WEAK CONVERGENCE THEOREM OF GENERALIZED SELF-ADAPTIVE ALGORITHMS FOR SOLVING SPLIT COMMON FIXED POINT PROBLEMS

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In the present paper, we introduce a self-adaptive algorithm for solving the split common fixed point problem of demicontractive operators in real Hilbert spaces. Weak convergence result is discussed under suitable assumptions. Some numerical experiments are also given to support our main theorem. Moreover, applications are given to the split common null point problem and the split feasibility problem.

Keywords: split common fixed point problem, demicontractive operator, self-adaptive algorithm.

MSC2010: 47J25, 47H10, 65K10.

1. Introduction

The split common fixed point problem and the split feasibility problem have received much attention due to its applications in image reconstruction, signal processing, intensity-modulated radiation therapy and computed tomography. Because the problem can be applied to solve several real-world problems as mentioned above, so many mathematicians proposed algorithms for solving the problem, see [4, 12, 13, 15, 16, 17].

Let H_1 and H_2 be two real Hilbert spaces with inner product $\langle \cdot, \cdot \rangle$ and norm $\| \cdot \|$. Let $S : H_1 \to H_1$ and $T : H_2 \to H_2$ be two nonlinear operators. Denote the fixed point sets of S and T by $Fix(S)$ and $Fix(T)$, respectively. The split common fixed point problem (SCFP) was firstly introduced by Censor and Segal [3], which is the problem of finding a point

$$x \in Fix(S) \text{ such that } Ax \in Fix(T), \tag{1}$$

where $A : H_1 \to H_2$ is a given bounded linear operator. They invented and proved, in finite dimensional spaces, the convergence of the following algorithm of two directed operators S and T for solving such a problem: for an arbitrary point x_0, generate a sequence $\{x_n\}$ recursively by the rule

$$x_{n+1} = S(I - \tau A^t(I - T)A)x_n, \quad n \geq 0,$$

where $\tau \in \left(0, \frac{2}{\lambda}\right)$, with λ being the largest eigenvalue of the matrix A^tA (A^t stands for matrix transposition).

The SCFP (1) for demicontractive operators was first investigated by Moudafi [8], who proved weak convergence of this problem, which the step-size of his algorithm was chosen in such a way that it depends on the norm of the bounded linear operator A. Later on, there has been growing interest in the SCFP (1) for demicontractive operators; for examples, see Jirakitpuwapat et al. [5], Maingé [7], Padcharoen et al. [9], Shehu and Cholamjiak [10], Tang et al. [11], Yao et al. [18].

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Recently, Yao et al. [14] presented the following iterative algorithm for the SCFP of two demicontractive operators S and T with constants $\beta \in [0,1)$ and $\mu \in [0,1)$, respectively.
Algorithm 1.1: Initialization: given an initial point $x_0 \in H_1$ be arbitrary, then compute x_{n+1} cyclically using

$$y_n = x_n - Sx_n + A^*(I-T)Ax_n, \tag{2}$$

$$x_{n+1} = x_n - \gamma\tau_n y_n, \ n \geq 0, \tag{3}$$

where τ_n is chosen self-adaptively as

$$\tau_n := \frac{\|x_n - Sx_n\|^2 + \|(I-T)Ax_n\|^2}{\|y_n\|^2} \tag{4}$$

with $\gamma \in (0, \min\{1-\beta, 1-\mu\})$ is a positive constant. If $y_n = 0$, then $x_{n+1} = x_n$ is a solution of SCFP (1), and the iterative process stops. Therefore, the weak convergence of Algorithm 1.1 can be obtained under some mild conditions.

Another interesting point of view to achieve a better algorithm is the result of Kanzow and Shehu [6]. In a real Hilbert space H, the inexact KrasnoselskiiMann scheme was modified for fixed point problems of nonexpansive operators $U : H \to K$, where $K \subseteq H$ is nonempty, closed and convex, as follows:
Algorithm 1.2: Initialization: given an initial point $x_0 \in H$ be arbitrary, then compute x_{n+1} cyclically using

$$x_{n+1} = \alpha_n x_n + \beta_n Ux_n + r_n, \ n \geq 0, \tag{5}$$

where r_n denotes the residual vector and $\alpha_n, \beta_n \in [0,1]$ such that $\alpha_n + \beta_n \leq 1$. They also proved the weak convergence result of Algorithm 1.3 under suitable assumptions.

Motivated by these research works, we construct a self-adaptive algorithm for solving the SCFP (1) and prove weak convergence theorem of the proposed algorithm under some suitable assumptions. Some numerical experiments have been presented to show the efficiency of our algorithm. Finally, we apply our result to solve split common null point problems and split feasibility problems.

2. **Mathematical preliminaries**

In this section, we give some mathematical preliminaries which will be used in the sequel. Let H be a real Hilbert space. We know that the metric projection P_C from H onto a nonempty, closed and convex subset $C \subseteq H$ is defined by

$$P_C x := \arg\min_{y \in C} \|x - y\|, \quad x \in H.$$

It is well known that P_C is characterized by the inequality, for $x \in H$

$$\langle x - P_C x, y - P_C x \rangle \leq 0, \quad \forall y \in C. \tag{6}$$

Next, we have the following equality:

$$2\langle x, y \rangle = \|x\|^2 + \|y\|^2 - \|x-y\|^2, \tag{7}$$

the subdifferential inequality:

$$\|x+y\|^2 \leq \|x\|^2 + 2\langle y, x+y \rangle, \tag{8}$$

and

$$\|tx + sy\|^2 = t(t+s)\|x\|^2 + s(t+s)\|y\|^2 - st\|x-y\|^2 \tag{9}$$

for all $x, y \in H, \ s, t \in \mathbb{R}$.

Definition 2.1. *An operator $T : C \to C$ is said to be demicontractive (or k-demicontractive) if there exists a constant $k \in [0, 1)$ such that*

$$\|Tx - x^*\|^2 \leq \|x - x^*\|^2 + k\|x - Tx\|^2,$$

or equivalently,

$$\langle x - Tx, x - x^* \rangle \geq \frac{1-k}{2}\|x - Tx\|^2,$$

for all $(x, x^*) \in H \times Fix(T)$.

We use $\rightharpoonup$ for weak convergence and $\to$ for strong convergence. Next, we give some important tools for proving our main results.

Definition 2.2. *Let $T : C \to H$ be an operator. Then T is said to be demiclosed at $y \in H$ if, for any sequence $\{x_n\}$ in C such that $x_n \rightharpoonup x \in C$ and $Tx_n \to y$ imply $Tx = y$.*

Lemma 2.1. [1] *Let $\{\sigma_n\}$ and $\{\gamma_n\}$ be nonnegative sequences satisfying $\sum_{n=1}^{\infty} \sigma_n < \infty$ and*

$$\gamma_{n+1} \leq \gamma_n + \sigma_n, \quad n \geq 0.$$

Then, $\{\gamma_n\}$ is a convergent sequence.

Lemma 2.2. (Opial) *Let D be a nonempty set of H and $\{x_n\}$ be a sequence in H such that the following two conditions hold:*
(a) for every $x \in D$, $\lim_{n\to\infty} \|x_n - x\|$ exists;
(b) every sequential weak cluster point of $\{x_n\}$ is in D.
Then $\{x_n\}$ converges weakly to a point in D.

3. Weak convergence theorem

In this section, we study the SCFP (1) under the following hypothesis.
$(HP1)$ H_1 and H_2 are two real Hilbert spaces;
$(HP2)$ $S : H_1 \to H_1$ and $T : H_2 \to H_2$ are two demicontractive operators with constants $\beta \in [0, 1)$ and $\mu \in [0, 1)$, respectively, and both $I - S$ and $I - T$ are demiclosed at zero;
$(HP3)$ $A : H_1 \to H_2$ is a bounded linear operator with its adjoint operator A^*;
$(HP4)$ The problem is consistent, i.e. its solution set, denoted by Ω, is nonempty.

Next, we construct the following self-adaptive algorithm to solve SCFP (1) and prove weak convergence of the proposed algorithm under some suitable conditions.
Algorithm 3.1: Initialization: given an initial point $x_0 \in H_1$ be arbitrary, then compute x_{n+1} cyclically using

$$y_n = x_n - Sx_n + A^*(I - T)Ax_n, \tag{10}$$

$$x_{n+1} = \alpha_n x_n + \beta_n(x_n - \tau_n y_n) + r_n, \; n \geq 0, \tag{11}$$

where τ_n is chosen self-adaptively as

$$\tau_n := \gamma \frac{\|x_n - Sx_n\|^2 + \|(I - T)Ax_n\|^2}{\|y_n\|^2} \tag{12}$$

with $\gamma \in (0, \min\{1 - \beta, 1 - \mu\})$ is a positive constant, r_n denotes the residual vector, and $\alpha_n, \beta_n \in [0, 1]$ such that $\alpha_n + \beta_n \leq 1$. If $y_n = 0$, then x_n is a solution of SCFP (1), and the iterative process stops.

Remark 3.1. *$z \in \Omega$ if and only if $\|z - Sz + A^*(I - T)Az\| = 0$, see Yao et al.* [14].

Theorem 3.1. *Let the following conditions hold:*

(*a*) $\liminf_{n\to\infty} \alpha_n\beta_n > 0$; (*b*) $\sum_{n=0}^{\infty} \|r_n\| < \infty$; (*c*) $\sum_{n=0}^{\infty}(1-\alpha_n-\beta_n) < \infty$.

Then the sequence $\{x_n\}$ *generated by Algorithm 3.1 converges weakly to a solution of SCFP* (1).

Proof. Let $z \in \Omega$. Firstly, we prove that $\lim_{n\to\infty} \|x_n - z\|$ exists. By the equivalence of demicontractive operators, we have

$$\begin{aligned}\langle y_n, x_n - z\rangle &= \langle x_n - Sx_n + A^*(I-T)Ax_n, x_n - z\rangle \\ &= \langle x_n - Sx_n, x_n - z\rangle + \langle (I-T)Ax_n, Ax_n - Az\rangle \\ &\geq \frac{1-\beta}{2}\|x_n - Sx_n\|^2 + \frac{1-\mu}{2}\|(I-T)Ax_n\|^2 \\ &\geq \frac{1}{2}\min\{1-\beta, 1-\mu\}\left(\|x_n - Sx_n\|^2 + \|(I-T)Ax_n\|^2\right). \end{aligned} \tag{13}$$

Using (13), we derive

$$\begin{aligned}\|x_n - \tau_n y_n - z\|^2 &= \|x_n - z\|^2 - 2\tau_n\langle y_n, x_n - z\rangle + \tau_n^2\|y_n\|^2 \\ &\leq \|x_n - z\|^2 - \gamma\min\{1-\beta, 1-\mu\}\frac{\left(\|x_n - Sx_n\|^2 + \|(I-T)Ax_n\|^2\right)^2}{\|y_n\|^2} \\ &\quad + \gamma^2\frac{\left(\|x_n - Sx_n\|^2 + \|(I-T)Ax_n\|^2\right)^2}{\|y_n\|^2} \\ &= \|x_n - z\|^2 - \gamma\left(\min\{1-\beta, 1-\mu\} - \gamma\right)\frac{\left(\|x_n - Sx_n\|^2 + \|(I-T)Ax_n\|^2\right)^2}{\|y_n\|^2}. \end{aligned} \tag{14}$$

So, since $\gamma \in (0, \min\{1-\beta, 1-\mu\})$, we have for all $n \geq 0$,

$$\|x_n - \tau_n y_n - z\| \leq \|x_n - z\|. \tag{15}$$

On the other hand, we see that

$$\begin{aligned}\|x_{n+1} - z\| &= \|\alpha_n(x_n - z) + \beta_n(x_n - \tau_n y_n - z) + r_n - (1-\alpha_n-\beta_n)z\| \\ &\leq \alpha_n\|x_n - z\| + \beta_n\|x_n - \tau_n y_n - z\| + \|r_n - (1-\alpha_n-\beta_n)z\| \\ &\leq (\alpha_n+\beta_n)\|x_n - z\| + (1-\alpha_n-\beta_n)\|r_n - z\| + (\alpha_n+\beta_n)\|r_n\| \\ &\leq \|x_n - z\| + (1-\alpha_n-\beta_n)M + \|r_n\|, \end{aligned}$$

for some $M > 0$. By conditions (*b*) and (*c*) together with Lemma 2.1, we determine that $\lim_{n\to\infty} \|x_n - z\|$ exists. This implies that $\{x_n\}$ is bounded. Here we show that $\lim_{n\to\infty} \|x_n - Sx_n\| = \lim_{n\to\infty} \|(I-T)Ax_n\| = 0$. From (9) and (15), we have

$$\begin{aligned}\|\alpha_n(x_n - z) + \beta_n(x_n - \tau_n y_n - z)\|^2 &= \alpha_n(\alpha_n+\beta_n)\|x_n - z\|^2 + \beta_n(\alpha_n+\beta_n)\|x_n - \tau_n y_n - z\|^2 \\ &\quad - \alpha_n\beta_n\tau_n^2\|y_n\|^2 \\ &\leq (\alpha_n+\beta_n)^2\|x_n - z\|^2 - \alpha_n\beta_n\tau_n^2\|y_n\|^2. \end{aligned} \tag{16}$$

Then from (8) and (16), we obtain

$$\begin{aligned}\|x_{n+1}-z\|^2 &= \|\alpha_n(x_n-z)+\beta_n(x_n-\tau_n y_n - z)+r_n-(1-\alpha_n-\beta_n)z\|^2\\ &\le \|\alpha_n(x_n-z)+\beta_n(x_n-\tau_n y_n-z)\|^2+2\langle r_n-(1-\alpha_n-\beta_n)z, x_{n+1}-z\rangle\\ &\le (\alpha_n+\beta_n)^2\|x_n-z\|^2-\alpha_n\beta_n\tau_n^2\|y_n\|^2+2\langle r_n-(1-\alpha_n-\beta_n)z, x_{n+1}-z\rangle\\ &\le \|x_n-z\|^2-\alpha_n\beta_n\tau_n^2\|y_n\|^2+2\langle r_n-(1-\alpha_n-\beta_n)z, x_{n+1}-z\rangle\\ &= \|x_n-z\|^2-\alpha_n\beta_n\tau_n^2\|y_n\|^2+2(1-\alpha_n-\beta_n)\langle r_n-z, x_{n+1}-z\rangle\\ &\quad +2(\alpha_n+\beta_n)\langle r_n, x_{n+1}-z\rangle\\ &\le \|x_n-z\|^2-\alpha_n\beta_n\tau_n^2\|y_n\|^2+2\left[(1-\alpha_n-\beta_n)\|r_n-z\|+(\alpha_n+\beta_n)\|r_n\|\right]\|x_{n+1}-z\|\\ &\le \|x_n-z\|^2-\alpha_n\beta_n\tau_n^2\|y_n\|^2+(1-\alpha_n-\beta_n)M_1+\|r_n\|M_2,\end{aligned}$$

for some $M_1, M_2 > 0$. That is

$$\alpha_n\beta_n\tau_n^2\|y_n\|^2 \le \|x_n-z\|^2-\|x_{n+1}-z\|^2+(1-\alpha_n-\beta_n)M_1+\|r_n\|M_2.$$

By our assumptions, we have that $\lim_{n\to\infty}\tau_n^2\|y_n\|^2=0$, which implies that

$$\lim_{n\to\infty}\frac{\left(\|x_n-Sx_n\|^2+\|(I-T)Ax_n\|^2\right)^2}{\|y_n\|^2}=0. \tag{17}$$

However, we observe that

$$\begin{aligned}\frac{\left(\|x_n-Sx_n\|^2+\|(I-T)Ax_n\|^2\right)^2}{\|y_n\|^2} &= \frac{\left(\|x_n-Sx_n\|^2+\|(I-T)Ax_n\|^2\right)^2}{\|x_n-Sx_n+A^*(I-T)Ax_n\|^2}\\ &\ge \frac{\left(\|x_n-Sx_n\|^2+\|(I-T)Ax_n\|^2\right)^2}{2\left(\|x_n-Sx_n\|^2+\|A\|^2\|(I-T)Ax_n\|^2\right)}\\ &\ge \frac{\|x_n-Sx_n\|^2+\|(I-T)Ax_n\|^2}{2\max\{1,\|A\|^2\}}.\end{aligned} \tag{18}$$

Combining (17) and (18), we immediately obtain

$$\lim_{n\to\infty}\|x_n-Sx_n\|=\lim_{n\to\infty}\|(I-T)Ax_n\|=0. \tag{19}$$

We next show that every weak cluster point of the sequence $\{x_n\}$ belongs to the solution set of SCFP (1). Let $\bar{z}$ be a sequential weak cluster point of $\{x_n\}$, that is, it has a subsequence $\{x_{n_k}\}$ fulfilling $x_{n_k} \rightharpoonup \bar{z}$ as $k\to\infty$. Since A is bounded linear operator, we obtain that $Ax_{n_k} \rightharpoonup A\bar{z}$ as $k\to\infty$. By the demiclosedness at zero of $I-S$ and $I-T$, we can conclude that $\bar{z}\in\Omega$.

Finally, by Opial's lemma (Lemma 2.2), we can conclude that $\{x_n\}$ converges weakly to a solution of SCFP (1). The proof is now completed. □

4. **Numerical experiments**

In this section, we provide some numerical experiments and illustrate its performance for supporting our main theorem.

Example 4.1. *Let $H_1 = H_2 = l_2$ with usual norm. Define*

$$S(v_1,v_2,v_3,\dots)=\begin{cases}\left(\frac{2}{3}v_1\sin\frac{1}{v_1},\frac{5}{6}v_2\sin\frac{4}{v_2},0,0,0,\dots\right) & \textit{if } v_1\neq 0 \textit{ and } v_2\neq 0,\\ (0,0,0,\dots) & \textit{otherwise,}\end{cases}$$

and $T(v_1,v_2,v_3,\dots) = -3(v_1,v_2,v_3,\dots)$, and let $A(v_1,v_2,v_3,\dots)=(0,v_1,v_2,\dots)$ for all $(v_1,v_2,v_3,\dots)\in l_2$.

Choose $\alpha_n = \frac{n+3}{3(n+1)}$, $\beta_n = \frac{2n}{3(n+1)}$ and $r_n = \left(\frac{1}{n+1}\right)^{20}$ for all $n \geq 0$ in Algorithm 3.1. It is not hard to show that S is 0-demicontractive but not nonexpansive and T is $\frac{1}{2}$-demicontractive and T is not quasi-nonexpansive. So, we set $\gamma = \frac{2}{5}$ and consider different choices of x_0 as follows:

Choice 1: $x_0 = e_1 + e_2 + 5e_3$;
Choice 2: $x_0 = 3e_1 - 6e_2 + 2e_3$;
Choice 3: $x_0 = -7e_1 + 4e_2 - 5e_3$;
Choice 4: $x_0 = -20e_1 + 25e_2 - 17e_3$,

where e_i is the sequences whose the i^{th} term is 1 and the other terms are zero, for $i \in \mathbb{N}$.

In the experiment, we choose the stopping criterion is $E_n := \|y_n\| < 10^{-16}$ such that $z := (0, 0, 0, \dots)$ is the solution of the SCFP (1). The following table shows numerical experiments of Algorithm 1.1 and Algorithm 3.1 for solving SCFP (1) with different choices of x_0.

Choice of x_0		Algorithm 1.1	Algorithm 3.1
Choice 1	No. of Iter.	41	27
	Elapsed Time (s)	0.0023	0.0007
Choice 2	No. of Iter.	43	27
	Elapsed Time (s)	0.0015	0.0006
Choice 3	No. of Iter.	43	25
	Elapsed Time (s)	0.0025	0.0006
Choice 4	No. of Iter.	46	24
	Elapsed Time (s)	0.0032	0.0010

TABLE 1. Numerical experiments of Example 4.1.

From Example 4.1, we observe that the sequence generated by our algorithms involving the residual vector provides less number of iterations and elapsed times than that of Yao et al. [14].

5. Applications

In this section, we apply our main result to obtain two new algorithms for solving the split common null point problem and the split feasibility problem.

5.1. The split common null point problem

Recall that a set-valued mapping $M : H_1 \to 2^{H_1}$ is called monotone if for all $x, y \in H_1$, $u \in M(x)$ and $v \in M(y)$ imply

$$\langle x - y, u - v\rangle \geq 0.$$

A monotone mapping M is said to be maximal if the graph $G(M)$ is not properly contained in the graph of any other monotone map, where $G(M) := \{(x, y) \in H_1 \times H_1 : y \in Mx\}$ for a multi-valued mapping M. The resolvent operator J^M_β associated with M and β is defined by

$$J^M_\beta(x) := (I + \beta M)^{-1}(x), \quad x \in H_1, \ \beta > 0.$$

It is known that the resolvent operator J^M_β is single-valued and 0-demicontractive and that a solution of the problem: find $x \in H_1$ such that $0 \in M(x)$ is a fixed point of J^M_β, for all $\beta > 0$, see [2]. Now, given set-valued maximal monotone mappings $B_1 : H_1 \to 2^{H_1}$, and $B_2 : H_2 \to 2^{H_2}$, respectively. The split common null point problem (SCNP) is the problem of finding a point $x \in H_1$ such that

$$0 \in B_1(x) \text{ and } 0 \in B_2(Ax). \tag{20}$$

By setting $S = J_\beta^{B_1}$ and $T = J_\beta^{B_2}$, we see that Algorithm 3.1 reduce to the following algorithm for studying SCNP (20).

Algorithm 5.1.1: Initialization: given an initial point $x_0 \in H_1$ be arbitrary, then compute x_{n+1} cyclically using

$$y_n = x_n - J_\beta^{B_1} x_n + A^*(I - J_\beta^{B_2})Ax_n, \tag{21}$$

$$x_{n+1} = \alpha_n x_n + \beta_n(x_n - \tau_n y_n) + r_n,\ n \geq 0, \tag{22}$$

where τ_n is chosen self-adaptively as

$$\tau_n := \gamma \frac{\|x_n - J_\beta^{B_1} x_n\|^2 + \|(I - J_\beta^{B_2})Ax_n\|^2}{\|y_n\|^2} \tag{23}$$

with $\gamma \in (0, 1)$ is a positive constant, r_n denotes the residual vector, and $\alpha_n, \beta_n \in [0, 1]$ such that $\alpha_n + \beta_n \leq 1$. If $y_n = 0$, then x_n is a solution of SCNP (20), and the iterative process stops.

We immediately obtain the following result by Theorem 3.1.

Theorem 5.1. *Suppose the conditions* (a) − (c) *in Theorem 3.1. Then the sequence* $\{x_n\}$ *generated by Algorithm 5.1.1 converges weakly to a solution of SCNP* (20).

5.2. The split feasibility problem

Given nonempty, closed and convex sets $C \subseteq H_1$ and $Q \subseteq H_2$. The split feasibility problem (SFP) is to find

$$x^* \in C \text{ such that } Ax^* \in Q. \tag{24}$$

By setting $S = P_C$, $T = P_Q$, $\kappa(x) := \frac{1}{2}\|(I - P_C)x\|^2$ and $\zeta(x) := \frac{1}{2}\|(I - P_Q)Ax\|^2$ for all $x \in H_1$. Then $\nabla\kappa(x) = (I - P_C)x$ and $\nabla\zeta(x) = A^*(I - P_Q)Ax$. Then Algorithm 3.1 reduce to the following algorithm for studying SFP (24).

Algorithm 5.2.1: Initialization: given an initial point $x_0 \in H_1$ be arbitrary, then compute x_{n+1} cyclically using

$$y_n = \nabla\kappa(x_n) + \nabla\zeta(x_n), \tag{25}$$

$$x_{n+1} = \alpha_n x_n + \beta_n(x_n - \tau_n y_n) + r_n,\ n \geq 0, \tag{26}$$

where τ_n is chosen self-adaptively as

$$\tau_n := 2\gamma \frac{(\kappa + \zeta)(x_n)}{\|y_n\|^2} \tag{27}$$

with $\gamma \in (0, 1)$ is a positive constant, r_n denotes the residual vector, and $\alpha_n, \beta_n \in [0, 1]$ such that $\alpha_n + \beta_n \leq 1$. If $y_n = 0$, then x_n is a solution of SFP (24), and the iterative process stops.

We get the following result by Theorem 3.1.

Theorem 5.2. *Suppose the conditions* (a) − (c) *in Theorem 3.1. Then the sequence* $\{x_n\}$ *generated by Algorithm 5.2.1 converges weakly to a solution of SFP* (24).

6. Conclusion

The problem of SCFP (1) is discussed, and we provide a self-adaptive algorithm, Algorithm 3.1, is presented for solving the problem. Convergence analysis of the algorithm shows that, under some simple and suitable control conditions, the sequence generated by Algorithm 3.1 converges weakly to a solution of the problem.

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