KURTOSIS IN BLACK-SCHOLES MODEL WITH GARCH VOLATILITY

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The famous Black-Scholes option pricing model is a mathematical description of financial market and derivative investment instruments [3]. In Black-Scholes model volatility is a constant function, where trading option is indeed risky due to random components such as volatility. The notion of non constant volatility was introduced in GARCH processes [6]. Recently a Black-Scholes model with GARCH volatility has been presented [10]. In this article we derive the kurtosis formula for underlying financial time series using BS-Model with GARCH volatility for the case of at the money option. We present the kurtosis formula in terms of the model's parameters. Also we compare our computational results by using another measure of kurtosis for different values of volatilities.

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1. Introduction

Fischer Black and Myron Scholes published an option valuation formula in their 1973's article [3] that today is known as Black-Scholes model. The model has some restrictions for example; a constant risk frees interest rate r and a constant volatility σ which do not seem to be realistic. Trading option is risky due to the possibly high random components such as volatility. The concept of non constant volatility was introduced by GARCH models [6]. The study of stock price models under the GARCH volatility is a new horizon in derivative investment instruments. Duan was the first to provide a solid theoretical foundation for GARCH option pricing [7]. Recently a Black-Scholes model with GARCH volatility has been introduced [10]. The volatility measures, the variation of price of financial instrument over time and implied volatility can be derived from the market price of a traded derivative. In financial literature researchers use GARCH models frequently in order to forecast the volatility of underlying stock market. The GARCH models have been used to investigate their performance and consistency of the parameters [17]. Taylor series approximations

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have been frequently used in option pricing. In Risk management particularly first and second order Taylor approximations are crucial. Taylor approximations have been also used frequently in Black-Scholes option pricing formula [5,13].

In this article we consider Black-Scholes model with GARCH volatility [10]. In Section 2 we provide fundamental theory and tools. In Section 3 we present our results of kurtosis formula for the case of at the money option (ATM). In section 4 we present our computational results and we compare our results with a new measure of kurtosis [2]. Section 5 concludes our results.

2. Preliminaries

Let $(\Omega, F_t, \mathrm{P})$ be the probability space then price of an asset S at time t is a Geometric Brownian Motion (GBM).

$$dS_t = rS_t d_t + \sigma S_t dW_t \tag{2.1}$$

where $\{W_t\}$ is a standard Brownian motion and σ is the volatility. We know that according to Black-Scholes option pricing model [3], A European call option for Black-Scholes model is given by

$$C_{BS} = S\phi(d_1) - Ke^{-r\tau}\phi(d_2) \tag{2.2}$$

where $d_1 = \dfrac{\log\left(\dfrac{S}{K}\right) + \left(r + \dfrac{\sigma^2}{2}\right)}{\sigma\sqrt{\tau}}$ and $d_2 = d_1 - \sigma\sqrt{\tau}$ and $\phi(.)$ is a cumulative distribution function for standardized Normal random variable and $\tau = T - t$ and S is a price of an asset, K is the strike price, r is the interest rate and T denotes the time to expiry.

Definition 1 [3] If S is the stock price, r is risk free interest rate then C is a European call option that, gives its holder the right, but not the obligation to buy the one unit of underlying asset for a predetermined price K at the maturity date T.

When variance of the log of stock returns changes with time i.e. $\sigma = \theta_t$ then a Black-Scholes model with GARCH volatility for a financial time series let say y_t, is given by [10].

$$dS_t = rS_t d_t + \theta_t S_t dW_t$$

$$y_t = \log\left(\frac{S_t}{S_{t-1}}\right) - E\left(\log\left(\frac{S_t}{S_{t-1}}\right)\right) = \theta_t Z_t$$

where $\{W_t\}$ is a standard Brownian motion and $\{\theta_t\}$ is a volatility process. The call option for the model is given by:

$$C_{BSG} = SE_{\theta_t}\left[\phi(d_1)\right] - Ke^{-r\tau}E_{\theta_t}\left[\phi(d_2)\right] \tag{2.3}$$

Theorem 1 [10] For a twice differentiable functions $f(x)$ and $g(x)$, the call price (2.3) can be written as:

$$\begin{aligned} C_{BSG} &= S\left(f\left(E\left(\theta_t^2\right)\right) + \frac{1}{2} f''\left(E\left(\theta_t^2\right)\right)\left(\frac{1}{3}k^{(y)} - 1\right)E^2\left(\theta_t^2\right)\right) - \\ &\quad Ke^{-rT}\left(g\left(E\left(\theta_t^2\right)\right) + \frac{1}{2} g''\left(E\left(\theta_t^2\right)\right)\left(\frac{1}{3}k^{(y)} - 1\right)E^2\left(\theta_t^2\right)\right) \end{aligned} \tag{2.4}$$

where $k^{(y)} = \dfrac{E\left(y_t^4\right)}{E\left(y_t^2\right)}$ is the kurtosis of the observed logreturns y_t and

$$f\left(E\left(\theta_t^2\right)\right) = \phi(d_1) = \phi\left(\frac{\log\left(\frac{S}{K}\right) + rT + \frac{1}{2}E\left(\theta_t^2\right)}{\sqrt{E\left(\theta_t^2\right)}}\right) \tag{2.5}$$

$$g\left(E\left(\theta_t^2\right)\right) = \phi(d_2) = \phi\left(\frac{\log\left(\frac{S}{K}\right) + rT - \frac{1}{2}E\left(\theta_t^2\right)}{\sqrt{E\left(\theta_t^2\right)}}\right) \tag{2.6}$$

where, θ_t is a stationary GARCH process having mean μ_θ and variance σ_θ^2. Option pricing based on GARCH models have been studied under the assumption that the innovations are standard normal (i.e. under normal GARCH).

Remark 1 The second derivatives of $f\left(E\left(\theta_t^2\right)\right)$ and $g\left(E\left(\theta_t^2\right)\right)$ are given by

$$f''\left(E\left(\theta_t^2\right)\right) = \frac{1}{\sqrt{2\pi}}\exp\left\{\frac{-\left(2H + E\left(\theta_t^2\right)\right)^2}{8E\left(\theta_t^2\right)}\right\}\cdot$$

$$\cdot\left[\frac{6H - E\left(\theta_t^2\right)}{8E^2\left(\theta_t^2\right)\sqrt{E\left(\theta_t^2\right)}} - \left(\frac{E^2\left(\theta_t^2\right) - 4H^2}{8E^2\left(\theta_t^2\right)}\right)\left(\frac{E^2\left(\theta_t^2\right) - 2H}{8E\left(\theta_t^2\right)\sqrt{E\left(\theta_t^2\right)}}\right)\right]$$

$$g''\left(E\left(\theta_t^2\right)\right)=\frac{1}{\sqrt{2\pi}}\exp\left\{\frac{-\left(2H-E\left(\theta_t^2\right)\right)^2}{8E\left(\theta_t^2\right)}\right\}$$

$$\left[\frac{6H+E\left(\theta_t^2\right)}{8E^2\left(\theta_t^2\right)\sqrt{E\left(\theta_t^2\right)}}-\left(\frac{E^2\left(\theta_t^2\right)-4H^2}{8E^2\left(\theta_t^2\right)}\right)\left(\frac{E^2\left(\theta_t^2\right)+2H}{8E\left(\theta_t^2\right)\sqrt{E\left(\theta_t^2\right)}}\right)\right]$$

where

$$H=\left(\log\left(\frac{S}{K}\right)+rT\right)$$

Definition 2 [6,11] A process of the form $\theta_t^2=\omega+\sum_{i=1}^{p}\alpha_i\varepsilon_{t-i}+\sum_{=1}^{q}\beta_j\theta_{t-j}^2$ is called GARCH (p,q) process where $\varepsilon_t=\theta_t Z_t$ and $Z_t\sim N\left(0,\sigma_\theta^2\right)$, $\omega>0$, $\alpha_i\geq 0,\beta_j\geq 0$. Similarly a process of the form

$\theta_t^2=\omega+\sum_{i=1}^{p}\alpha_i\left(\varepsilon_{t-i}+\gamma\right)^2+\sum_{j=1}^{q}\beta_j\theta_{t-j}^2$ is called AGARCH-(I)-(p,q) process and AGARCH-(I)-(0,1) is given by $\theta_t^2=\omega+\sum_{i=1}^{p}\alpha_i\left(\varepsilon_{t-i}+\gamma\right)^2$,where γ is an extra parameter of asymmetry.

3. Main Results

An option is called at the money (ATM) if the strike price K is the same as the current spot price S of the underlying asset. We know the Black-Scholes call price when volatility is a GARCH process. The following theorem presents the general results for Black-Scholes model with GARCH volatility when option is at the money.

Proposition 1 For the call option C_{BSG} we obtain the following relations.

i. $\frac{\partial d_1}{\partial\theta_t}=1-\frac{d_1}{\theta_t},\frac{\partial d_2}{\partial\theta_t}=-1-\frac{d_2}{\theta_t}$ and $d_1-d_2=\theta_t$

ii. $\frac{\partial d_1}{\partial S}=\frac{\partial d_2}{\partial S},\frac{\partial d_1}{\partial K}=\frac{\partial d_2}{\partial K},\frac{\partial d_1}{\partial r}=\frac{\partial d_2}{\partial r},\frac{\partial d_1}{\partial T}=\frac{\partial d_2}{\partial T}$ and $\frac{\partial d_1}{\partial\theta_t}-\frac{\partial d_2}{\partial\theta_t}=1$

iii. $Ke^{-rT}\dfrac{\partial\phi(d_2)}{\partial d_2}-S\dfrac{\partial\phi(d_1)}{\partial d_1}=0$

Proof We know that the value of Call option for BS-model with GARCH volatility is give by equation (2.3). The relations i and ii can be easly obtaind by following the values of d_1 and d_2 where the relation iii can be obtained using the value of K and $\dfrac{\partial\phi(d_1)}{\partial d_1}=\dfrac{1}{\sqrt{2\pi}}e^{-\frac{d_1^2}{2}}, \dfrac{\partial\phi(d_2)}{\partial d_2}=\dfrac{1}{\sqrt{2\pi}}e^{-\frac{d_2^2}{2}}$

$$Ke^{-rT}-\frac{\partial\phi(d_2)}{\partial d_2}-S\frac{\partial\phi(d_1)}{\partial d_1}=Se^{\left\{-\left(d_1\theta_t-rT-\frac{\theta_t^2}{2}\right)\right\}}.e^{-rT}.\frac{1}{\sqrt{2\pi}}e^{\frac{-d_2^2}{2}}-S\frac{1}{\sqrt{2\pi}}e^{\frac{-d_1^2}{2}}$$

$$=\frac{S}{\sqrt{2\pi}}e^{\left\{-\left(d_1\theta_t-rT-\frac{\theta_t^2}{2}\right)-rT\right\}}.e^{\frac{-(d_1-\theta_t)^2}{2}}-S\frac{1}{\sqrt{2\pi}}e^{\frac{-d_1^2}{2}}=0$$

Theorem 2 Consider the equations (2.5) and (2.6) when stock price S and strike price K are identical i.e. $S=K$ and $r=0$, then we can write as:

$$f\left(E\left(\theta_t^2\right)\right)=\phi(d_1)=\phi\left(\frac{\sqrt{E\left(\theta_t^2\right)}}{2}\right) \text{ and}$$

$$g\left(E\left(\theta_t^2\right)\right)=\phi(d2)=\phi\left(\frac{-\sqrt{E\left(\theta_t^2\right)}}{2}\right)$$

Then we have following results:

i. $f''\left(E\left(\theta_t^2\right)\right)=\dfrac{1}{\sqrt{2\pi}}e^{\frac{-E\left(\theta_t^2\right)}{8}}\left[\dfrac{-1}{8E\left(\theta_t^2\right)\sqrt{E\left(\theta_t^2\right)}}-\dfrac{1}{32\sqrt{E\left(\theta_t^2\right)}}\right]$

ii. $g''\left(E\left(\theta_t^2\right)\right)=\dfrac{1}{\sqrt{2\pi}}e^{\frac{-E\left(\theta_t^2\right)}{8}}\left[\dfrac{1}{8E\left(\theta_t^2\right)\sqrt{E\left(\theta_t^2\right)}}+\dfrac{1}{32\sqrt{E\left(\theta_t^2\right)}}\right]$

Proof : Following the given condition for $f\left(E\left(\theta_t^2\right)\right)=\phi\left(\frac{\sqrt{E\left(\theta_t^2\right)}}{2}\right)$ and $g\left(E\left(\theta_t^2\right)\right)=\phi\left(\frac{-\sqrt{E\left(\theta_t^2\right)}}{2}\right)$ we obtain :

$$f''\left(E\left(\theta_t^2\right)\right)=\frac{-1}{8E\left(\theta_t^2\right)\sqrt{E\left(\theta_t^2\right)}}\varphi\left(\frac{\sqrt{E\left(\theta_t^2\right)}}{2}\right)+\frac{1}{16E\left(\theta_t^2\right)}\varphi'\left(\frac{\sqrt{E\left(\theta_t^2\right)}}{2}\right)$$

$$g''\left(E\left(\theta_t^2\right)\right)=\frac{1}{8E\left(\theta_t^2\right)\sqrt{E\left(\theta_t^2\right)}}\varphi\left(\frac{-\sqrt{E\left(\theta_t^2\right)}}{2}\right)+\frac{1}{16E\left(\theta_t^2\right)}\varphi'\left(\frac{\sqrt{E\left(\theta_t^2\right)}}{2}\right)$$

where $\varphi\left(E\left(\theta_t^2\right)\right)=\frac{1}{\sqrt{2\pi}}e^{\frac{-\left(E\left(\theta_t^2\right)\right)^2}{2}}$, therefore $\varphi'\left(E\left(\theta_t^2\right)\right)=-E\left(\theta_t^2\right)\varphi\left(E\left(\theta_t^2\right)\right)$ and thus we obtain the required form of the $f''\left(E\left(\theta_t^2\right)\right)$ and $g''\left(E\left(\theta_t^2\right)\right)$.

Theorem 3 Consider the call option of BS-model with GARCH volatility

$$C_{BSG}=S\left(f\left(E\left(\theta_t^2\right)\right)+\frac{1}{2}f''\left(E\left(\theta_t^2\right)\right)\left(\frac{1}{3}k^{(y)}-1\right)E^2\left(\theta_t^2\right)\right)-$$
$$Ke^{-rT}\left(g\left(E\left(\theta_t^2\right)\right)+\frac{1}{2}g''\left(E\left(\theta_t^2\right)\right)\left(\frac{1}{3}k^{(y)}-1\right)E^2\left(\theta_t^2\right)\right)$$

Then for the case of ATM the value of the kurtosis is in terms of the models parameters is given by

$$k^{(y)}=\frac{768\left(\sqrt{\frac{2\pi}{E\left(\theta_t^2\right)}}.\frac{C_{BSG}}{S}-1\right)}{\left(E\left(\theta_t^2\right)+4\right)\left(E\left(\theta_t^2\right)-8\right)}+3$$

where C_{BSG} is the value of call option for BS-model with GARCH volatility, S is the stock price and θ_t is the GARCH volatility.

Proof Using Theorem 1 for the case of ATM option and interest rate $r=0$ we obatin

$$C_{BSG}=S\left[f\left(E\left(\theta_t^2\right)\right)-g\left(E\left(\theta_t^2\right)\right)+\frac{k_1}{2}\left(f''\left(E\left(\theta_t^2\right)\right)-g''\left(E\left(\theta_t^2\right)\right)\right)E^2\left(\theta_t^2\right)\right] \tag{3.1}$$

In the above equation (3.1) $k_1=\frac{k^{(y)}}{3}-1$. Using the property , $d_1-d_2=\sqrt{E\left(\theta_t^2\right)}$ and expansion $\phi\left(E\left(\theta_t^2\right)\right)\cong\frac{1}{2}+\frac{1}{\sqrt{2\pi}}\left[E\left(\theta_t^2\right)-\frac{E^3\left(\theta_t^2\right)}{6}+\frac{E^5\left(\theta_t^2\right)}{40}-..\right]$, we find

$$\begin{aligned}f\left(E\left(\theta_t^2\right)\right)-g\left(E\left(\theta_t^2\right)\right)&\cong\left[\frac{1}{2}+\frac{d_1}{\sqrt{2\pi}}\right]-\left[\frac{1}{2}+\frac{d_2}{\sqrt{2\pi}}\right]\\&\cong\frac{1}{\sqrt{2\pi}}\left(d_1-d_2\right)=\frac{1}{\sqrt{2\pi}}\sqrt{E\left(\theta_t^2\right)}\end{aligned} \tag{3.2}$$

Now we find $f''\left(E\left(\theta_t^2\right)\right)$ and $g''\left(E\left(\theta_t^2\right)\right)$. Let us suppose $H=\left(\log\left(\frac{S}{K}\right)+rT\right)$, then we can write as :

$$f''\left(E\left(\theta_t^2\right)\right)=\frac{1}{\sqrt{2\pi}}\left(e^{\frac{-E\left(\theta_t^2\right)}{8}}\right)\left[\frac{-1}{8E\left(\theta_t^2\right)\sqrt{E\left(\theta_t^2\right)}}-\frac{1}{32\sqrt{E\left(\theta_t^2\right)}}\right]$$

Using the expansion $e^{\frac{-E\left(\theta_t^2\right)}{8}}=1-\frac{E\left(\theta_t^2\right)}{8}+..$ we obtain

$$f''\left(E\left(\theta_t^2\right)\right)\cong\frac{1}{\sqrt{2\pi}}\left(1-\frac{E\left(\theta_t^2\right)}{8}\right)\left[\frac{-4-E\left(\theta_t^2\right)}{32E\left(\theta_t^2\right)\sqrt{E\left(\theta_t^2\right)}}\right]=\frac{1}{\sqrt{2\pi}}\left[\frac{\left(E\left(\theta_t^2\right)-8\right)\left(E\left(\theta_t^2\right)+4\right)}{256E\left(\theta_t^2\right)\sqrt{E\left(\theta_t^2\right)}}\right]$$

i.e.

$$f''\left(E\left(\theta_t^2\right)\right)\cong\frac{1}{\sqrt{2\pi}}\left[\frac{\left(E\left(\theta_t^2\right)\right)^2-4E\left(\theta_t^2\right)-32}{256E\left(\theta_t^2\right)\sqrt{E\left(\theta_t^2\right)}}\right]$$

Similarly we can find

$$g''\left(E\left(\theta_t^2\right)\right)\cong\frac{1}{\sqrt{2\pi}}\left(1-\frac{E\left(\theta_t^2\right)}{8}\right)\left[\frac{4+E\left(\theta_t^2\right)}{32E\left(\theta_t^2\right)\sqrt{E\left(\theta_t^2\right)}}\right]$$

i.e.

$$g''\left(E\left(\theta_t^2\right)\right)\cong\frac{-1}{\sqrt{2\pi}}\left[\frac{\left(E\left(\theta_t^2\right)\right)^2-4E\left(\theta_t^2\right)-32}{256E\left(\theta_t^2\right)\sqrt{E\left(\theta_t^2\right)}}\right]$$

Now we can obtain

$$f''\left(E\left(\theta_t^2\right)\right)-g''\left(E\left(\theta_t^2\right)\right)=\frac{1}{\sqrt{2\pi}}\left[\left[\frac{\left(E\left(\theta_t^2\right)\right)^2-4E\left(\theta_t^2\right)-32}{256E\left(\theta_t^2\right)\sqrt{E\left(\theta_t^2\right)}}\right]+\left[\frac{\left(E\left(\theta_t^2\right)\right)^2-4E\left(\theta_t^2\right)-32}{256E\left(\theta_t^2\right)\sqrt{E\left(\theta_t^2\right)}}\right]\right]$$

$$=\frac{1}{\sqrt{2\pi}}\left[\frac{\left(E\left(\theta_t^2\right)\right)^2-4E\left(\theta_t^2\right)-32}{128E\left(\theta_t^2\right)\sqrt{E\left(\theta_t^2\right)}}\right]$$

Now we put the values of $f\left(E\left(\theta_t^2\right)\right)-g\left(E\left(\theta_t^2\right)\right)$ and $f''\left(E\left(\theta_t^2\right)\right)-g''\left(E\left(\theta_t^2\right)\right)$ in equation (3.1) and we obtain :

$$C_{BSG}=S\left[\frac{1}{\sqrt{2\pi}}\sqrt{E\left(\theta_t^2\right)}+\frac{k_1}{2}\left(\frac{1}{\sqrt{2\pi}}\left[\frac{\left(E\left(\theta_t^2\right)\right)^2-4E\left(\theta_t^2\right)-32}{128E\left(\theta_t^2\right)\sqrt{E\left(\theta_t^2\right)}}\right]\right)E^2\left(\theta_t^2\right)\right]$$

$$=\frac{S}{\sqrt{2\pi}}\left[\frac{256E\left(\theta_t^2\right)+k_1\left(\left(E\left(\theta_t^2\right)^3\right)-4\left(E\left(\theta_t^2\right)\right)^2-32E\left(\theta_t^2\right)\right)}{256\sqrt{E\left(\theta_t^2\right)}}\right]$$

$$Sk_1\left(\left(E\left(\theta_t^2\right)^3\right)-4\left(E\left(\theta_t^2\right)\right)^2-32E\left(\theta_t^2\right)\right)+256SE\left(\theta_t^2\right)-256\sqrt{2\pi}C_{BSG}\sqrt{E\left(\theta_t^2\right)}=0$$

which is equivalent to

$$k_1 = \frac{256\sqrt{2\pi}C_{BSG}\sqrt{E\left(\theta_t^2\right)} - 256SE\left(\theta_t^2\right)}{S\left(\left(E\left(\theta_t^2\right)\right)^3 - 4\left(E\left(\theta_t^2\right)\right)^2 - 32E\left(\theta_t^2\right)\right)}$$

$$\frac{k^{(y)}}{3} - 1 = \frac{256\left(\sqrt{\frac{2\pi}{E\left(\theta_t^2\right)}}.\frac{C_{BSG}}{S} - 1\right)}{\left(E\left(\theta_t^2\right)\right)^2 - 4E\left(\theta_t^2\right) - 32} \text{ i.e. } k^{(y)} = \frac{768\left(\sqrt{\frac{2\pi}{E\left(\theta_t^2\right)}}.\frac{C_{BSG}}{S} - 1\right)}{\left(E\left(\theta_t^2\right)\right)^2 - 4E\left(\theta_t^2\right) - 32} + 3$$

and we have obtained finally our result.

Proposition 2 If θ_t is a stationary GARCH process having mean μ_θ and variance σ_θ^2 then we

i. For the case of GARCH(1,1) , $E\left(\theta_t^2\right) = \frac{\omega}{\left[1-\left(\alpha_1\sigma_z^2 + \beta_1\right)\right]}$

ii. For the case of AGARCH(I)-(0,1), $E\left(\theta_t^2\right) = \frac{\omega}{1-\alpha_1\sigma_z^2}$

Proof We know that for GARCH(1,1) process

$$\theta_t^2 = \omega + \alpha_1\varepsilon_{t-1}^2 + \beta_1\theta_{t-1}^2$$
$$E\left(\theta_t^2\right) = \omega + \alpha_1 E\left(\theta_{t-1}^2\right)E\left(z_t^2\right) + \beta_1 E\left(\theta_{t-1}^2\right)$$

Using the second order stationarity condition we obtain the required result where for the case of AGARCH(I)-(0,1) we can follow the similar procedure.

4. Computational Results

We consider different values of Call option C_{BSG} and different stock prices S for BS-model with GARCH volatility proposed in Gong et al. (2010). We consider the kurtosis formula for different values of volatility. The kurtosis measure has three types in the literature, which are defined in terms of the standardized variable, quintile measures and order statistics. The classical definition of kurtosis belongs to the first type. Since the introduction of kurtosis [14], various new measures of kurtosis have been introduced see [2]. In 2003 Blest introduced a new measure of kurtosis that makes an explicit adjustment for the skewness of the distribution.

$$k_1^* = k_1 - 3\left[\left(1 + f^*\right)^2 - 1\right] \quad (4.1)$$

where

$$f^* = \left(\sqrt{0.25\gamma_1^2 + 1} + 0.5\gamma_1\right)^{1/3} - \left(\sqrt{0.25\gamma_1^2 + 1} - 0.5\gamma_1\right)^{1/3} \quad (4.2)$$

and γ_1 is the standardized third moment given by $\gamma_1 = \frac{\mu_3}{\sigma_3}$, where μ is the mean and σ denotes standard deviation. Aggregate stock market returns display negative skewness, A large body of literature has aimed to explain this stylized fact about the distribution of aggregate stock returns see [4, 8,9, 10]. Firm stock returns display positive skewness [1]. We consider the new measure of kurtosis k_1^* as given above in equation (4.1) in order to compare our results of kurtosis k_1 . We consider γ_1 = 0.5 and 1 for calculating k_1^*.

Volatility	Call Price	Kurtosis k_1 Classical	Stock Price	Kurtosis k_1^* γ_1 =0.5	Kurtosis k_1^* γ_1 = 1
0.3	25.33	19.473	400	19.307	18.818
0.3	20.40	20.848	405	20.682	20.193
0.3	15.58	22.162	410	21.996	21.507
0.4	25.33	20.202	400	20.036	19.547
0.4	20.40	21.382	405	21.216	20.727
0.4	15.58	22.508	410	22.342	21.853

From our computational results we can see that we obtain different value of k_1^* when γ_1 have different values for k_1 with the same value. In addition when call price is high, then value of k_1 is less. We can see when volatility is increased then corresponding values of k_1 are also increased and $k_1^* < k_1$ always because k_1^* is adjusted with skewness γ_1.

5. Conclusions

In this article, some extensions of the Black-Scholes model with GARCH volatility have been derived. We have used Taylor approximation to obtain the kurtosis formula. The proposed kurtosis formula contains the parameters of BS-models with GARCH volatility and particularly in the case of GARCH volatility we can use various extensions of GARCH models. The calibration results show that kurtosis value for the underlying stock process is greater then 3 in the case of at the money option, which strongly affirms the non Gaussian, return distribution. A comparison with another new measure of kurtosis with adjusted skewness is also given.

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R E F E R E N C E S

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