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GENERAL METHOD TO DETERMINE THE FREQUENCY DOMAIN IN WHICH LEFT HANDED STRUCTURES BEHAVE AS A DOUBLE NEGATIVE MEDIUM

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In this paper we present a general method of analysis used to determine the frequency domain in which any Composite Right Left Handed - CRLH structure behaves as a double negative medium, with both equivalent electric permittivity and magnetic permeability negative. This aspect is highly important when designing metamaterial transmission lines with multiband behaviour. In order to explain and validate this method, the most complex CRLH structure, which is the extended one (E-CRLH) is considered. The analytical expressions are given as a function of the lumped elements used to model and design the structure.

Keywords: Double negative medium, metamaterials, Extended Composite Right Left Handed transmission line-ECRLH TL, multiband band behavior

1. Introduction

The importance of double negative artificial media (with both the permittivity and permeability negative) has been proved by the numerous applications in the last few years. The artificial metamaterial transmission lines have played an important role in the development of most microwave passive devices [1], [2].

The metamaterials are in fact ladder structures made of unit cells which respect the homogenous condition, so they can be considered artificial materials rather than simple filters. Metamaterial transmission lines are modelled using lumped elements, each unit cell being smaller than a quarter of a wavelength in order to apply the analysis from the distributed elements theory [2].

So, in order to better understand and use the properties of these transmission lines, it is important to use analytical methods that have a high level of generality and are also very handy to work with. One of the aspects that is taken into account when designing metamaterial transmission lines is to determine as precisely as possible the frequency domain in which the transmission line behaves as a LH medium [3]. This allows designing it as a multi-band device due

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to its dual behaviour: the conventional one or the Right Handed one, respectively the artificial one also called Left Handed [4].

The method presented in this paper maps Maxwell's equations written for a conventional media over the Telegraphers' equations written for metamaterial transmission lines. This is how there are obtained the frequency domains in which the transmission line acts as a double negative media and can be used as a multi-band component.

2. Transmission matrix formalism

The artificial metamaterial transmission lines are implemented by chaining different types of unit cells. In this case the most appropriate formalism to characterise such a structure is the transmission matrix formalism. In order to write the transmission matrix for a unit cell, we will divide the cell into smaller two-ports for which we can determine the matrix easily and then we will multiply these matrixes, neglecting the infinite small terms [5]. In this manner we will obtain the transmission matrix for the unit cell and later on the matrix for the whole transmission line.

We will take as an example the case of the unit cell for an E-CRLH (Extended CRLH) transmission line, the most complex structure from literature. The unit cell is a combination of a Composite Right Left Handed - CRLH and Dual Composite Right Left Handed - D-CRLH one. So, it is characterized by eight lumped elements, four of them from CRLH unit cell and four from D-CRLH one.

In Fig. 1 there is presented the design of a infinite small E-CRLH unit cell with lumped elements per unit length. The super-indexes "c", respectively "d" refers to the character of the elements: C-CRLH (Conventional CRLH), respectively D-CRLH (Dual CRLH) [6]. The input voltage and current, respectively the output voltage and current, as well as the impedances in this scheme are expressed using Laplace formalism.

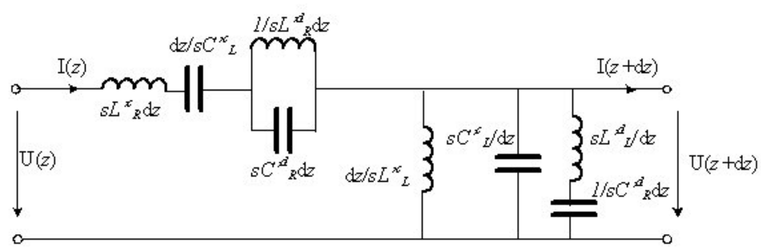

Fig. 1. The unit cell for an E-CRLH transmission line

The unit cell can be divided as shown in Fig. 2. In fact, there are emphasised the equivalent impedance for the longitudinal section, presented in Fig. 3 and given by Z_{long}, respectively transversal section, presented in Fig. 4 and given by Z_{tr}.

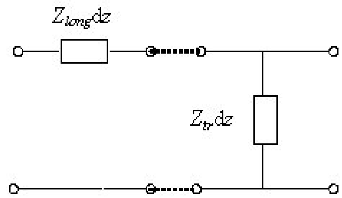

Fig. 2 The unit cell for an E-CRLH transmission line divided into two-ports chained.

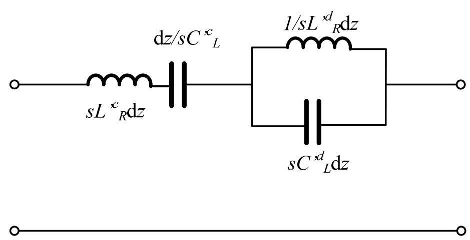

Fig.3 The two-port corresponding to the longitudinal section

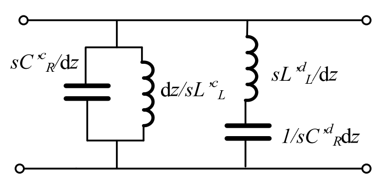

Fig.4 - The two-port corresponding to the transversal section

The expressions for the equivalent impedance for the longitudinal and transversal sections are given by:

$$Z_{long} = \left(sL'^{c}_{R} + \frac{1}{sC'^{c}_{L}} \right) + \frac{1}{\left(sC'^{d}_{L} + \frac{1}{sL'^{d}_{R}} \right)},$$

$$Z_{tr}=\left(sC'^{c}_{R}+\frac{1}{sL'^{c}_{L}}\right)+\frac{1}{\left(sL'^{d}_{L}+\frac{1}{sC'^{d}_{R}}\right)} \tag{1}$$

We can determine the transmission matrix for the unit cell from Fig.2, knowing that:

$$(A)=\begin{pmatrix}1 & Z_{long}\mathrm{d}z\\ 0 & 1\end{pmatrix}\cdot\begin{pmatrix}1 & 0\\ \frac{\mathrm{d}z}{Z_{tr}} & 1\end{pmatrix} \tag{2}$$

and replacing the expressions from relations (1) into relation (2). If we neglect the infinite small terms obtained after multiplication of the two matrixes, we can write a relation between the voltages and currents at the input and at the output of the unit cell:

$$\begin{pmatrix}\mathrm{U}(s,z)\\ \mathrm{I}(s,z)\end{pmatrix}=$$

$$=\begin{pmatrix}1 & \left[\frac{1}{\left(sC'^{d}_{L}+\frac{1}{sL'^{d}_{R}}\right)}+\left(sL'^{c}_{R}+\frac{1}{sC'^{c}_{L}}\right)\right]\mathrm{d}z\\ \mathrm{d}z\cdot\left[\left(sC'^{c}_{R}+\frac{1}{sL'^{c}_{L}}\right)+\frac{1}{\left(sL'^{d}_{L}+\frac{1}{sC'^{d}_{R}}\right)}\right] & 1\end{pmatrix}\cdot\begin{pmatrix}\mathrm{U}(s,z+\mathrm{d}z)\\ \mathrm{I}(s,z+\mathrm{d}z)\end{pmatrix}. \tag{3}$$

After some manipulations, relation (3) can be rewritten:

$$-\frac{\mathrm{dU}(s,z)}{\mathrm{d}z}=\left(sL'^{c}_{R}+\frac{1}{sC'^{c}_{L}}+\frac{sL'^{d}_{R}}{1+s^{2}L'^{d}_{R}C'^{d}_{L}}\right)\mathrm{I}(s,z),$$

$$-\frac{\mathrm{dI}(s,z)}{\mathrm{d}z}=\left(sC'^{c}_{R}+\frac{1}{sL'^{c}_{L}}+\frac{sC'^{d}_{R}}{1+s^{2}C'^{d}_{R}L'^{d}_{L}}\right)\mathrm{U}(s,z). \tag{4}$$

So, at a closer inspection, relation (3) is in fact the matrix form for the Telegraphers' equations, which justifies the assumption that the unit cell described in Fig. 1 is an infinite small piece of an artificial E-CRLH transmission line.

3. Metamaterial transmission lines seen as a double negative material

Let us consider a medium characterised by the constitutive parameters (ε, μ) and write Maxwell's equations [7], [8]:

$$rot\overline{E} = -\mu \frac{\partial \overline{H}}{\partial t},$$
$$rot\overline{H} = \varepsilon \frac{\partial \overline{E}}{\partial t}. \qquad (5)$$

For plane waves, which propagate only along the Oz axis, E and H are functions of position, z and time, t:

$$-\frac{\partial E_y}{\partial z} = \mu \frac{\partial H_x}{\partial t},$$
$$-\frac{\partial H_x}{\partial z} = \varepsilon \frac{\partial E_y}{\partial t}. \qquad (6)$$

Considering initial conditions null, the Laplace transformation is applied to the equations system (6):

$$-\frac{\mathrm{dE}_y(s,z)}{\mathrm{d}z} = s\mu \mathrm{H}_x(s,z),$$
$$-\frac{\mathrm{dH}_x(s,z)}{\mathrm{d}z} = s\varepsilon \mathrm{E}_y(s,z). \qquad (7)$$

After some manipulations, ignoring the second order infinite small terms, one gets:

$$\mathrm{E}_y(s,z) - \mathrm{E}_y(s,z+\mathrm{d}z) = s\mu \mathrm{d}z \mathrm{H}_x(s,z),$$
$$\mathrm{H}_x(s,z) - \mathrm{H}_x(s,z+\mathrm{d}z) = s\varepsilon \mathrm{d}z \mathrm{E}_y(s,z). \qquad (8)$$

The last equation system, (8) can be rewritten as follows. So, for a small distance, dz, we can write Maxwell's equations for an electromagnetic wave that propagates in a transverse electromagnetic mode (TEM) using Laplace formalism as follows:

$$\begin{pmatrix} \mathrm{E}_y(s,z) \\ \mathrm{H}_x(s,z) \end{pmatrix} = \begin{pmatrix} 1 & s\mu \mathrm{d}z \\ s\varepsilon \mathrm{d}z & 1 \end{pmatrix} \cdot \begin{pmatrix} \mathrm{E}_y(s,z+\mathrm{d}z) \\ \mathrm{H}_x(s,z+\mathrm{d}z) \end{pmatrix} \tag{9}$$

In the case of the E-CRLH transmission line, if the unit cell is small enough to respect the homogenous condition, then we can map relations (3) and (9). In fact, mapping the two matrix equations resume to identify the elements of the transmission matrix to the ones of the constitutive parameters' matrix:

$$s\varepsilon \leftrightarrow \left(sC'^{c}_{R} + \frac{1}{sL'^{c}_{L}} \right) + \frac{1}{\left(sL'^{d}_{L} + \frac{1}{sC'^{d}_{R}} \right)},$$

$$s\mu \leftrightarrow \frac{1}{\left(sC'^{d}_{L} + \frac{1}{sL'^{d}_{R}} \right)} + \left(sL'^{c}_{R} + \frac{1}{sC'^{c}_{L}} \right). \tag{10}$$

In order to determine the frequency domain in which the E-CRLH transmission line acts as double negative medium, first of all we need to make the transformation $s \to j\omega$ and then to solve the inequality system given by the conditions $\varepsilon(\omega) < 0, \mu(\omega) < 0$, replacing the constitutive parameters with the expressions using the components of the unit cell:

$$C'^{c}_{R} - \frac{1}{\omega^2 L'^{c}_{L}} + \frac{C'^{d}_{R}}{1-\omega^2 L'^{d}_{L} C'^{d}_{R}} < 0,$$
$$L'^{c}_{R} - \frac{1}{\omega^2 C'^{c}_{L}} + \frac{L'^{d}_{R}}{1-\omega^2 L'^{d}_{R} C'^{d}_{L}} < 0. \tag{11}$$

After some manipulations, one obtains:

$$\frac{\omega^4 L'^{c}_{L} C'^{c}_{R} L'^{d}_{L} C'^{d}_{R} - \omega^2 \left(L'^{c}_{L} C'^{c}_{R} + L'^{d}_{L} C'^{d}_{R} + C'^{d}_{R} L'^{c}_{L} \right) + 1}{\omega^2 L'^{c}_{L} \left(\omega^2 L'^{d}_{L} C'^{d}_{R} - 1 \right)} < 0$$

$$\frac{\omega^4 L'^c_R C'^c_L L'^d_R C'^d_L - \omega^2\left(L'^c_R C'^c_L + L'^d_R C'^d_L + C'^d_L L'^c_R\right)+1}{\omega^2 C'^c_L\left(\omega^2 L'^d_R C'^d_L - 1\right)} < 0 \tag{12}$$

The solution for the system above is given by:

$$\omega \in \left(0;\min\{\omega_1;\omega_5\}\right)\cup\left(\max\{\omega_3;\omega_7\};\min\{\omega_2;\omega_6\}\right) \tag{13}$$

Where:

$$\omega_1 = \sqrt{\frac{\left(L'^c_L C'^c_R + L'^d_L C'^d_R + C'^d_R L'^c_L\right) - \sqrt{\left(L'^c_L C'^c_R + L'^d_L C'^d_R + C'^d_R L'^c_L\right)^2 - 4L'^c_L C'^c_R L'^d_L C'^d_R}}{2L'^c_L C'^c_R L'^d_L C'^d_R}},$$

$$\omega_2 = \sqrt{\frac{\left(L'^c_L C'^c_R + L'^d_L C'^d_R + C'^d_R L'^c_L\right) + \sqrt{\left(L'^c_L C'^c_R + L'^d_L C'^d_R + C'^d_R L'^c_L\right)^2 - 4L'^c_L C'^c_R L'^d_L C'^d_R}}{2L'^c_L C'^c_R L'^d_L C'^d_R}},$$

$$\omega_3 = \frac{1}{\sqrt{L'^d_L C'^d_R}},\ \omega_4 = 0,$$

$$\omega_5 = \sqrt{\frac{\left(L'^c_R C'^c_L + L'^d_R C'^d_L + C'^d_L L'^c_R\right) - \sqrt{\left(L'^c_R C'^c_L + L'^d_R C'^d_L + C'^d_L L'^c_R\right)^2 - 4L'^c_R C'^c_L L'^d_R C'^d_L}}{2L'^c_R C'^c_L L'^d_R C'^d_L}},$$

$$\omega_6 = \sqrt{\frac{\left(L'^c_R C'^c_L + L'^d_R C'^d_L + C'^d_L L'^c_R\right) + \sqrt{\left(L'^c_R C'^c_L + L'^d_R C'^d_L + C'^d_L L'^c_R\right)^2 - 4L'^c_R C'^c_L L'^d_R C'^d_L}}{2L'^c_R C'^c_L L'^d_R C'^d_L}},$$

$$\omega_7 = \frac{1}{\sqrt{L'^d_R C'^d_L}},\ \omega_8 = 0.$$

So, for the frequency domain given by (13), the E-CRLH transmission line acts as a double negative media, so it acts as a Left Handed material [9]. This property justifies the name of metamaterial transmission line and allows designing multi-band microwave devices by exploiting both the right handed and left handed properties [10].

4. Conclusions

In this paper, it is presented a method to determine the bandwidth where a CRLH structure behaves as a double negative medium. In fact, the main idea is to consider a LH handed structure from two perspectives: as a material for which Maxwell's laws apply and an artificial transmission line for which Telegraphers' equations can be considered. This duality allows knowing for which frequency domain, the transmission line can be used as a LH medium or a RH one. Multiband devices can be created using these dual properties.

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R E F E R E N C E S

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