SINGLE SENSOR 3-AXIS ATTITUDE DETERMINATION SYSTEM FOR CUBESATS

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The classical methods used for satellites 3-axis attitude determination require at least two onboard independent sensors. The paper presents a 3-axis attitude determination method that can be implemented on board small satellites using measurements from a single sensor. The simulations results are presented for the cubesat satellite type orbit with the apogee at 1450 km, the perigee at 300 km and an inclination of 69.5 degrees, achieved with the Vega rocket maiden flight on 13 February 2012.

Keywords: attitude determination, nanosatellite, cubesat

List of symbols

A	states matrix of the Kalman filter
B, B_x, B_y, B_z	magnetic field vector and its components
$\dot{B}, \dot{B}_x, \dot{B}_y, \dot{B}_z$	magnetic field derivative vector and its components
$\hat{B}, \hat{B}_x, \hat{B}_y, \hat{B}_z$	estimate of the magnetic field vector and its components
$\hat{\dot{B}}, \hat{\dot{B}}_x, \hat{\dot{B}}_y, \hat{\dot{B}}_z$	estimate of the magnetic field derivative vector and its components
B_b	magnetic field vector expressed in the body frame (satellite frame)
B_I	magnetic field vector expressed in the inertial frame
C	measurement matrix of the Kalman filter
I_{xx}, I_{yy}, I_{zz}	principal inertia moments
DADMOD	Deterministic Attitude Determination using Magnetometer-Only Data
ECEF	Earth Center Earth Fix frame
EKF	extended Kalman Filter
q	spacecraft attitude quaternion
q^c	conjugate of the spacecraft attitude quaternion

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q_0, q_1, q_2, q_3	quaternion elements of q
w	process noise
x	state vector of the Kalman filter
y	measurements vector of the Kalman filter
θ, ϕ, ψ	Euler angles
$\omega_x, \omega_y, \omega_z$	satellite's angular speed of the satellite

1. Introduction

The absolute attitude of a spacecraft can be determined when two independent vectors are known, each of them expressed in an inertial frame and in a spacecraft (body fixed) frame, forming the basis of the TRIAD algorithm [1], [2]. Other early attitude determination studies have resulted in the QUEST method and additional solutions to Wabha's problem, all of them representing developments in solving the attitude determination issue [3].

In his book [4], Wertz presents various aspects of vector-based attitude determination and basics attitude quaternion derivation used to reduce the complexity of having a nine element attitude matrix. The most common algorithms use a combination of the following sensors: magnetometer, gyroscope, horizon sensor, sun sensor, star tracker and inertial measurement unit. For small satellites, the mass, volume and power budgets are the most important restrictions which lead either to the use of low power microprocessors and very advanced levels of PCB's miniaturization, either to the lack of a consistent payload.

A step forward was made by Santoni and Bolotti showing that the solar panels necessary for power generation can be used as a sensing system for attitude determination [5]. After that, the DADMOD method [6] was one of the first algorithms that give a solution using a single sensor for a complete attitude determination, followed by Psiaki's Kalman filtering model [7] that showed errors of 3 degrees after 100 seconds of simulation.

The modeling of the proposed method was developed around a low cost 3-axis resistive magnetometer [8] together with a MSP430 microprocessor. The method uses two extended Kalman filters (EKF) for estimates with respect to the satellite frame and a simple finite differencing model for obtaining magnetic field derivative with respect to the inertial frame. The first EKF filter is used to estimate the magnetic field on orbit and its first derivative. These estimates are then converted in measurements for the second filter in order to obtain a fully estimate of the states: attitude quaternion q and angular rates $\omega_x, \omega_y, \omega_z$.

2. Attitude determination using a single onboard sensor

Due to the drastic restrictions imposed by the Cubesat standard [9], the use of a single attitude determination sensor has become a challenge especially when the final target is to maintain the overall accuracy.

The proposed method computes the satellite's attitude using as unique sensor an onboard 3-axis magnetometer [8]. The algorithm partially used in the method is known in literature as DADMOD [6] which requires two independent sets of measurements, each expressed in two reference frame systems: ECEF – and the body reference frame.

The first measurements vector is composed from the magnetic field values expressed in the body reference frame (direct measurements of the HMR3400 onboard instrument) and from the calculated values onboard satellite using the IGRF database expressed in the ECEF [10]. The second vector is quasi-independent with respect to the first one because it is composed from first magnetic field derivative expressed in the same reference frames. The improvement to the DADMOD method consists in using the Kalman filters in order to reduce the error caused by the disturbance noises.

The magnetic field measurements (from Honeywell HMR3400 onboard tri-axis magnetometer) were simulated adding white Gaussian noise to the true values of the magnetic field obtained from the IGRF database [10].

For the second pseudo-measurement vector one uses a pre-filter with a 2nd order Markovian process:

$$\frac{d^2}{dt^2} B = w \quad , \tag{1}$$

where B represents the magnetic field and w is the process noise (in this case Gaussian white noise with zero mean and 0.2 covariance). Using a 3rd order Markovian process, one can estimate the first derivative and the second derivative of the magnetic field and even the magnetic field value. The model used for the Kalman pre-filter has the following equations

$$\begin{aligned} \frac{d}{dt} B &= \dot{B} \\ \frac{d}{dt} \dot{B} &= \ddot{B} \end{aligned} \tag{2}$$

The measurements are:

$$y = \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} \tag{3}$$

The state vector is represented by:

$$x = \begin{bmatrix} B_x & B_y & B_z & \dot{B}_x & \dot{B}_y & \dot{B}_z \end{bmatrix}^T \quad (4)$$

and the states matrix together with the measurements matrix are as follows:

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (5)$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \quad (6)$$

One can easily check that the pair (C, A) is observable which property guarantees that the estimation problem is feasible. In the case studies presented further to illustrate the proposed method, a polar circular orbit with altitude at 400 km for duration of 3600 seconds (this means about 67% of an entire orbit) is considered. The satellite model used in simulation is the simplified model of the Goliat nanosatellite (mass 1Kg, dimensions 10x10x10 cm) [11]. The satellite inertia tensor in flight configuration (with both antennas deployed) considered in the simulation is (expressed in $kg \cdot m^2$):

$$\begin{aligned} &I_{xx} = 0.00528280271 \quad &&\mathrm{I}_{xy} = -0.00002191777 \quad &&\mathrm{I}_{xz} = -0.00005884673 \\ &I_{yx} = -0.00002191777 \quad &&\mathrm{I}_{yy} = 0.005132957.01 \quad &&\mathrm{I}_{yz} = -0.00003785080 \\ &I_{zx} = -0.00005884673 \quad &&\mathrm{I}_{zy} = -0.00003785080 \quad &&I_{zz} = 0.00202597332 \end{aligned} \quad (7)$$

The initial conditions for the satellite attitude quaternion and the satellite angular speed vector were chosen as follows:

$$q = \begin{bmatrix} 0.7071 & 0.0000 & 0.7071 & 0.0000 \end{bmatrix}^T \quad (8)$$

$$\omega = \begin{bmatrix} 2 & 2 & 2 \end{bmatrix}^T \ {}^0/\sec \quad (9)$$

which correspond in Euler angles attitude of

$$\begin{aligned} \theta &= 45^0 \\ \phi &= 90^0 \\ \psi &= 45^0 \end{aligned} \tag{10}$$

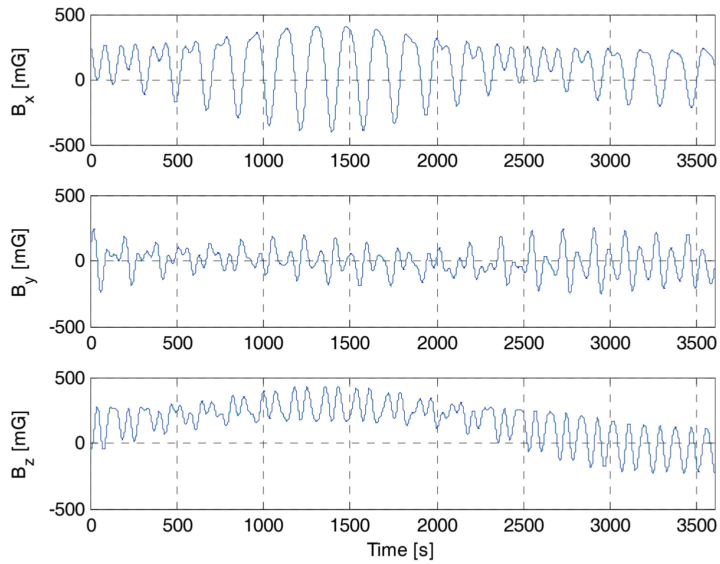

Fig. 1 – Estimated values of the magnetic field on each axis

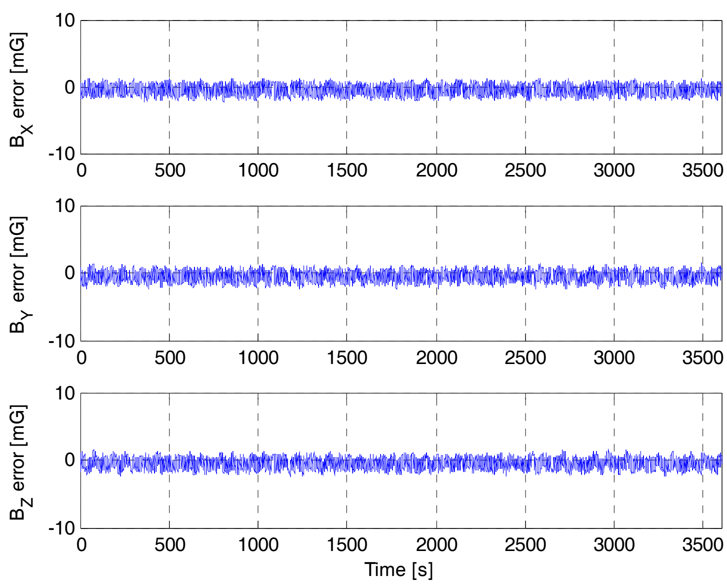

Fig. 2 – The estimated error of the magnetic field

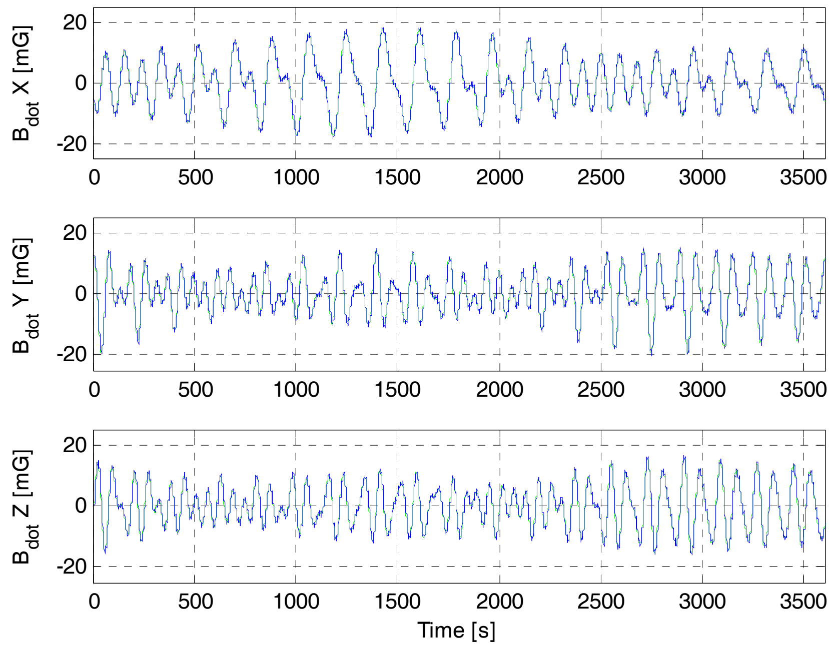

Fig. 3 – First derivative of the magnetic field

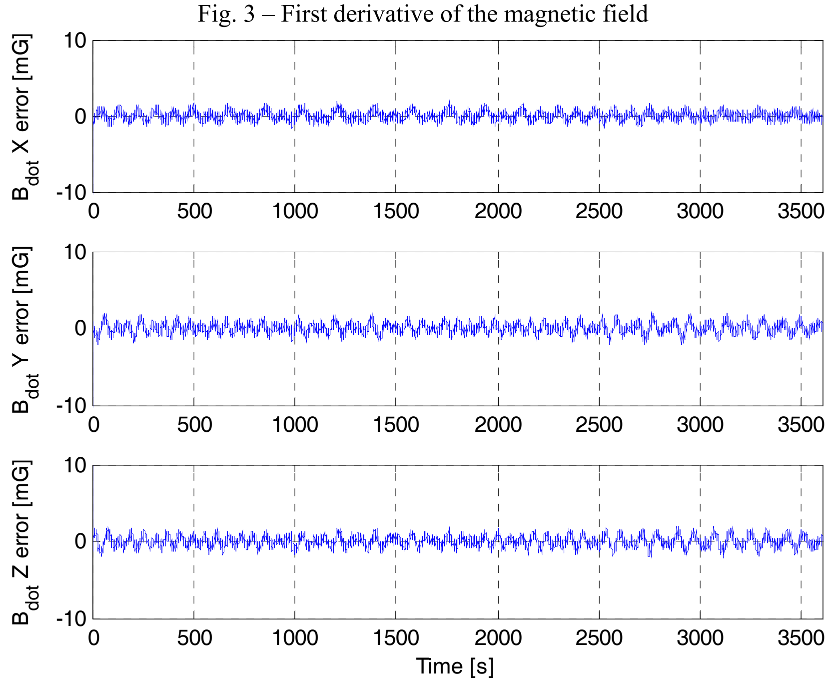

Fig. 4 – First derivative error of the magnetic field

The attitude filter is the second extended Kalman filter (EKF) that uses the estimates of the magnetic field and its derivative provided by the first EKF in order to obtain the angular quaternion estimate q together with estimates of the angular rates $\omega_x, \omega_y, \omega_z$.

The measurement vector for the second Kalman filter is:

$$y = \begin{bmatrix} \hat{B}_{b_x} & \hat{B}_{b_y} & \hat{B}_{b_z} & \hat{\dot{B}}_{b_x} & \hat{\dot{B}}_{b_y} & \hat{\dot{B}}_{b_z} \end{bmatrix}^T \tag{11}$$

where $\hat{B}_{b_x}, \hat{B}_{b_y}, \hat{B}_{b_z}, \hat{\dot{B}}_{b_x}, \hat{\dot{B}}_{b_y}, \hat{\dot{B}}_{b_z}$ are the estimates of the magnetic field and its derivative obtained from the first EKF filter.

The extended model for the second filter, including the kinematics and the dynamics of the satellite in terms of quaternion, angular speed and Euler's angles, is given by [12].

$$\frac{d}{dt}\begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} \frac{1}{2} q\omega \\ \frac{-\left(I_z - I_y\right)\omega_y\omega_z}{I_x} \\ \frac{-\left(I_x - I_z\right)\omega_x\omega_z}{I_x} \\ \frac{-\left(I_y - I_x\right)\omega_x\omega_y}{I_x} \end{bmatrix} \tag{12}$$

where $q_{0,}\, q_{1,}\, q_{2,}\, q_3$ are the attitude quaternion elements noted above as q.

Linearizing the right side terms around the equilibrium states $q = \begin{bmatrix} 0.7071 & 0.0000 & 0.7071 & 0.0000 \end{bmatrix}^T$ and $\omega = \begin{bmatrix} 2 & 2 & 2 \end{bmatrix}^T \, {}^0/\sec$, direct computations show that the state matrix corresponding to the linear approximation of the above system has the following expression:

$$A=\begin{bmatrix} 0 & -\frac{1}{2}\omega_x & -\frac{1}{2}\omega_y & -\frac{1}{2}\omega_z & -\frac{1}{2}q_1 & -\frac{1}{2}q_2 & -\frac{1}{2}q_3 \\ \frac{1}{2}\omega_x & 0 & \frac{1}{2}\omega_z & -\frac{1}{2}\omega_y & \frac{1}{2}q_0 & -\frac{1}{2}q_3 & \frac{1}{2}q_2 \\ \frac{1}{2}\omega_x & -\frac{1}{2}\omega_z & 0 & \frac{1}{2}\omega_x & \frac{1}{2}q_3 & -\frac{1}{2}q_0 & -\frac{1}{2}q_1 \\ \frac{1}{2}\omega_z & \frac{1}{2}\omega_y & -\frac{1}{2}\omega_x & 0 & -\frac{1}{2}q_2 & \frac{1}{2}q_1 & \frac{1}{2}q_0 \\ 0 & 0 & 0 & 0 & 0 & \frac{-\left(I_z-I_y\right)\omega_z}{I_x} & \frac{-\left(I_z-I_y\right)\omega_y}{I_x} \\ 0 & 0 & 0 & 0 & \frac{-\left(I_x-I_z\right)\omega_z}{I_y} & 0 & \frac{-\left(I_x-I_z\right)\omega_x}{I_y} \\ 0 & 0 & 0 & 0 & \frac{-\left(I_y-I_x\right)\omega_y}{I_z} & \frac{-\left(I_y-I_x\right)\omega_x}{I_z} & 0 \end{bmatrix} \tag{13}$$

The magnetic field is calculated from measurements using the quaternion transformation instead of the regular direct cosine matrix

$$\left\langle 0,\hat{B}_b\right\rangle = q^c\left\langle 0,\hat{B}_I\right\rangle q \tag{14}$$

where q is the spacecraft attitude quaternion, and q^c is the attitude quaternion conjugate: $q^c=\left[q_0,-q_1,-q_2,-q_3\right]$. The time derivative of the magnetic field results in:

$$\left\langle 0,\dot{\hat{B}}_b\right\rangle = \dot{q}^c\left\langle 0,\hat{B}_I\right\rangle q + q^c\left\langle 0,\dot{\hat{B}}_I\right\rangle q + q^c\left\langle 0,\hat{B}_I\right\rangle \dot{q} \tag{15}$$

Using the propagating dynamic equation together with equation (14) and equation (15), the states matrix may be computed using the finite differencing method on the following equation that relates the system's measurements to the states.

$$y=\begin{bmatrix} q^c\left\langle 0,\hat{B}_I\right\rangle q \\ \dot{q}^c\left\langle 0,\hat{B}_I\right\rangle q + q^c\left\langle 0,\dot{\hat{B}}_I\right\rangle q + q^c\left\langle 0,\hat{B}_I\right\rangle \dot{q} \end{bmatrix} = \begin{bmatrix} q^c\left\langle 0,\hat{B}_I\right\rangle q \\ \frac{1}{2}q^c\omega\left\langle 0,\hat{B}_I\right\rangle q + q^c\left\langle 0,\dot{\hat{B}}_I\right\rangle q + q^c\left\langle 0,\hat{B}_I\right\rangle \frac{1}{2}q\omega \end{bmatrix} \tag{16}$$

where above multiplications are quaternion multiplication [13] and $\langle 0, B_I \rangle$ is the magnetic field expressed in the inertial reference frame with a null value on the first position: $\langle 0, \hat{B}_I \rangle = \begin{bmatrix} 0 & \hat{B}_x & \hat{B}_y & \hat{B}_z \end{bmatrix}$. After multiplication the first value will be always a null one and it will be removed for simplification.

3. Conclusions and future work

When analyzing the results obtained during simulations, the following observations can be made.

The wavering observed in Figure 1 and Figure 3 is due to the satellite motion around its center of mass considered in this simulation in equation (9). Figure 2 and Figure 4 show that the error between the true magnetic field and the measured one is maintain within the $-2 \div +2mG$ domain. This concludes that the first extended Kalman filter estimates the magnetic field and its derivative in such a manner that these estimates are adequate as inputs for the second extended Kalman filter. Keeping the same orbit inclination but with different initial condition the second EKF filter estimates satisfactory the attitude quaternion and angular rates of the satellite. The next step in the research studies will be a reliability analysis by comparing the results of the filters after varying the orbit inclination plane which should modify the dynamics of the magnetic field and therefore the controllability of the system.

R E F E R E N C E S

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