THE PRODUCTION FUNCTION APPROACH IN THE CONTEXT OF OPTIMIZING EFFICIENCY

Ionel BOSTAN[1] and Ioan HURJUI[2]

The purpose of this paper is to emphasize the production functions facets specific for the industrial production system. Consequently, we will investigate the manner in which the specific function and its properties are reporting to technologies. Taking into consideration that its increase might induce a definite path towards sustainable development of the system, in the final part we will briefly present some evidence related to the average yield calculation for the each production factor (N, K; Y), where: Y – production output, K – the volume of capital factor and N – the aggregate volume of labor factor.

1. Introduction

The sustainable development issue of industrial, agricultural, transportation, or tourism systems has been relatively frequent addressed by the existent literature [1-2]. Relying on production functions, the purpose of this article is to investigate the interdependence between certain variables belonging to an industrial system (enterprise) able to encourage the insertion on the sustainable development orbit [3]. These represent the functional link between the result of a production related activity and the factors that influence it [4], thoroughly describing the multiple correlations that may occurr between the output and the main production factors.

We want to demonstrate and to bring to the attention of those concerned with investigating the efficiency of using the productive factors in industrial systems, in terms of quality, this calculation model (which we refer further), which enables a good characterization in terms performance/ efficiency of those factors. Also, we think that through the way we highlighted certain elements through adjustments/ systematization that we used, the manner of presentation, etc., we managed to make things easy to be understood by professional economists category industrial companies.

[1]Professor, Ștefan cel Mare University of Suceava, Doctoral School of Economics, Romania, ionel_bostan@yahoo.com
[2]Professor, Ștefan cel Mare University of Suceava, Doctoral School of Economics, Romania, hurjui.ioan@yahoo.com

Practically, the production functions show, with the help of relations such as

$$y = \otimes (x_1, ..., x_n), \tag{1}$$

the results of an activity, based on the production factors used ($x_1, ..., x_n$ - variable inputs).

The first to calculate production functions were Cobb and Douglas (CD) [4-6], using the model:

$$Y = \otimes (K, N) \tag{2}$$

where: Y – production output, K – the volume of capital factor and N – the aggregate volume of labor factor.

The three-dimensional representation of the CD type functions is a vertically inverted cone surface (Fig. 1).
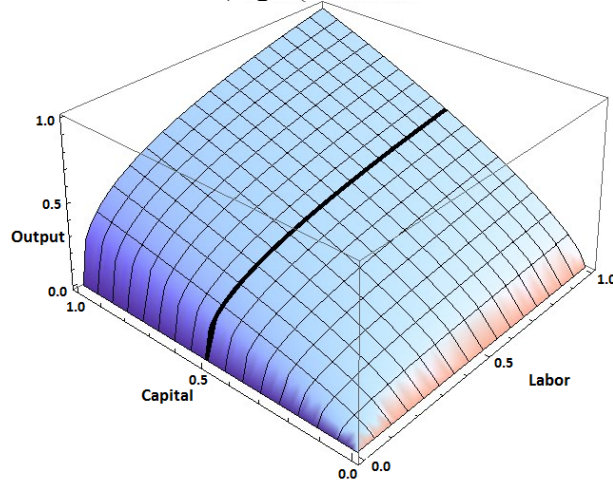

Fig. 1. Cobb-Douglas Production Function [7]

The substitution of the factors is given by asymptotic isoquants to the axes, and the total factorial surface encompasses [3-8]:

● the output volume cannot be infinite;

● the substitution of the factors can be completed within certain boundaries.

2. Production functions and the reference to technology

Regarding production functions [8],

$$\otimes: R_+^n \rightarrow R_+ \tag{3}$$

the following assumptions are made (Table 1).

Notations: R_+^n - *set of vectors* $x = (x_1, ..., x_n)$ *where* $x_i \geq 0$*, considered as the set of production factors;* R_+ - *the set of nonnegative values of the real line, where* $\{x \in R / x \geq 0\}$.

Technology, considered from the point of view of production factors, is seen as the triplet

$$\tau = (N, K; Y) \tag{4}$$

Table 1

Group of hypotheses and related conditions [3; 8; 9]

Name	Related conditions
● Inputs and outputs are real and positive quantities	(a) $f(x) = 0$, for any vector $x = (x_1, ..., x_n)$ having at least one component that equals zero;
● For a given volume of production there will be chosen a technology that minimizes consumption for each factor	(b) $\dfrac{\partial f}{\partial x_i} \geq 0$, (i = 1, ..., n);
● For a fixed combination of factors there will be chosen a technology that maximizes the production volume	(c) $\dfrac{\partial^2 f}{\partial x_i^2} \leq 0$, (i = 1, ..., n).

Defined as "the complex collection of knowledge, tools and procedures, managed in order to manufacture a certain product from certain a materials, in a local context and under favourable economic circumstances" [10], the technology a company disposes of can be assessed by means of the production function [11]. It shows the ways in which inputs can be combined in order to get the desired output.

Technologies (defined by the elements presented in Table 2) are subsequently integrated within technological processes.

Table 2

Typical characteristics of technologies [10]

Type	Specific features
Theoretical knowledge	Bring about transformations (in physics, chemistry, materials science, etc)
Tools/ Machinery	Support the processing of the raw materials into the desired output
Procedures	Refer to the ways in which toold can be used in order to achieve the desired goal

The set of all possible technologies for a productive system is denoted by T and can be expressed as [12-13]:

$$T= \{\tau/\tau = (N, K; Y)\} \tag{5}$$

The set T meets the following assumptions:

● *if* $\tau = (N, K; Y) \in T$, then $N \geq 0$, $K \geq 0$, $Y \geq 0$;

● *if* $\tau = (N_1, K_1; Y_1) \in T$, then every real number $\lambda > 0$ corresponds to an output, denoted by Y_λ, and $\tau_\lambda = (\lambda N_1, \lambda K_1; Y_\lambda) \in T$.

Approaching specific properties

Nowadays, the professional literature is frequently tackling the topic of production functions – seen as "inputs/outputs" [9] – with a small number of factors. Practically, reference is made to the two factor production functions: work (a variable factor measured as individual/ time unit) and capital (which, in the short term, is a fixed factor, while on the long term it is a variable one) [14].

The properties of production functions (2) are [3; 8]:

● The absence of any input in the context of a given technology leads to a zero output. Actually [10], the inputs are purchased on the inputs markets and their value usually consists in the overheads of the company. The output is sold on the goods and services markets and, in exchange, their value is collected - the revenue;

● The production function is monotonically increasing in both arguments, meaning that X and Y are two nonnegative vectors expressing a certain usage level of the n inputs. Thus the production function has a non-descending efficiency.

$$\otimes (X + Y) \geq \otimes (X) + \otimes (Y) \tag{6}$$

● If in the homogeneous production functions of degree h

$$\otimes (\lambda x_1, ..., \lambda x_n) = \lambda^n \otimes (x_1, ..., x_n) \tag{7}$$

the quantity $(x_1, ..., x_n)$ for each input is multiplied by λ, then the size of output increases λ times;

● Production functions are differentiable, and their first-order partial derivatives are positive,

$$\frac{\partial f}{\partial x_i} \geq 0 \qquad (i = 1, ..., n) \tag{8}$$

This differential indicates how much will the output increase due to a unitary increase in the amount of the *i* resource used in production, assuming that the other resources remain quantitatively unchanged.

● Second order partial derivatives of production functions are negative

$$\frac{\partial^2 f}{\partial x_i^2} < 0 \qquad (i = 1, ..., n) \tag{9}$$

which means that for rising values of $\otimes$, the same increase in the volume of production factors causes relatively smaller increases in the output.

3. The calculation of efficiency for factors used in industrial systems

The usual tendency of every company is to reach a high productivity level of the inputs, resulting in a higher output with the same amount of effort (input consumption) or the same output with less effort. This issue is generally regarded as particularly important, since we are dealing with a widespread exhaustion of natural resources.

The efficient integration of the inputs towards achieving the maximum output with minimum effort (resources) is expressed by productivity or input efficiency (performance) (Table 3).

Table 3

Efficiency (performance) approach [15]

Type of approach	Type of efficiency	Characteristics
According to the ay of measuring the output	Physical	Measures the natural efficiency of input use, expressed in physical units (natural or natural - conventional units)
	Numerical	Express performance in financial and monetary terms
According to type of manifestation	Gross	Measures the general output in relation to the used factor of production. In this case, production is seen as "final output".
	Net	Focuses on removing the value of the external purchases and the installed capital cost from the final output.

Technical capital, material and energy resources are nowadays greatly influenced by scientific and technical progress, when [15]: "manufacturing techniques and technologies are improved, state-of-the-art technologies replace the old ones, the work force structure is being changed and all these changes ultimately contribute to improving workforce productivity in the economic sector". The actual increase in productivity/ efficiency consists either in the reduced amount of labour per product unit or in the more efficient use of the material factors of production [16].

Considering the production function (2), we can calculate the average yield for each production factor [3]:

- for *K*:

$$\overline{\eta}_K = \frac{f(K_1, N_0) - f(K_0, N_0)}{K_1 - K_0} \qquad (10)$$

- for *N*:

$$\bar{\eta}_L = \frac{f(K_0, N_1) - f(K_0, N_0)}{N_1 - N_0} \tag{11}$$

Where indices 0 and 1 represent two consecutive moments in the volume evolution of the considered production factors.

If for relations (12) and (13) $K_0 = 0$ and $N_0 = 0$ and given the property 1), the above yields become:

$$\text{- for K } \bar{\eta}_K = \frac{f(K_1, N_0)}{K_1} \tag{12}$$

$$\text{- for N } \bar{\eta}_L = \frac{f(K_0, N_1)}{N_1} \tag{13}$$

which are differential yields for the two factors.

With their help, the efficiency of factors' usage can be more precisely characterized, in terms of quality [9].

The average yield includes quality deficiencies of the average; first of all it dims the evolution between points 0 and 1. Differential yield characterizes the efficiency of a certain level of factor usage, gives quality information on the function's behavior in each point and highlights any disproportions and deviations from this behavior.

There is a link between the two indicators which can be described with the help of Euler's formula for homogeneous functions [3]:

- for functions of degree 1:

$$Q = f(K, N) = \frac{\partial f}{\partial K} K + \frac{\partial f}{\partial N} N \tag{14}$$

- for functions of degree h:

$$hf(K, N) = \frac{\partial f}{\partial K} K + \frac{\partial f}{\partial N} N \tag{15}$$

where Q – the output volume.

With the help of relations (16) and (17) it can be shown that in the case of an homothetic production function, the average yield of a production factor equals its differential yield, if the other factor's differential yield is 0.

Considering, one after the other, $\partial \otimes / \partial K = 0$ and $\partial \otimes / \partial N = 0$, we get

$$\frac{f(K, N)}{N} = \frac{\partial f}{\partial N} \tag{16}$$

and

$$\frac{f(K, N)}{K} = \frac{\partial f}{\partial K} . \tag{17}$$

The elasticity of production in relation to the volume of used resources shows the increase in production for a 1% increase in the quantity of a certain resource *i*.

We emphasize that some works [17] distinguish between the *technical elasticity* of substitution, as a measure of potential substitution and *the price elasticity* of substitution, as a measure of potential substitution.

4. Conclusions

This paper was directed at identifying certain essential aspects entailed when calculating the yield of the factors used in industrial (production) systems, resorting to the mathematical conformation functions (production functions).

In this respect, we have approached technology as a triplet, in the form $t = (N,K;Y)$. Subsequently, after considering the production function $y = Ä(x_1,...,x_n)$ (the outcome of an activity depending on the production factors used in manufacturing certain goods), we have identified the method of calculating the average yield of each production factor (N, K; Y).

Our references also focus on two consecutive moments in the evolution of the output volume, an aspect that has been highlighted by means of the 0 and 1 indices. The presented calculation model offers the possibility to characterize the efficiency of using productive factors, in terms of quality, in industrial systems.

However, we must distinguish between average and differential yield, although the link between the two indicators can be characterized by Euler's formula for homogeneous functions. If the first type dims the evolution between points 0 and 1 by leveling disproportions that may appear in this range, the second characterizes the efficiency of a certain usage level of the production factor, taking into account the particularities of its evolution throughout the function's existence.

In essence, the production level increases as long as differential yields of resources are positive, the average yield increases if its rate is positive and the differential yield is greater than the average one.

Acknowledgments

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