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MODAL ANALYSIS ON CANTILEVER PLATES WITH ELASTIC BOUNDARY SUPPORTS FOR OPTIMIZING THE PLACEMENT OF ACTUATORS AND SENSORS IN ACTIVE VIBRATION CONTROL APPLICATIONS

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The knowledge of the systems response to certain excitation conduct to the possibility of designing structures that have improved characteristics, and when using active control systems, this response is important for optimal positioning of sensors and actuators. It will be studied the case of vibrations reduction of a thin cantilever plate with different elastic boundary constrains, where it is necessary to perform modal tests to identify the modal parameters of the structure, especially to improve the system observability and controllability of using active vibration control systems. Mode shapes obtained from the finite element analysis FEA will be compared to those determined by tests and so the decision could be made for best placing the sensors and excitation elements.

Keywords: active vibration control, modal analysis, *FEA*, state-space model, actuators and sensors placement, observability and controllability

1. Introduction

There are dynamic systems, such as those with applications in the automotive or aerospace industries, where thin flexible structures are used in order to meet the requirements of low weight and increased resistance, and thus to sustain important dynamic displacements. Their operation in regimes that may endanger the integrity of the structure leads to the need of using the active control systems, especially on frequency domains where passive control proves to be insufficient [1]. Following modal tests, Frequency Response Function *FRF* is obtained, and this solution is used to identify the system's natural frequencies, modes and especially the damping coefficients, which would be extremely difficult to be determined by other methods involving an analytical calculation.

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The *FRF* of a structure depends on the geometry and characteristics of the material from which the structure is made, the constraints to which it is subjected and the forces that act upon it [1]. If optimization of the response is considered during design, geometry and structure characteristics improvements can be achieved, and in the case of active control also the forces that act upon it. A modal test is performed in order to find a description concept for the dynamic movements of a system. In this article, a study was made starting from the objective of designing a feedback control model with modal based control obtained by an optimal placement of actuators and sensors. It is well known that in a feedback type active control approach, only the low frequency modes could be somehow controlled. High densities of modes conduct to a very complicated control system which is hard to be applied in reality [2].

The results obtained after *FEA* simulation will be compared with those obtained from the experimental study. An eventual algorithm model for finding the optimal placement of sensors and actuators could lead to o better stabilization of the structure dynamics. Such optimal placement of sensors and actuators as mentioned generally by *Baruh* [13] are mainly done for a better action of forces and moments applied on the structure by the control system for stabilization of dynamic behavior. Another control characteristics improvement by right placement of sensors and actuators is reducing deflection by mode shapes control, increasing the energy dissipation or better controllability and observability [14].

This article is focused on finding the best placement of actuators and sensors used on active vibration control under the special case of elastic clamped boundary cantilever using comparative FEA and modal testing procedure and future studies will continue with applying this concept to different type of active control algorithms.

2. Modal analysis and active control

Generally, the modal analysis is a procedure used in the field of structural dynamics, with aim of finding the modal parameters as the natural frequencies ω, local damping ς and mode shape Φ. [2] In an Active Vibration Control (*AVC*) system of a continuous structure, the energy is introduced in the system by an external or internal force and also by the *AVC* actuator which is equilibrating the input force, and it is dissipated by the passive damping elements which are connected directly to the structure or are part of boundary condition. The modal parameters, natural frequencies and mode shapes can be calculated analytically, numerically or after some experiments, but damping can be only the result of experiments. One important property is that one mode could be influenced by the presence of other modes, so, when we study the behavior of few modes inside of a frequency band, we should take into consideration also the number of modes placed twice the frequency band outside the studied band. The energy of this „outside" modes is somehow present in the studied modes [3].

In this article it was adopted a structural vibration approach studied in the way of modal vibrations. Because the boundary conditions are included in this study, also a reduction of transmitted energy is aimed. According with classic theory [6, 7, 8], the behavior of a plate with elastic edges under forced vibration is according with equation:

$$D\left[\frac{\partial^4 w(x,y,t)}{\partial x^4}+2\,\frac{\partial^4 w(x,y,t)}{\partial x^2\partial y^2}+\frac{\partial^4 w(x,y,t)}{\partial y^4}\right]=\rho h\omega^2\,\frac{\partial^2 w(x,y,t)}{\partial t^2}+f(x,y,t), \quad (1)$$

where it is noted with D –flexural rigidity, w – deflection, ρ – density, ω- angular frequency, h- thickness of the plate.

For a rectangular plate with dimensions of $L \times l \times h$, the linear k_x and rotational K_x elastic constants on elastic boundary conditions are according with equations:

$$k_{x=0}=-\frac{D}{w(x,y)}\left(\frac{\partial^3 w(x,y)}{\partial x^3}+2\,\frac{\partial^3 w(x,y)}{\partial x\partial y^2}-\vartheta\,\frac{\partial^3 w(x,y)}{\partial x\partial y^2}\right), \quad (2)$$

$$k_{x=l}=\frac{D}{w(x,y)}\left(\frac{\partial^3 w(x,y)}{\partial x^3}+2\,\frac{\partial^3 w(x,y)}{\partial x\partial y^2}-\vartheta\,\frac{\partial^3 w(x,y)}{\partial x\partial y^2}\right), \quad (3)$$

$$K_{x=0}\,\frac{\partial w(x,y)}{\partial x}=D\left(\frac{\partial^2 w(x,y)}{\partial x^2}+\vartheta\,\frac{\partial^2 w(x,y)}{\partial y^2}\right),\;\; K_{x=l}\,\frac{\partial w(x,y)}{\partial x}=-\,D\left(\frac{\partial^2 w(x,y)}{\partial x^2}+\vartheta\,\frac{\partial^2 w(x,y)}{\partial y^2}\right). \quad (4)$$

These equations could be used for an ideal case where the elastic boundaries are linearly distributed on the edge. If the plate is clamped on the edge, as it is in the most real cases, with different forces which are distributed on clamping area, such equations are not describing with good accuracy the phenomenon. Different clamping forces could induce on the plate structure different frequencies responses. The response of a mode of a plate under forced vibration can be defined by the equation:

$$m(x,y,t)\,\frac{\partial^2 w(x,y,t)}{\partial t^2}+c\,\frac{\partial w(x,y,t)}{\partial t}+kw(x,y,t)=f(x,y,t), \quad (5)$$

where w represents the displacement at (x, y) point, $c = \alpha m + \beta k$ is the proportional damping depending on stiffness and mass distribution [15].

3. FEA analysis

For modelling the structure response with *FEA*, we use the matrix equation

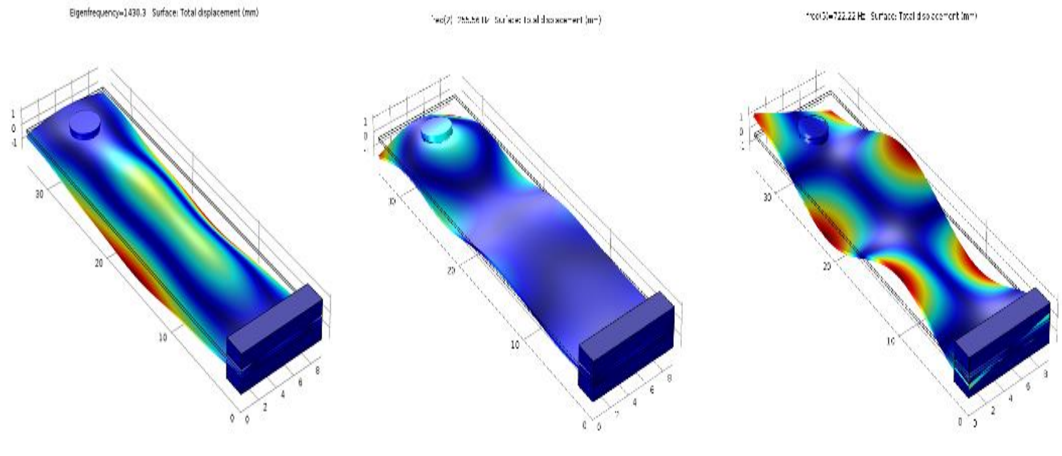
$$M\ddot{z}+C\dot{z}+Kz=f, \quad (6)$$

with $\boldsymbol{z}=[z_1, z_2, \ldots z_n]^T$, the vector of displacements, $\boldsymbol{M}$ the mass matrix, $\boldsymbol{C}$ the damping matrix and $\boldsymbol{K}$ the stiffness matrix. Simulations with *FEA* will help us to define the frequency domains which contain the modes under the aimed control, finding the resonance frequencies and to obtain some suggestion or approximations for placement of actuators and sensors by locating the nodes and anti-nodes, under the various edge loading forces. The frequency response in the case of forced excitation applied on systems with proportional damping, where other modes has a contribution on each other, is

$$\alpha_{jk}(\omega) = \sum_{i=1}^{N} \frac{\psi_i(x_j)\,\psi_i(x_k)}{\omega_i^2 m_i \left[1 - \left(\frac{\omega}{\omega_i}\right)^2 + 2j\left(\frac{\omega}{\omega_i}\right)\zeta_i\right]}, \tag{7}$$

where ψ_i is a mode shape, x_i is the location of the applied force, x_k is the location of the response point and ζ_i is the damping ratio for a certain mode [15].

A *FEA* analysis and simulation should be performed before the decision upon which modes we intend to include in the design of the system. Such a simulation, compared with experimental modal analysis is a better method which should be done for finding the best location of actuators and sensors, for optimal modal control in the case of complicated elastic boundary condition. In some studies there were compared the results between *FEA* simulations and experimental modal analysis for cantilever plates with fixed constrains and the results were promising [1, 4, 5]. On this study, it was made a simulation with COMSOL *FEA* software for identifying the low frequencies modes and the mode shapes given by natural frequencies (Fig. 1).



(a) (b) (c)

Fig. 1 *FEA* mode shapes of clamped cantilever plate 1430.3 Hz (a), 209.2 Hz (b), 722.2 Hz (c)

As modal parameters, even the geometry and local damping coefficients remain the same, the stiffness and mode shapes are changing under different clamping forces applied on the elastic boundaries. From the point of view of active vibration control, this could lead to changing the observability and controllability coefficients, if a state-space model is considered [14].

4. State-space model

For processing and control methods and finally for modal filtering realization, we have to re-express the modal equations in a state-space form [10, 12]. The design and optimization of control systems can be done using such state-space techniques. The basic concept of frequency analysis for control systems is the transfer function *TF*. A continuous, linear and with no variation on time system, it is defined as Laplace's transformation of the output *y(s)* and Laplace transform of its input *u(s).* The variable of Laplace transform is complex and is written in the form $s=\sigma + j\omega$ (where j $=\sqrt{-1}$). Equations of state space are written in the general form:

$$\dot{x} = Ax + Bu, \qquad y = Cx + Du. \tag{8}$$

Applying the Laplace transform, state equations are written in the form:

$$(sI - A)\, x(s) = Bu(s), \qquad y(s) = C\, x(s) + D\, u(s). \tag{9}$$

The roots of the equation det*(s I - A)*=0, are named as *poles* and the *TF* is:

$$G(s) = \frac{y(s)}{u(s)} = C\,(sI - A)^{-1}\, B + D = C\, \frac{\alpha_1 s^{k-1} + \alpha_2 s^{k-2} + \cdots \alpha_k}{s^k + \alpha_1 s^{k-1} + \cdots \alpha_k}\, B + D. \tag{10}$$

In the case of a causal system, the connection between the input and output quantities is given by what is called the *state of the system*. The state of a linear system is expressed in a matrix form by the equations

$$\dot{\boldsymbol{x}} = \boldsymbol{Ax} + \boldsymbol{Bu}, \qquad \boldsymbol{y} = \boldsymbol{Cx} + \boldsymbol{Du}, \tag{11}$$

where parameters ***A, B, C, D*** matrix shape the state of linear systems.

In the case of discrete systems, like those used in data acquisition and processing, the sampling of variables is done in time moments *k* and their value of the states, inputs and outputs at those moments is *x(k), u(k)* and *y(k).* Output quantities at a discrete moment *k* depend on the state of the system at that moment, a condition that is related to a previous *k-1*, which gives the causal character between input and output. In the case of linear systems with a single variable, the state equations are in the form of equations with differences

$$\boldsymbol{x}(\boldsymbol{k}+\boldsymbol{1}) = \boldsymbol{Ax}(\boldsymbol{k}) + \boldsymbol{Bu}(\boldsymbol{k}), \qquad \boldsymbol{y}(\boldsymbol{k}) = \boldsymbol{Cx}(\boldsymbol{k}) + \boldsymbol{Du}(\boldsymbol{k}). \tag{12}$$

The input-output models directly express the linkage between the *u* and *y* variables, in the form of differential equations for continuous time or differences for discrete time systems, as in digitization process. For a discrete time system, the transfer function is the ratio of the *z-transforms* of the output and input variables deducted under null initial conditions, similar with Laplace transform made for continuous systems. Writing the equation in differences and by applying the *z-transform* under null initial conditions, we obtain the *TF* [1, 11]:

$$H(z) = \frac{b_1 z^{-1} + b_2 z^{-2} \ldots + b_n z^{-n}}{1 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_n z^{-n}}. \tag{13}$$

5. Poles and zeros

FRF of a system which is a physical structure has poles and zeros which give its dynamic movement. Similar results, but is *s*-domain it can be obtained for Transfer Function *TF* using Laplace transform. From this, we can deduce the relation between *FRF* and *TF*. As both represent a ratio between inputs and outputs with aim of changing the differential equation in polynomial expressions, *poles* are the solutions of denominator and *zeros* of the nominator. As interpretation of this, *poles* represent the natural frequencies and *zeros* represent the position of the nodes and depends also on the boundary conditions [11].

$$H(s) = \frac{Y(s)}{U(s)} = \frac{b_{n-1}s^{n-1} + b_{n-2}s^{n-2} + \ldots + b_0}{s^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \ldots + a_0} = b_{n-1} \frac{\prod_{i=1}^{n-1}(s - z_i)}{\prod_{i=1}^{n-1}(s - p_i)}, \tag{14}$$

where $z_i, i = \overline{1, n-1}$, are *zeros* and $p_i, i = \overline{1, n}$, the *poles* of the function. The poles of a transfer function *G(s)* is the value of the coefficient *s* so that the solution of the transfer function equation has a value other than zero for an input *u(s)* equal to zero and from (9)

$$(sI - A)u(s) = 0, \tag{15}$$
$$|sI - A| = s^2 - \omega_n^2 = 0,$$

result $s = \pm\, j\omega_n$. The poles are the systems eigen-frequencies and are influenced by the physical shape and properties of the materials and the boundary conditions and have an important role in system *stability*. The poles have complex form and the real part is related to the stability of the system and the imaginary part is the oscillatory component of the system response. If the real part of all poles is negative, the solution of the state equation tending asymptotically to zero and the system is becoming asymptomatic stable. If the real part of any of the poles is zero, the system is stable. If the real part of at least one pole is positive, the system response increases exponentially without additional disturbance, and the system becomes unstable [12]. Control system zeros are the frequencies for which a non-zero input produces a null output and from (9) we have the matrix form:

$$\begin{bmatrix} sI - A & -B \\ C & 0 \end{bmatrix} \begin{bmatrix} x(s) \\ C(s) \end{bmatrix} = 0. \tag{16}$$

The poles depend only on the physical parameters of the system, and the zeros depend on both the physical parameters and the position of the inputs, outputs or actuators and sensors. Zeros are dead zones in which the energy transfer is not realized, even if there is an input of energy. If a sensor is placed in such a dead zone, the sum of the system’s natural modes will be null and it will not be recorded by the sensors. Characteristics of the poles and zeros of *TF* of the continuous systems are the same also in the case of discrete systems. Zeros-poles representation on plan *s* or *z* it is done with *TF* of form:

$$H(s) = K \cdot \frac{\prod_{i=1}^{n-1}(s - z_i)}{\prod_{i=1}^{n}(s - p_i)}, \text{ respective } \quad H(z) = K \cdot \frac{\prod_{i=1}^{n-1}(z - z_i)}{\prod_{i=1}^{n}(z - p_i)}. \qquad (17)$$

In the case of active vibro-acoustic control systems, the control parameters aim to modify the mechanical disturbance parameters, without an initial direct dependence between them. We have to place the poles, in the case of a stable system, in the left half of the complex plane [12].

Controllability and *observability* matrix were introduced in 1960 by *R. Kalman* [14] with the scope of obtaining a method of better controlling a dynamic system under better observation of its behavior, or finding the dynamic response of a system by interpreting the measurements outputs. If we know the model and the state of the system *x(k)* at a certain step *k*, it could be found the control raw *u(k), u(k+1),...u(k+n-1)* and with this it could be obtain a desired output after a certain number of steps. By knowing the *TF* of a system which describe its response, then we could have upon the system the *controllability* and *observability*, in certain conditions. From this, we assume that the right placement of actuators could lead to a better *controllability* by right placement of poles on the complex plane. We can say that a system under control is *observable* if its state *x(k)* at any step could be obtained from the systems model and from the measurements of outputs or in other words, the behavior of the system could be obtained by knowing the signals from the sensors. Using the modal analysis and state-space model, modal filters could be made for controlling the system [8].

For knowing if a system is *controllable* or if it is not *controllable*, the rank of the controllability matrix $\mathbf{C=[B \quad AB \quad ....A^{2n-1}B]}$ should be a *full rank* one or *rank**C**=2n*, where **A** is a state matrix and **B** is the input vector as multiple of $(1, 0, ...,0)^T$. The level of controllability is given by the optimal placement of actuators and sensors. Also the *observability* matrix $\mathbf{O=[C \quad CA \quad C\ A2 \quad CA3....\ CA^{2n-1}]^T}$ should be a full rank one, or *rank**O**=2n*, for the system to be observable [11, 12]. Although these conditions don't give any information regarding the observability of the modes, it gives the conclusion that the system is observable or not [16].

6. Experiment and results

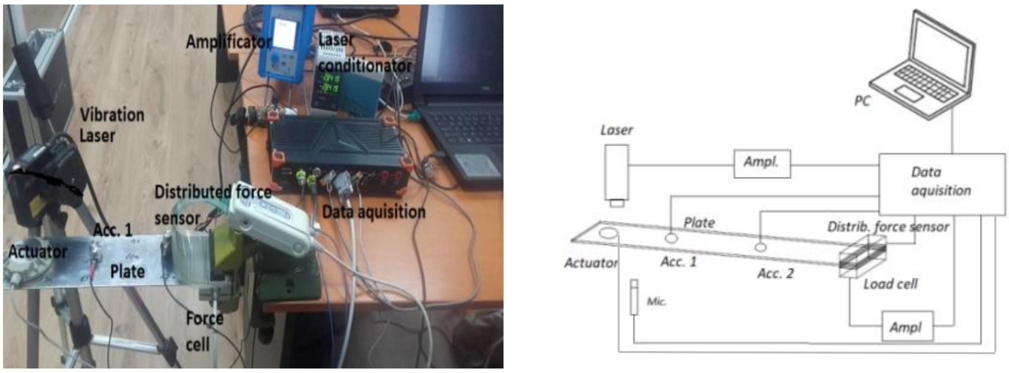
In the experiment, the *TF* will be determined and will be located the most suitable points for positioning sensors and actuators. For this purpose, as in Fig. 2, an electrodynamic exciter attached to a simple structure, one laser velocimeter, two accelerometers and a microphone will be used to determine both the structure response to harmonic sweep excitation and the acoustic response generated by the excited structure. Exciter and sensors mounted at various points of the structure will be tested to obtain an optimal location. Also load cell for force measurement and piezoresistive matrix sensor for mapping the distribution of the force on the boundary domain it is used. Measurements were made and Bode frequency response were plotted for amplitude and phase, the coherence diagram between the excitation signal and the structure and acoustic response, and Nyquist diagrams in the complex field as in Fig. 3, 4. It was used a sweep frequency signal from 100 to 2000 Hz. As structure was used an aluminium plate with dimensions of 330 × 90 × 2 mm with E = 71000 [N/mm^2], Poisson 0.37 and ρ = 2710 [kg/m^3], rigidly clamped using also an elastic thin layer on one side and it was marked on it 18 places for possible placements of sensors and actuators. On the clamped side it was placed a distributed force matrix sensor from company Tekscan which came with its own amplifier and signal conditioning, for measuring and mapping the force. The clamping total force applied was changed between three different levels, 1 N, 1.5 N and 2 N and measured with a piezoresistive sensor produced by company Laumas. Two accelerometers and one microphone from Bruel&Kjaer were used. The electrodynamic actuator HIAX was used for actuating the generated input function and also forces. All the signals acquired from the sensors and also the actuator control was made with a DEWEsoft Sirius 8 channel system as in Fig. 2. With DEWEsoft software it was made the modal analysis and also the function generation for the actuator. The main objective of the article was to find the modal parameters and as a consequence, the optimal poles and zeros and finally the location of sensors and actuators, if there are different boundary constraints. In analytical calculation, the boundaries are considered mostly as ideal, mainly because of complexities and difficulties of modeling the constraints. In this study the approach was for using an elastic boundary at one edge with different clamping forces mapped with a matrix piezoresistive sensor. Using smart matrix force sensors for mapping the distribution of the force on the interface between the structure and the fixed base or between two structures, the frequency response function was found as being different than the classic rigidly fixed (clamped). In the case of active vibration control, modal testing under different boundary conditions it is a more realistic method for finding more precise the modal parameters (natural frequencies, damping and mode shapes) and this could lead to a better *observability* and *controllability* of the system and better dynamic response and behavior. Circle matching or circle-fit method [2, 13] which consists in a series of operations around the resonance mode peak has the aim of more precise fixing of resonance frequency. The theory under the calculation of circle-fit for structures with structural damping on these tests has as basis the equation

$$\alpha(\omega) = \frac{1}{\omega_r^2\left[1-\left(\frac{\omega}{\omega_r}\right)^2 - i\eta_r\right]}, \tag{18}$$

where α is the response function , ω_r is the natural frequency, and η_r is the structural loss [15]. It has the aim of obtaining more precise information about the resonance frequency and damping values, damping which cannot be obtained in other ways than experimentally. Because the frequency resolution is not so precise, it is hard to find the exact value where the real part of *FRF* is passing the abscissa axis of complex plane. This zero passing point is happening at the natural frequency and at this value the structural loss is

$$\eta_r = \frac{{\omega_1}^2 - {\omega_2}^2}{{\omega_r}^2} \ . \tag{19}$$

The advantage of circle-fit method (Fig. 4, 5) is that it includes the influence of other modes on the one to whom the determination is made. There is the case of systems which have many degrees of freedom, where is an influence of a number of adjacent modes on the resonance mode for which is made the analysis. On the plate was marked 45 points, equidistant covering the surface (ex. A10-S16-S22 means that the actuator is placed at point 10 and accelerometers at point 16 and 22). The *TF* between actuator and accelerometers was noted with AS1 or AS2.



(a) (b)

Fig. 2 Experiment configuration (a) working stand, (b) experiment concept

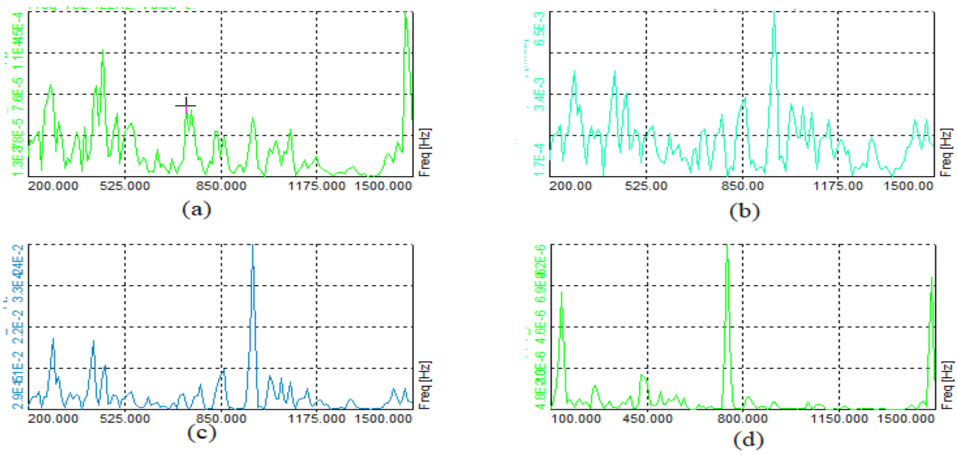


Fig. 3 Cross-correlation actuator-accelerometer A/S1 (a) and A/S2 (b), auto-correlation accelerometer S1 (c) and S2 (d)

As comparison, *FEA* / *pole fitting* modes with certain placement of actuators A and sensors S with which was made actually the experiment and obtained the transfer function. The frequency response function is obtained after the measurements and processing the signal, as a ratio between the cross-spectrum of force-excitation and acceleration-response signal S_{AS} and auto-spectrum (Fig.2) of the signal obtained from the force-exciter S_{AA}.

$$H(\omega) = \frac{S_{AS}(\omega)}{S_{AA}(\omega)}. \tag{20}$$

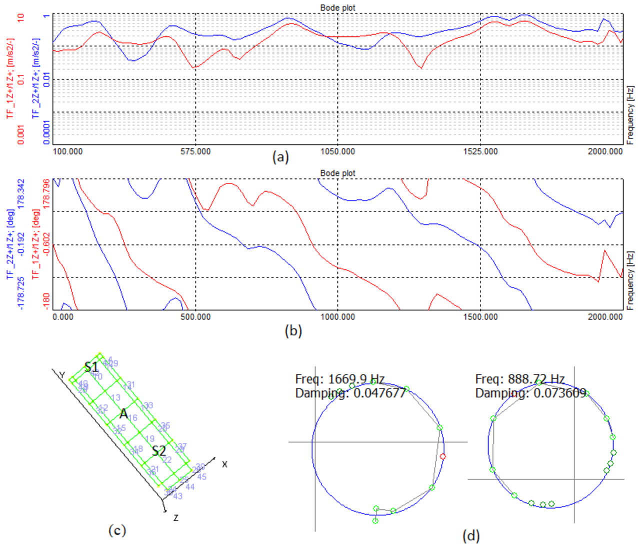


Fig. 4 Transfer function on amplitude Bode plot (a) and phase Bode plot (b), location of actuator A and accelerometers S1 and S2 (c), poles circle-fit and damping, for 1.5 N total boundary force

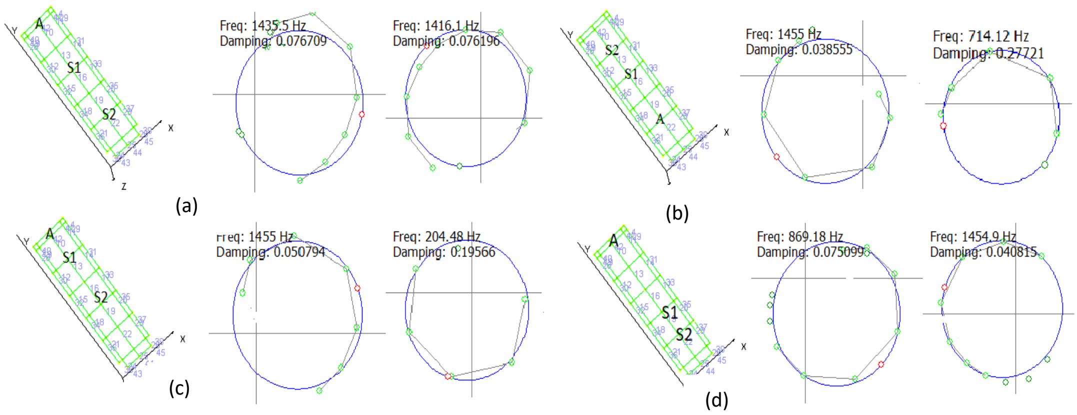


Fig. 5 Actuator-sensors placement A10-S16-S22 (a), S13-S16-A22 (b), A10-S13-S19 (c), A10-S19-S22 (d) and corresponding poles circle-fit and damping

Table 1

Comparison of FEA mode shapes and experimental modal analysis results

Actuator-sensors locations	Boundary clamping force	FEA modes [Hz]	Experimental poles fitting AS1/AS2 [Hz]	Damping experimental response AS1/AS2
A10-S16-S22	1N	1430.3 / 1411.7	1435.5 / 1416.1	0.076709 / 0.076196
A10-S13-S19	1N	1451 / 209.2	1455 / 204.48	0.050794 / 0.19566
A10-S16-S22	1.5N	1427 / 1421.4	1420.7 / 1415.2	0.077432 / 0.075402
A10-S13-S19	1.5N	1457 / 217.6	1462 / 211.5	0.057325 / 0.09369
A10-S16-S22	2N	1415 / 1439.2	1425.4 / 1432	0.073264 / 0.071475
A10-S13-S19	2N	1477 / 255.5	1471 / 251.3	0.068934 / 0.074974
S13-S16-A22	1N	1451 / 722.2	1455 / 714.12	0.038555 / 0.27721
A10-S19-S22	1N	863.2 / 1447.9	869.18 / 1454.9	0.075099 / 0.040815
S13-S16-A22	1.5N	1469 / 714.3	1461.3 / 723.4	0.056349 / 0.13477
A10-S19-S22	1.5N	849.6 / 1469.6	852.2 / 1477.3	0.096634 / 0.063597
S13-S16-A22	2N	1518.5 / 794.1	1521.3 / 772.4	0.075093 / 0.093664
A10-S19-S22	1.5N	881 / 1494.6	891.7 / 1479.7	0.076354 / 0.093345

In the *Table 1* we have included the results of different locations of actuators and sensors according with poles placements obtained experimentally and similar mode shapes obtained with FEA, under different clamping boundary force. The poles experimentally obtained and circumscribed in the fitting circle gives the position of sensors and actuators and this position it is compared with position of maximum displacement on mode shapes obtained with FEA. For this case of structural damping, the frequency response function *FRF* was used. The method of poles matching the circle was adopted because of the assumption that the resonance of the system takes place at the point of maximum of the vibration amplitude and the damping corresponding to this peak can be determined from the frequency band around the peak resonance mode, considered at half level of power which is with 3dB lower than the peak.

7. Conclusion

Modal analysis and state-space methods are good approaches to determine the position of poles and zeros and thus, the level of stability, observability and controllability of the control system. The placement of poles and zeros are giving the positions where could be applied the sensors and actuators. An optimal control implies a correct positioning of the sensors and actuators and for this, determination of the frequency response of the structure is compulsory. Thus, this article approached the modal analysis as a good solution for a small system where it was supposed that the mode shapes are well separated and a feedback control could be used. Feedback type active vibration control has the optimal effect when it is tuned to the resonant low-frequencies of the vibro-acoustic structure or space. For this analysis it was considered a frequency band much broader, because each mode inside the researched band is influenced by modes which are placed outside the band, so the energy introduced by these modes should be considered also. The aim was to determine the resonance frequency and structure response using modal analysis and from this, the mode shapes and damping, under different elastic boundary clamped edge. The main objective of this study was to get the response of the structure on the condition where on the clamped area an exterior force was applied at different levels for "augmenting" the elastic behavior and finally the response of the structure. It can be also appreciated that effective modal control can be achieved on the low frequency range, including on the feedback loop the control of the clamping force.

R E F E R E N C E S

[1] *Jimin He, Zhi Fu,* Modal Analysis, Butterworth-Heinemann, pp.123-138, 2001

[2] *Hansen C., Snyder S., Qiu X., Brooks L., Moreau D.,* Active control of noise and vibration, Vol 2, pp.1155-1213, Taylor & Francis Group, 2012

[3] *George Fox lang,* Understanding Modal Vectors, Sound and Vibration Magazine, The Society for Experimental Mechanics, 2017

[4] *Sonowane A., Talmale P.,* Modal Analysis of Single Rectangular Cantilever Plate by Mathematically, FEA and Experimental, International Research Journal of Engineering and Technology, 2017

[5] *Kumble D., Kumar C., Sontakke r., Gaikvad R.,* Analytical and Experimental Analysis of Cantilever Beam under Forced Vibration, IJESC, vol.6, Issue no. 3, 2016

[6] *Bhusnar M., Sarawade S.,* Modal Analysis of Rectangular Plate with Lap Joints to Find Natural Frequencies and Mode Shapes , Journal of Mechanical & Civil Engineering, 2016

[7] *Gupta V., Sharma M., Thakur N.,* Optimization Criteria for Optimal Placement of Piezoelectric Sensors and Actuators on a Smart Structure: A Technical Review, Journal of Intelligent Material Systems and Structures, 2010

[8] *Leissa A. W.,* Vibration of plates, NASA, 1969

[9] *Leissa A. W., Laura P. A., Gutierez R. H.,*Vibrations of Rectangular Plates With Nonuniform Elastic Edge Supports, Journal of Applied Mechanics, Vol. 47, 1980

[10] *Li W. L., Zhang X., Du J., Liu Z,* An exact series solution for the transverse vibration of rectangular plates with general elastic boundary supports, Journal of sound and vibration 321, 2009

[11] *D. Inman,* Vibration with Control, John Wiley and Sons, 2006

[12] *Hansen C., Snyder S., Qiu X., Brooks L., Moreau D.,* Active control of noise and vibration, Vol 1, pp.301-303, Taylor & Francis Group, 2012

[13] *Baruh, H.* Placement of sensors and actuators in structural control. *Control and Dynamic Systems,* (1992).

[14] *R. Kalman,* On the general theory of control systems, Research Institute for Advanced Study, 1960

[15] *D. J. Ewins*, Modal Testing: Theory, Practice and Application. 2nd Edn. Research Studies Press, Baldock, pp.309, 2000

[16] *William S. Levine,* Control Systems Advanced Methods, CRC Press, pp. 55-67, 2011