plate, Heywood formulation is in excellent agreement with FE results with a maximum error of 0.032.
2. This study reveals that for both isotropic and orthotropic cases, global stress concentration factor values increase as the *(d/W)* ratio increases and this indicates an increase in *SCF* for a reduction in the diameter. On the other hand, the net stress concentration factor for both isotropic and orthotropic plate decreases as the *(d/W)* ratio increases and this indicates a decrease in *SCF* for a reduction in the diameter.
3. For isotropic plates, there is a very good agreement between analytical and FE results of K_{Tg} and K_{Tn} *SCF*, when *(d/W)* ratio is less than 0.4 for all *(L/W)* ratios. However, in the case of *(d/W)* ratio greater than 0.4 one can see a difference between the numerical and analytical values of *SCF* for *(L/W)* ratios of 1, 1.1 and 1.3 due to the proximity of the hole. For *(L/W)* ratios more than 1.5, an excellent agreement between all curves is observed.
4. For orthotropic plates, there is a good agreement between the analytical and finite element results of global and net *SCF* when *(d/W)* ratio is less than 0.5 for all *(L/W)* ratios. However, in the case of *(d/W)* ratio more than 0.5 one can see a difference between the numerical and analytical curves for *(L/W)* ratios equal to 1, 1.1,1.3,1.5 and 1.7 because of the proximity of the hole and the effect of the material orthotropy. For *(L/W)* ratios greater than 1.7, a good agreement between all curves can be noticed.
5. For isotropic plates, for *(L/W)* equal to 2, there is an excellent agreement between Heywood, Howland formulations and FE results.
6. For orthotropic plates, it can be seen that for *(L/W)* equal to 2 there is also a very good accuracy between Heywood formulation and FE results.

R E F E R E N C E S

[1] *J. Rezaeepazhand, M. Jafari*, "Stress concentration in metallic plates with special shaped cutout", International Journal of Mechanical Sciences, **vol. 52**, no. 1, 2010, pp. 96-102

[2]. *S. Daghboudj, H. Satha*, "Determination of the in-plane shear rigidity modulus of a carbon non-crimp fabric from bias-extension data test", Journal of Composite Materials., **vol. 48**, no. 22, 2014, pp. 2729-2736

[3]. *P.M.G.P. Moreira, S. D. Pastrama and P.M.S.T. de Castro*, " Comparative Three Dimensional Fracture Analyses of Cracked Plates", U.P.B. Sci. Bull., Series D, **vol.69,** no. 1 69, 2007, pp.43-58

[4]. *R.B. Heywood*, Designing by Photoelasticity, Chapman and Hall, 1952.

[5]. *W.D. Pilkey*, Peterson's Stress Concentration Factors, John Wiley & Sons, 2008.

[6]. *R.E. Peterson*, Stress Concentration Factors, John Wiley & Sons, 1997.

[7]. *N.I. Muskhelishvili*, Some Basic Problems of the Mathematical Theory of Elasticity, Groningen, 1953.

[8]. *S. Timoshenko,* J.*N. Goodier*, Theory of Elasticity, McGraw-Hill, Book Company, New York, 1951.

[9]. *R.C.J. Howland,* "On the Stresses in the Neighborhood of a Circular Hole in a Strip under Tension", Philosophical Transactions of the Royal Society of London, **vol. 229,** no. (670-680), 1930, pp. 49-86

[10]. *C. Hwai Chung, M. Bin*, "On stress concentrations for isotropic/orthotropic plates and cylinders with a circular hole", Composites Part B: Engineering, **vol. 34**, no. 2, 2003, pp. 127-134

[11]. *N.K. Jain, N.D. Mittal,* "Finite element analysis for stress concentration and deflection in isotropic, orthotropic and laminated composite plates with central circular hole under transverse static loading", Materials Science and Engineering: A, **vol. 498**, no.(1-2), 2008, pp.115-124

[12]. *N. Troyani, C. Gomes and G. Sterlacci,*"Theoretical stress concentration factors for short rectangular plates with centered circular holes", ASME Journal of Mechanical Design, **vol. 124,** no. 3, 2002, pp. 126-128

[13]. *L.Toubal, M.Karama and B.Lorrain,* "Stress concentration in a circular hole in composite plate", Composite Structures., **vol. 68**, no. 1, 2005, pp. 31-36

[14]. N.K *Jain, N.D.Mittal*, "Effect of fibre orientation on stress concentration factor in a laminate with central circular hole under transverse static loading", Indian Journal of Engineering & Material Sciences., **vol. 15**, no. 6, 2008, pp. 452-458

[15]. *Z. Hashem, M. Bahador, B. Pedram, M.Gudarzi,* "On Stress Concentration Factor for Randomly Oriented Discontinuous Fiber Laminas with Circular/Square Hole", Journal of Science and Engineering, **vol. 3,** no. 1, 2013, pp. 7-18

[16]. *M.M. Mhallah, C. Bouraoui,* "Determination of Stress Concentration Factor for Orthotropic and Isotropic Materials Using Digital Image Correlation (DCI)", In: Multiphysics Modelling and Simulation for Systems Design and Monitoring, Applied Condition Monitoring., **vol. 2**, 2015, pp. 517-530

[17]. *S.G. Leknitskii, S.W Tsai and T. Cheron,* Anisotropic Plates, Cordon and Breach Science Publishers, 1968.

[18]. *S.C. Tan,* "Finite Width Correction Factors for Anisotropic Plate Containing a Central Opening, " Journal of Composite Materials, **vol. 22**,no. 11, 1988, pp. 1080-1097

[19]. *M. Enayat, L.S. David,* "Boundary Element Study of a Loaded Hole in an Orthotropic Plate", Journal of Composite Materials., **vol. 20**, no. 4, 1986, pp. 375-389

[20]. *J.H. Nicholas, M. Christoph,* "Stress Concentration Factors for Cylindrically Orthotropic Plates", Journal of Composite Materials., **vol. 16**, no. 4, 1982, pp. 313-317

[21]. *B. Kambiz, R. Iraj, and R. Ferhad,* "Investigation of Stress Concentration Factor for Finite-Width Orthotropic Rectangular Plates with a Circular Opening Using Three-Dimensional Finite Element Model",Journal of Mechanical Engineering., **vol. 54,** no. 2, 2008, pp. 140-147

[22]. *S.G. Leknitskii,* Theory of Elasticity of an Anisotropic Elastic Body, 1st Ed Holden-Day, San Francisco, 1963.

[23]. *A.N.Tawakol,* "Stress concentration analysis in functionally graded plates with elliptic holes under biaxial loadings", Ain Shams Engineering Journal., **vol. 5,** no. 3, 2014, pp. 839-850

[24]. *B.C.L. Vanam, M. Aajyalakshmi and R. Inala*, "Static analysis of an isotropic rectangular plate using finite element analysis (FEA) ", Journal of Mechanical Engineering Research., **vol. 4**, no. 4, 2012, pp. 48-162

[25]. ***ABAQUS user manual documentation V-6.11, 2011.