U.P.B. Sci. Bull., Series D, Vol. 74, Iss. 4, 2012 ISSN 1454-2358

ESTIMATION OF THE COST FUNCTION USING BAYESIAN APPROACH

Dan Cristian ION[1], Claudiu GEACAR[2]

Administrarea aeroporturilor prezintă numeroase particularităţi, motiv pentru care managerii aeroporturilor au fost nevoiţi să facă tranziţia de la unităţi administrative controlate şi subvenţionate de către stat la afaceri generatoare de profit, fără o bază teoretică şi fără posibilitatea de a apela la un corp de experţi în domeniu. Această lucrare îşi propune să completeze metodele pentru evaluarea performanţelor operaţionale ale aeroporturilor şi utilizează ca metodă de măsurare a eficienţei estimarea funcţiei cost prin metoda Bayesiană. Funcţia integrează variabilele de intrare cu cele de ieşire ale aeroporturilor, constituind un instrument viabil pentru analiza comparativă ca parte a procesului de benchmarking.

Since the airport management has numerous particularities, airport managers had to make transition from administrative units controlled and subsidized by the state to profit-generating business without a theoretical basis and without opportunity to call on a body of experts. This paper aims to fill the gap represented by the lack of methods for the assessing operational performance of the airports and it proposes as a method for measuring the efficiency an estimation of the cost function using the Bayesian approach. This function integrates various inputs and outputs of airports, being a viable tool for comparative analysis as part of benchmarking process.

Key words: airport benchmarking, airport efficiency, cost function

1. Introduction

In the last three decades, airport industry has undergone profound changes at the administrative level. If all airports were initially seen as infrastructure, managed by the state authorities, they became self-sustaining business. The change occurred as the links between central authorities and airports began to weaken, airport managers gaining freedom to focus on the commercial side of business. However, there were many questions about how this freedom can be used to change the airport profile toward commercial. In most countries in the late '70s, the airports were seen as some minor extensions of the administrative structure and, as a result, received little attention. As budget deficits were

[1] PhD student, Faculty of Aerospace Engineering, University POLITEHNICA of Bucharest, Romania, e-mail: cba_ion@yahoo.com
[2] PhD student, Faculty of Aerospace Engineering, University POLITEHNICA of Bucharest, Romania, e-mail: claudiugeacar@gmail.com

growing, airports have had to finance themselves in a higher measure, showing their commercial orientation [1,2,3]. At this point it became obvious the acute lack of theoretical knowledge on the management of airports as profit-oriented business. Only in recent years some articles dealing with various features on commercial management of airports have appeared, though still there was no consistent work to analyze the whole conceptual framework of this problem. Airport managers had to transform these subsidized infrastructure elements into successful business, without a theoretical basis and without opportunity to appeal to a body of experts. Unlike other businesses, airport problem is unique. The initial investment is huge, the most of the assets are items that cannot be moved, reconvert or sold in case of bankrupcy. Moreover, the demand for airport services is out of managerial control, as the airlines, not the airports, are the ones who decide where, when and how the need for transportation will be satisfied. Maximizing profit under these conditions is a very difficult task for a manager.

2. Airport benchmarking

Rapid and continuous changes occurred in the airport industry business lead to a complex range of challenges [7] which the managers have to deal with: infrastructure congestions, safety, privatisation of the air traffic operators, mergers and alliances between airlines, continuous growth of the low cost carriers, etc. In the absence of any standard evaluation tool, all these pressures lead to the use of benchmarking techniques [2,5,6].

Basically, the benchmarking process consist in a comparative analysis by reporting to the competitor with the best results. To do this, we have to establish the criteria for assessing the performance. As in case of any other commercial entity, we start by determining the main inputs and outputs. In the case of an airport, the inputs are capital stock (mainly consisting of runways, terminals and boarding gates) and operating costs composed by labour costs and soft costs[3]. If we can consider the capital as a fixed input, the operating costs are variable. As for an airport outputs, the main ones are passengers, number of landings and take-offs and the non-aeronautical revenues [2,4] (those generated by activities not directly related to aircraft operation, generally resulting from commercial activities within the terminal or rents).

As one can see, airports are using more inputs to produce few different outputs. This causes difficulties in defining a coherent overall measure. Usually,

[3] We call soft costs all inputs other then labour and capital. These include costs of outsourced services, consulting services, utilities, maintenance and staff travel expenses. Soft costs reflect the fact that airports are outsourcing services in different proportions. In practice, soft costs represent between 27% and 94% of the non-capital costs of an airport. [11]

in airport industry, universities and the media, partial measures are used in order to assess differences of performance. In general, partial productivity measures for airports reveal the relationship between one output and a specific input. For example, the number of passengers per boarding gate is a partial measure of productivity. [11]

A variety of partial performance indicators are used to evaluate the performance of airports. Because they are easy to use, require simple calculations and needs only limited data, these measures are very popular in benchmarking studies. However, productivity of a given input depends on the size of other inputs. As a result, a report indicating high productivity of one input may be caused by a low productivity of others. Therefore, any interpretation of partial measures must be made with caution.

Due to this fact, an overall performance measure is needed. In the following chapter we are going to estimate the cost function for a given airport by aggregating all inputs and outputs using variable cost shares as weights [8].

3. Cost function estimation

Stochastic Cost Frontier Function Estimation

The short-run cost function for airport *i* at time *t* is [9,10]:

$$C_{it} = f(Y_{it}, W_{it}, X_{it}, t)$$

where Y_{it} denote the output vector; W_{it} denotes the variable input price vector, X_{it} denotes the fixed input vector, and *t* denotes time. Specifically, we have three output measures included in Y_{it} , namely:

y_{1it} : number of passengers

y_{2it} : number of landings and take-offs

y_{3it} : the non-aeronautical-revenue measured in a certain currency

The input price vector W_{it} includes:

w_{1it} : labor price

w_{2it} : soft costs input

The fixed input vector includes three measures for capital stock:

x_{1it} : number of runways

x_{2it} : number of gates

k_{3it} : terminal size

Translog Cost Frontier

In translog specification for the cost function, we expand $\ln C(Y_{it}, W_{it}, X_{it}, t)$ by a second-order Taylor series at the point $\ln Y_{it} = 0$, $\ln W_{it} = 0$, $\ln X_{it} = 0$ and $t = 0$ to get

$$\ln C_{it} = \alpha + \sum_{j=1}^{2}\beta_j \ln y_{jit} + \sum_{j=1}^{2}\lambda_j \ln x_{jit} + \sum_{j=1}^{2}\delta_j \ln w_{jit} + \frac{1}{2}\sum_{j=1}^{2}\sum_{n=1}^{2}\phi_{jn} \ln y_{jit} \ln y_{nit}$$

$$+\sum_{j=1}^{2}\sum_{n=1}^{2}\pi_{jn}\ln y_{jit}\ln w_{nit}+\sum_{j=1}^{2}\sum_{n=1}^{2}v_{jn}\ln y_{jit}\ln x_{nit}+\frac{1}{2}\sum_{j=1}^{2}\sum_{n=1}^{2}\tau_{jn}\ln w_{jit}\ln w_{nit}$$
$$+\sum_{j=1}^{2}\sum_{n=1}^{2}\zeta_{jn}\ln x_{jit}\ln w_{nit}+\frac{1}{2}\sum_{j=1}^{2}\sum_{n=1}^{2}\psi_{jn}\ln x_{jit}\ln x_{nit}+\theta_1 t+\frac{1}{2}\theta_2 t^2 \quad (1)$$
$$+\sum_{j=1}^{2}\theta_{j3}t\ln y_{jit}+\sum_{j=1}^{2}\theta_{j4}t\ln w_{jit}+\sum_{j=1}^{2}\theta_{j5}t\ln x_{jit}$$

Remembering this property

$$\frac{\partial C_{it}}{\partial w_{jit}}=\frac{\partial C_{it}}{\partial \ln C_{it}}\frac{\partial \ln C_{it}}{\partial \ln w_{jit}}\frac{\partial \ln w_{jit}}{\partial w_{jit}}=\frac{C_{it}}{w_{jit}}\frac{\partial \ln C_{it}}{\partial \ln w_{jit}} \quad (2)$$

Applying the Shephard's Lemma [12], we can get two share equations

$$S_{1it}\equiv\frac{\partial \ln C_{it}}{\partial \ln w_{1it}}=\delta_1+\sum_{j=1}^{2}\pi_{j1}\ln y_{jit}+\sum_{j=1}^{2}\tau_{j1}\ln w_{jit}+\sum_{j=1}^{2}\zeta_{j1}\ln x_{jit}+\theta_{14}t \quad (3a)$$

$$S_{2it}\equiv\frac{\partial \ln C_{it}}{\partial \ln w_{2it}}=\delta_2+\sum_{j=1}^{2}\pi_{j2}\ln y_{jit}+\sum_{j=1}^{2}\tau_{j2}\ln w_{jit}+\sum_{j=1}^{2}\zeta_{j2}\ln x_{jit}+\theta_{24}t \quad (3b)$$

The Hessian matrix of the cost function with respect to the inputs' prices is then

$$\nabla_W^2 C_{it}(Y_{it},W_{it},X_{it},t)=\begin{bmatrix}-\frac{C_{it}}{w_{1it}}\left(\frac{S_{1it}}{w_{1it}}-\tau_{11}\right) & \frac{C_{it}}{w_{1it}}\tau_{21}\\ \frac{C_{it}}{w_{2it}}\tau_{12} & -\frac{C_{it}}{w_{2it}}\left(\frac{S_{2it}}{w_{2it}}-\tau_{22}\right)\end{bmatrix} \quad (4)$$

To get a well-defined cost function, we need the following constraints:

Symmetric constraints:

$\phi_{12}=\phi_{21}$ (5a)

$\tau_{12}=\tau_{21}$ (5b)

$\Psi_{12}=\Psi_{21}$ (5c)

Homogeneity constraints:

Because the cost function is homogeneous of degree 1 with respect to the inputs prices, we can have

$\delta_1+\delta_2=1$ (6a)

$\pi_{11}+\pi_{12}=0$ (6b)

$\pi_{21}+\pi_{22}=0$ (6c)

$\tau_{11}+\tau_{12}=0$ (6d)

$\tau_{12}+\tau_{22}=0$ (6e)

$$\zeta_{11} + \zeta_{12} = 0 \tag{6f}$$
$$\zeta_{21} + \zeta_{22} = 0 \tag{6g}$$
$$\theta_{14} + \theta_{24} = 0 \tag{6h}$$

Concavity constraints:

The cost function is concave with respect to inputs prices, so the Hessian matrix in (7) is negative by semi-defined. We have

$$-\frac{C_{it}}{w_{1it}}\left(\frac{S_{1it}}{w_{1it}} - \tau_{11}\right) \le 0 \Rightarrow \tau_{11} \le \frac{S_{1it}}{w_{1it}} \tag{7a}$$

$$-\frac{C_{it}}{w_{2it}}\left(\frac{S_{2it}}{w_{2it}} - \tau_{22}\right) \le 0 \Rightarrow \tau_{22} \le \frac{S_{2it}}{w_{2it}} \tag{7b}$$

$$\frac{C_{it}^2}{w_{1it}w_{2it}}\left(\frac{S_{1it}}{w_{1it}} - \tau_{11}\right)\left(\frac{S_{2it}}{w_{2it}} - \tau_{22}\right) - \frac{C_{it}^2}{w_{1it}w_{2it}}\tau_{12}^2 \ge 0 \Rightarrow \tau_{12}^2 \le \left(\frac{S_{1it}}{w_{1it}} - \tau_{11}\right)\left(\frac{S_{2it}}{w_{2it}} - \tau_{22}\right) \tag{7c}$$

Other constraints:

Since the fixed inputs generally don't change with time, we can set

$$\theta_{15} = \theta_{25} \tag{8}$$

Substituting all these constraints into equations (1) and (3a), we get

$$\widetilde{C}_{it} = \alpha_0 + \sum_{j=1}^{2}\beta_j \ln y_{jit} + \sum_{j=1}^{2}\lambda_j \ln x_{jit} + \frac{1}{2}\phi_{11}(\ln y_{1it})^2 + \frac{1}{2}\phi_{22}(\ln y_{2it})^2 +$$
$$\phi_{12}\ln y_{1it}\ln y_{2it} + \sum_{j=1}^{2}\sum_{n=1}^{2} v_{jn}\ln y_{jit}\ln x_{nit} + \frac{1}{2}\psi_{11}(\ln x_{1it})^2 + \frac{1}{2}\psi_{22}(\ln x_{2it})^2 +$$
$$\psi_{12}\ln x_{1it}\ln x_{2it} + \theta_1 t + \frac{1}{2}\theta_2 t^2 + \sum_{j=1}^{2}\theta_{j3} t\ln y_{jit} + \delta_1 \ln\widetilde{w}_{it} + \pi_{11}\ln y_{1it}\ln\widetilde{w}_{it} + \tag{9}$$
$$\pi_{21}\ln y_{2it}\ln\widetilde{w}_{it} + \tau_{11}\hat{w}_{it} + \zeta_{11}\ln x_{1it}\ln\widetilde{w}_{it} + \zeta_{21}\ln x_{2it}\ln\widetilde{w}_{it} + \theta_{14} t\ln\widetilde{w}_{it}$$
$$S_{1it} = \delta_1 + \sum_{j=1}^{2}\pi_{j1}\ln y_{it}^{j} + \tau_{11}\ln\widetilde{w}_{it} + \sum_{j=1}^{2}\zeta_{j1}\ln x_{it}^{j}\theta_{14} t$$

with the constraint of

$$\tau_{11} \le \min\left(\frac{S_{1it}}{w_{1it}}, \frac{S_{2it}}{w_{2it}}, \frac{S_{1it}S_{2it}}{w_{1it}w_{2it}}\right) \tag{11}$$

In the above equations

$$\widetilde{C}_{it} = \ln C_{it} - \ln w_{2it}$$

$$\ln \widetilde{w}_{it} = \ln\left(\frac{w_{1it}}{w_{2it}}\right)$$

$$\hat{w}_{it} = \frac{1}{2}(\ln w_{1it})^2 - \ln w_{1it} \ln w_{2it} + \frac{1}{2}(\ln w_{2it})^2 = \frac{1}{2}(\ln w_{1it} - \ln w_{2it})^2$$

Econometric Estimation

For econometric estimation, we add the following two error terms on the two equations – equations (9) and (10), respectively.

$$\zeta_{it}^{C} = v_i^C + \mu_i + \varepsilon_{it}^C \quad (12)$$

$$\xi_{it}^{S} = v_i^S + \varepsilon_{it}^S \quad (13)$$

where

$$V_i \equiv \begin{bmatrix} v_i^C \\ v_i^S \end{bmatrix} \approx N\left(0, \sum\right) \quad (14)$$

$$\log(\mu_i) \approx N(Z_i \eta, \sigma_\mu^2) \quad (15)$$

$$\varepsilon_{it}^C \approx N(0, \sigma_{c\varepsilon}^2) \quad (16)$$

$$\varepsilon_{it}^S \approx N(0, \sigma_{S\varepsilon}^2) \quad (17)$$

Equation (14) captures the individual heterogeneity and the correlation between cost and share equations; equation (15) captures the inefficiency, which may be explained by the observables like ownership structures. We estimate the model using the Bayesian approach.

A restricted model:

The number of parameters in the translog cost specification is large. Also, it is hard to add the constraints of non-increasing and convex in fixed inputs, because we do not have data on the shadow prices of fixed inputs. We then estimate the following restricted translog cost frontier:

$$\ln C(Y_{it}, W_{it}, X_{it}, t) = \alpha + \sum_{j=1}^{2} \lambda_j \ln x_{jit} + \theta_1 t + \theta_2 t^2 + G(\ln Y_{it}, \ln W_{it}) \quad (18)$$

It is log-linear in the fixed inputs. Taking the second order Taylor expansion for the $G(\cdot)$ function at $\ln Y_{it} = 0.$ and $\ln W_{it} = 0$, we get

$$\ln C_{it} = \alpha + \sum_{j=1}^{2}\beta_j \ln y_{jit} + \sum_{j=1}^{2}\lambda_j \ln x_{jit} + \sum_{j=1}^{2}\delta_j \ln w_{jit} + \frac{1}{2}\sum_{j=1}^{2}\sum_{n=1}^{2}\phi_{jn}\ln y_{jit}\ln y_{nit}$$
$$+ \sum_{j=1}^{2}\sum_{n=1}^{2}\pi_{jn}\ln y_{jit}\ln w_{nit} + \frac{1}{2}\sum_{j=1}^{2}\sum_{n=1}^{2}\tau_{jn}\ln w_{jit}\ln w_{nit} + \theta_1 t + \theta_2 t^2 \quad (19)$$

Substituting the symmetric and homogeneity constraints into the cost function, we have:

The cost function:

$$\widetilde{C}_{it} = \alpha + \sum_{j=1}^{3}\beta_j \ln y_{jit} + \sum_{j=1}^{3}\lambda_j \ln x_{jit} + \frac{1}{2}\phi_{11}(\ln y_{1it})^2 + \frac{1}{2}\phi_{22}(\ln y_{2it})^2$$
$$+ \frac{1}{2}\phi_{33}(\ln y_{3it})^2 + \phi_{12}\ln y_{1it}\ln y_{2it} + \phi_{13}\ln y_{1it}\ln y_{3it} + \phi_{23}\ln y_{2it}\ln y_{3it} \quad (20)$$
$$+ \phi_1 t + \phi_2 t^2 + \delta_1 \ln\widetilde{w}_{it} + \pi_{11}\ln y_{1it}\ln\widetilde{w}_{it} + \pi_{21}\ln y_{2it}\ln\widetilde{w}_{it}$$
$$+ \pi_{31}\ln y_{3it}\ln\widetilde{w}_{it} + \tau_{11}\hat{w}_{it}$$

The labour share function:

$$S_{1it} = \delta_1 + \sum_{j=1}^{3}\pi_{j1}\ln y_{it}^{j} + \tau_{11}\ln\widetilde{w}_{it} \quad (21)$$

with the following constraints on monotonicity and concavity (convexity)

$$\tau_{11} \le \min\left(\frac{S_{1it}}{w_{1it}}, \frac{S_{2it}}{w_{2it}}, \frac{S_{1it}S_{2it}}{w_{1it}w_{1it}}\right) \quad (22)$$

$$\lambda \le 0 \quad \text{for } j=1,2,3 \quad (23)$$

In the above equations,

$$\widetilde{C}_{it} = \ln C_{it} - \ln w_{2it}$$
$$\ln\widetilde{w}_{it} = \ln\left(\frac{w_{1it}}{w_{2it}}\right)$$
$$\hat{w}_{it} = \frac{1}{2}(\ln w_{1it})^2 - \ln w_{1it}\ln w_{2it} + \frac{1}{2}(\ln w_{2it})^2 = \frac{1}{2}(\ln w_{1it} - \ln w_{2it})^2$$

4. Conclusions

This paper goal was to fill the gap represented by the lack of methods for the assessing operational performance of the airports and proposes as a method for measuring the efficiency an estimation of the cost function using the Bayesian approach. This function integrates various inputs and outputs of airports using variable cost shares as weights and represents a viable tool for comparative analysis as part of benchmarking process. Based on the theoretical developments presented above some illustrative numerical case studies are in progress based on recent available data. These results will be presented in the forthcoming papers.

Acknowledgements

The work has been funded by the Sectorial Operational Programme Human Resources Development 2007-2013 of the Romanian Ministry of Labour, Family and Social Protection through the Financial Agreement POSDRU/6/1.5/S/19.

REFERENCES

[1] *Doganis, Rigas,* The airport business, Taylor & Francis, 2005;
[2] *Graham, Anne,* Managing Airport*s,* third edition, Elsevier, 2008;
[3] *Graham, Anne,* Airport benchmarking: a review of the current situation, Benchmarking: an international journal, 12 (2). pp. 99-111, 2005;
[4] *Zenglein, Max J.; Müller, Jürgen,* Non-Aviation Revenue in the Airport Business – Evaluating Performance Measurement for a Changing Value Proposition, 2009 GAP Project website, www.gap-projekt.de;
[5] *Civil Aviation Authority (UK)*, The Use of Benchmarking in the Airport Reviews, 2000;
[6] *Airports Council International (ACI)*, Airport benchmarking to maximise efficiency, 2006;
[7] *Barrett, Sean D.*, Airport competition in the deregulated European aviation market, Journal of Air Transport Management 6 (2000), pp.13-27;
[8] *Caves, D.W., Christensen, L.R., Diewert, W. E.*, Multilateral Comparisons of Output, Input and Productivity Using Superlative Index Numbers, Economic Journal, 92 (1982), pp. 73-86;
[9] *Xia Zhao,* Essays on the Bayesian Estimation of Stochastic Cost Frontier, Louisiana State University, 2003;
[10] *Subal C. Kumbhakar, C. A. Knox Lovell,* Stochastic Frontier Analysis, Cambridge University Press, 2000;
[11] *Air Transport Research Society*, Airport Benchmarking Report 2010: Global standards for airport excellence, ATRS 2010;
[12]*Hal Varian*, Microeconomic Analysis, third edition, New York: W.W. Norton & Co. (1992).