TRAFFIC SIGNAL SELF-ORGANIZING CONTROL BASED ON MINIMUM EXPECTED TRAFFIC FLOW DELAY RULE

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To settle the problem of traffic control under complex and unstable traffic condition, a traffic signal self-organizing control method based on minimum expected traffic flow delay rule was proposed. Based on intersection traffic flow model and traffic flow discrete model, the traffic demand prediction model under unsteady random fluctuation traffic condition was established. The traditional traffic signal control method based on fixed period was discretized into a non-fixed traffic signal cycle model with the phase green light duration as control parameter. Taking the minimum expectation of traffic flow delay as the goal, according to short-term prediction of local traffic demand, a real-time online solution model for phase green light duration of the local intersection was established. And this model was used as traffic signal self-organizing control rule. According to the self-organizing control rule, the local intersection can solve the optimal phase duration in real time. The global optimization is realizing through the interaction of adjacent intersections with time step iterations. Simulations shows, under the same traffic flow inputs, the control performance is significantly better than that of SCATS system. Considering the unsteady random characteristics of urban traffic flow, the optimal phase green light duration is solved in real time at the local intersection according to the self-organizing control rule, so as to meet the traffic demand under complex and unsteady traffic conditions.

Keywords: real-time control, self-organizing control, traffic signal control, traffic control

1. Introduction

The complexity of urban traffic signal control lies in the unsteady random characteristics of traffic flow and rapid changes of traffic flow state. The main factors leading to the unsteady random characteristics of traffic flow are mainly from three aspects. (a) Openness of urban transport systems. (b) Randomness of traffic flow itself. It mainly includes the randomness of driver's driving behavior and traffic demand. (c) Non-linear relationship between adjacent intersections. The output traffic flows of the upstream intersections are the input traffic flows of the downstream intersections, and there are significant nonlinear relationships between the traffic flows of adjacent intersections, which leads to the unsteady characteristics of traffic flow. If the unsteady random characteristics of traffic

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flow are ignored and the randomness of traffic demand is neglected, the traffic signal control performance may be poor [1].

Traditional traffic signal control methods usually adopt offline mode, according to the average state of traffic flow optimizing traffic signal cycle, green ratio, phase offset and other parameters. However, the urban traffic flow state changes rapidly and the traffic signal optimization based on the average traffic flow state is difficult to accurately match the current traffic state, which is prone to low traffic efficiency and traffic oversaturation. Gershenson [2] pointed out that urban traffic states are constantly changing, and urban traffic signal control is not an optimization problem, but an adaptive control problem in response to traffic state changes. Centralized coordinated control of traffic signal is a typical NP-complete problem [3, 4]. Its model is complex, and its calculation cost is high, so it is difficult to adapt to the changes of traffic flow state in time. Adopting distributed control is an effective method to improve the flexibility and real-time performance of traffic signal control [5-8]. Traffic signal self-organizing control belongs to distributed control model. Compared with other distributed control models, self-organized control emphasizes the expression of interaction relationship between individuals of the system and global optimization through the interaction between individuals. Lammer et al. [9] established traffic signal phase switching rules based on phase priority. Aiming at minimizing traffic delay, and considering the loss time of phase switching, a calculation model of traffic signal phase priority was established, and the phase with the highest priority remained green. The local intersection allocates phase time in real time, according to phase priority, and has good adaptability to random fluctuation of traffic flow. The self-organizing control model can produce "green wave" control effect under certain conditions. Cesme et al. [10] established a self-organizing control rule for the second extension of phase green time on the basis of vehicle actuated control. When the queuing vehicles are released, the phase green light extension time is calculated according to the evaluation index of the vehicle average loss time to achieve approximate "green wave" effect and form dynamic coordinated control. Zou et al. [11], learning from ant colony algorithm, set up a model of vehicle releasing digital pheromones on the driving path. According to the accumulated digital pheromone ratio of the phase, the green signal ratio of each phase is allocated in real time at local intersection, so as to meet the traffic demand under unsteady conditions. Self-organizing traffic signal control describes local traffic demands based on the interaction between traffic flow and traffic signal [12], and then allocates intersection resources (phase green time) according to rules [13, 14]. Self-organizing control can describe the complex characteristics of the system from bottom to top based on the expression of the microscopic interaction relationship of the subsystem and has significant advantages for traffic signal control in the complex and unsteady traffic states [15-17].

In recent years, artificial intelligence methods such as reinforcement learning [18-19] and deep learning [20] have been used to solve the traffic signal decision problem under complex traffic conditions and good research results have been achieved. However, the signal control models based on deep learning and reinforcement learning do not have explicit control models, so it is difficult to fully explain their coordination mechanism and control effect. The traffic signal self-organizing control has its own advantages because of the explicit control models.

In this paper, traffic flow platoon dispersion model was used to describe the unsteady random characteristics of urban traffic flow. Compared with the Continuous Traffic Flow Model [21, 22] and Cell Transmission Model (CTM)[23], urban traffic flow states can be described and predicted more accurately. In order to configure traffic signal more flexibly and accurately, this paper abandoned the traditional traffic signal cycle-based method and established a traffic signal self-organizing control model based on real-time online solution of phase green light duration. Based on the short-term prediction of traffic flow, a real-time online solution model of phase green light duration was established to minimize the expected delay of traffic flow at local intersection, to meet the random traffic demand under unsteady fluctuation traffic states. Through the interaction between adjacent intersections and time-step iterations, the overall performance of the system is constantly improved, and the self-organizing control of traffic signal is realized.

2. Real-time control of urban traffic signal

2.1 Traffic signal control

Urban traffic signal control can be divided into static control and real-time (vehicle actuated) control. Static control means that the traffic signal scheme of the intersection is set in advance and the traffic signal scheme (cycle length, green ratio, phase offset, phase sequence, etc.) is fixed. Static control can be divided into single intersection control and multi-intersection coordination control. Taking multi-intersection coordination control as an example, the usual method is to optimize the green ratio, phase offset and other parameters of each intersection based on average traffic flow states, so as to reduce the number of vehicle's stops and traffic delay. However, the urban traffic has unsteady random characteristics, and the traffic signal scheme based on the optimization of specific traffic flow state is difficult to accurately match the actual traffic demand. When the traffic flow fluctuates greatly, traffic signals cannot accurately match traffic states and the traffic efficiency is low.

The schemes of traffic signal real-time control are unfixed, which can be changed in real time according to current traffic demand. Taking full vehicle actuated control as an example, the phase duration of the traffic signal can be adjusted in real time according to the current traffic demand at the local intersection. However, full vehicle actuated control mainly determines traffic signal according to the traffic conditions of the local intersection. Due to the lack of coordination control model and traffic signal optimization model, it is difficult to achieve good "green wave" control effect, so it is mainly applicable to the control of isolated intersections. In order to improve the real-time control performance, various models of distributed control [5-8] and self-organized control [9-17] were proposed to optimize traffic signal real-time control.

2.2 Traffic signal control based on phase duration

Intersections allocate their spatial resources through traffic signal phase. The traffic signal real-time control problem can be summed up as the real-time decision of phase duration. Phase time can be divided into three stages: phase initiation stage, queuing vehicle dissipation stage and green light extension stage. After phase green light turned on, the queuing vehicles start to accelerate, and this stage is the phase initiation stage. After that, the queuing vehicles pass through the intersection at saturation flow rate, and this stage is the queuing vehicles dissipation stage. After the queuing vehicles dissipation, the outgoing traffic flow rate begins to decline, and the green light efficiency begins to decrease. This stage is the green light extension stage, at which time it is necessary to determine when to end the current phase, that is, determine the current phase duration. If the green light extension stage is too long, the efficiency of the current phase green light will decrease and the vehicles queuing delay of the other phases will increase. If the green light extension stage is too short, the effect of "green wave" is weakened, and the number of vehicle stops is increased. The micro-objective of real-time traffic signal control is to determine the balance point between green light efficiency and continuous traffic flow ("green wave") and solve the optimal phase time duration.

Aiming at the problem of urban traffic signal real-time control, this paper establishes self-organizing control rules based on real-time calculation of phase green light duration through short-term prediction of local intersection traffic demand. Based on the multi-agent architecture, the local intersection calculates current phase optimal green light duration in real time according to the self-organizing control rules, so as to meet the random traffic demand of unsteady traffic flow. Traffic signal control is the optimizing process of allocation intersection space resources according to traffic demand, which is realized by traffic signal phase. The relationship between traffic signal cycle (the interval between two consecutive turn-on of a certain phase, which is an unfixed value), phase green time and phase interval time is described by Formula (1) :

$$T = \sum_{i \in \beta} (\tau_i + g_i) \ , g_i \in [g_i^{\min}, g_i^{\max}], i \in \beta \tag{1}$$

T is the traffic signal cycle length; g_i is the green light duration of phase *i*; $g_i^{\min}$ and $g_i^{\max}$ are the allowed minimum green light duration and maximum green light duration of phase *i*; τ_i is the interval time between the previous phase green light turning off and the green light of phase *i* turning on. τ_i is generally composed of all-red time and yellow light time and it is usually a constant. β is the traffic signal phase set of the current intersection.

3. Urban traffic flow model

3.1 Intersection traffic flow model

The traffic flow in each direction at the intersection is independent in time and space, and the change of queuing vehicles in each direction is shown in (2) :

$$\frac{dQ_j(t)}{dt} = q_j^{\text{in}}(t) - q_j^{\text{out}}(t) \quad , \quad j \in \gamma \tag{2}$$

$$q_j^{\text{in}}(t) = \sum_{i \in \phi_j} q_{ij}^{\text{in}}(t) + A_j(t) \tag{3}$$

$Q_j(t)$ ($Q_j(t) \geqslant 0$) is the number of vehicles queuing in direction *j*, the unit is pcu(Passenger Car Unit); $q_j^{\text{in}}(t)$ is the incoming traffic flow rate in direction *j*, the unit is pcu/s. $q_j^{\text{out}}(t)$ is the traffic flow departing rate in direction *j*; ϕ_j is the set of traffic flows entering *j* direction from the adjacent intersections; γ is the set of traffic flows at the current intersection; $q_{ij}^{\text{in}}(t)$ is the traffic flow rate flowing from the adjacent intersection *i* into the direction *j* at the current intersection; $A_j(t)$ is the flow rate of vehicles entering from (or departing to) the external environment. The delay time of the queuing deceleration process at the traffic stop line and the acceleration process when the green light turned on is approximately treated as the phase loss time. The phase loss time is approximately equal to the phase interval time τ_i, which is processed as red light. Literature [9] treated τ_i as phase set-up time, and similar processing method is adopted in this paper. By phase loss time approximate treatment, the departure flow rates of intersection can be simply expressed:

$$q_j^{\text{out}}(t) = \begin{cases} 0 & \text{(red)} \\ q_j^{\max} & (\text{green}, Q_j(t) > 0) \\ q_j^{in}(t) & (\text{green}, Q_j(t) = 0) \end{cases} \tag{4}$$

q_j^{out} is the departure flow rate in direction *j*; $q_j^{\max}$ is the saturation flow rate of direction *j*; red indicates that the current traffic signal phase status is red; green indicates that the current traffic signal phase status is green.

The number of queuing vehicles represents the traffic demand of the intersection. When there are queuing vehicles, traffic flow delay will occur. $w_j(t)$ represents the delay time of traffic flow in direction j, $W(t)$ represents the delay time of overall traffic flow at the intersection, and the relationship between the number of queuing vehicles and the delay time is:

$$\frac{dw_j(t)}{dt} = Q_j(t) \tag{5}$$

$$\frac{dW(t)}{dt} = \sum_{i\in\gamma} Q_i(t) \tag{6}$$

3.2 Phase random traffic demand

The discrete phenomenon occurs when the traffic flow travels from the upstream intersection to the downstream intersection, and the traffic flow continuous model cannot accurately describe the state of the traffic flow. Pacey model [24-25] and Robertson model [26] respectively use normal distribution and geometric distribution to describe the discrete process of traffic flow. Pacey model proposed that travel time of vehicles from upstream section to downstream section conforms to normal distribution. The probability density distribution function is:

$$d(x) = \frac{D}{t^2\sigma\sqrt{2\pi}}\exp[\frac{-(D/x - D/\bar{x})^2}{2\sigma^2}]$$

x is the travel time; D is the distance between upstream section and downstream section; $\bar{x}$ is the average travel time; σ is the standard deviation of vehicle speeds. The values of $\bar{x}$ and σ are usually obtained by traffic data statistics. The distance between adjacent intersections is known, that is, the distance between the upstream section and downstream section is determined. If the departure flow rates of upstream intersections can be obtained, the arrival flow rates of the downstream intersection can be predicted according to the Pacey model. With the arrival flow rates prediction, the traffic demand can be predicted.

$d(t)$ is used to represent the travel time probability density function of traffic flow from the upstream intersection to the downstream intersection, and it is modeled by Pacey model. According to (2)~ (4) and Pacey model:

$$\begin{aligned} E\{Q_j(t)\} &= N_j(t_0) + E\{\int_{t_0}^{t}(q_j^{\text{in}}(t) - q_j^{\text{out}}(t))dt\} \\ &= N_j(t_0) + \mathrm{E}\{\sum_{i\in\phi_j}\int_{t_0}^{t}(\alpha_{ij}d_{ij}(t)q_{ij}^{\text{out}}(t) + A_j(t))dt\} - \mathrm{E}\{\int_{t_0}^{t} q_j^{\text{out}}(t)dt\} \end{aligned} \tag{7}$$

$E\{*\}$ represents the mathematical expectation; $N_j(t_0)$ is the number of queuing vehicles at the moment t_0; $d_{ij}(t)$ is the probability density function of travel time from the upstream intersection i to direction j at the current intersection; $q_{ij}^{\text{out}}(t)$ is the departing traffic flow rate from the adjacent intersection *i* to the *j* direction at current intersection; α_{ij} is the proportion of $q_{ij}^{\text{out}}(t)$ flow to *j* direction at the current intersection and it is usually obtained by traffic data statistics. $q_{ij}^{\text{out}}(t)$ is controlled by the traffic signal of intersection *i*. Formula (7) expresses the interactive relationships of traffic signal and traffic demand between adjacent intersections.

The urban traffic flow states are unsteady and stochastic, so it is very difficult to accurately predict the traffic demand in a long period of time. However, the phase duration of traffic signal is very short. If sensors are installed in each lane at the intersection and adjacent intersections can communicate with each other to get the departing flow rates, the number of vehicles queuing at the phase initial moment ($N_j(t_0)$) can be detected by sensors, and the traffic demand (number of vehicles waiting to pass) during the phase time can be relative accurately predicted according to (7).

4. Traffic signal self-organizing control

4.1 Sate equations of traffic signal control basic unit

The local intersection traffic signal controller is modeled as an agent of traffic signal control, and the following assumptions are made:

(1) Sensors are installed in each lane at the intersection which can detect the number of vehicles queuing in each lane and the traffic flow rate of each lane in real time.

(2) Adjacent intersections can communicate with each other. The traffic signal states, queuing vehicle states and departing flow rates information of adjacent intersections can be obtain by the local intersection.

Based on the assumption that intersections can communicate with each other, and sensors are installed in each lane at intersections, the intersections form an interlinked and interactive union. The local intersection and its adjacent (directly connected) intersections are regarded as the basic unit of traffic signal control. The basic unit state equations of traffic signal control are established, and the optimal phase green light duration of local intersection is studied by short-term traffic demand prediction. Taking the standard intersection as an example, as shown in Fig. 1, intersection *m* and its four adjacent intersections *m*1 ~ *m*4 constitute the basic traffic signal control unit. The queuing vehicle state equations of intersection *m* can be established:

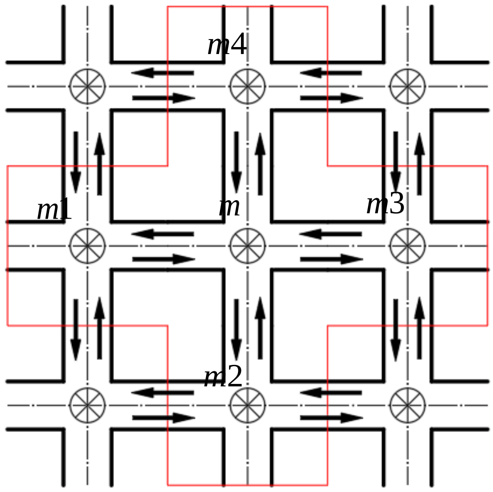

Fig. 1. Basic unit of traffic signal self-organizing control

$$\dot{\boldsymbol{Q}}(t) = f(\boldsymbol{Q}(t),\boldsymbol{u}(t),\xi(t),t),\ \boldsymbol{Q}(t)\in\Omega_Q,\ \boldsymbol{u}(t)\in\Omega_{\boldsymbol{u}},$$
$$\boldsymbol{Q}(t_0)=\boldsymbol{Q}_0,\ t\in\left[t_0,t_n\right] \tag{8}$$

$\boldsymbol{Q}(t)$ is the l-dimension state vector of queuing vehicles, each component represents the number of queuing vehicles in corresponding direction, and l is the number of traffic flow directions. $\boldsymbol{u}(t)$ is the y-dimension traffic signal state vector, y is the phase number, and each component represents the corresponding phase traffic signal state. Traffic signal has only red and green states. Red means red light and green means green light. $\Omega_{\boldsymbol{u}}$ is the set of allowable traffic signal states, and Ω_Q is the set of queuing vehicle states; $\xi(t)$ is l-dimensional random vector, representing the random characteristics of traffic flow; t_0 is the start moment of traffic signal control, t_n is the end moment. Formula (8) is established by Formulas (2) to (4).

The objective of intersection signal control is to optimize the allocation of intersection space resources and minimize traffic delay. According to the objective, signal control performance evaluation function of intersection m can be expressed:

$$I(\boldsymbol{u}(t)) = E\{(t-t_0)^{-1}\int_{t_0}^{t} g(\boldsymbol{Q}(t),\boldsymbol{u}(t),t)dt\},\quad t\in[t_0,t_n] \tag{9}$$

$$g\left(\boldsymbol{Q}(t),\boldsymbol{u}(t),t\right)=\frac{dW(t)}{dt}=\sum_{i\in\gamma}Q_i(t)$$

$I(\boldsymbol{u}(t))$ is a performance evaluation function, which means the expectation of traffic flow delay per unit time at the intersection. The smaller its value is, the higher intersection space utilization efficiency is. It is assumed that $\boldsymbol{u}^*(t)$ is the optimal traffic signal state vector and $I_{min}=I(\boldsymbol{u}^*(t))$. I_{min} is the optimal value of Formula (9). If t_n-t_0 >>T, it is difficult to obtain $\boldsymbol{u}^*(t)$ by offline solving.

Because the changes of traffic flow state are very complex, it is difficult to forecast the traffic demand for a long time. However, the approximate ***u****(t) can be obtained by online iterative solving.

4.2 Optimal phase duration

Because the traffic signal phase is discrete in time, it is only in a certain phase(τ_k classified as phase k) at any time. The sub-control performance evaluation function of the traffic signal phase duration is established to determine the optimal phase duration. The approximate ***u****(t) can be obtained by online iterative solving sub-control performance evaluation function.

The initial moment of the current phase is set as t_{i-1}, the phase of current traffic signal is assumed as phase k, and the sub-control objective of the traffic signal is to get the optimal value of g_k^* to minimize the overall traffic flow delay at the intersection within current phase. g_k^* is the optimal duration of current phase green light. According to Formula (9), the sub-control performance evaluation function is established:

$$I(\boldsymbol{u}_i(t)) = E\{(t-t_{i-1})^{-1}\int_{t_{i-1}}^{t} g(\boldsymbol{Q}(t),\boldsymbol{u}(t),t)dt\} \ ,t \in [t_{i-1}+\tau_k+g_k^{\min}, t_{i-1}+\tau_k+g_k^{\max}] \qquad (10)$$

$I(\boldsymbol{u}_i(t))$ is the sub-control performance evaluation function in time period [t_{i-1},t_i].The current phase optimal duration can be expressed as:

$$t_i - t_{i-1} = \tau_k + g_k^*$$

$$t_i = \{t \mid \min I(\boldsymbol{u}_i(t)), t \in [t_{i-1}+\tau_k+g_k^{\min}, t_{i-1}+\tau_k+g_k^{\max}]\}, t_{i-1} \in [t_0,t_n], t_i \in [t_0,t_n] \qquad (11)$$

τ_k (constant) is the interval time from the end of the previous phase to the beginning of current phase green light turned on. t_i is the phase end moment, and g_k^* is the optimal green light duration of the current phase. $g_k^{\min}$ and $g_k^{\max}$ are respectively the permitted minimum green light duration and the maximum green light duration of phase k.

Taking the derivative of (10) to study its optimal value:

$$dI(\boldsymbol{u}_i(t))/dt = \frac{g(\boldsymbol{Q}(t),\boldsymbol{u}(t),t)(t-t_{i-1}) - \int_{t_{i-1}}^{t} g(\boldsymbol{Q}(t),\boldsymbol{u}(t),t)dt}{(t-t_{i-1})^2} \qquad (12)$$

According to relevant theories of advanced mathematics, the possible value of t_i is:

$$t_i = t_{i-1}+\tau_k+g_k^{\min} \text{ or } t_i = t_{i-1}+\tau_k+g_k^{\max} \text{ or } \{t_i = t \mid dI(\boldsymbol{u}_i(t))/dt = 0\} \qquad (13)$$

According to the mean value theorem of integrals:

$$\int_{t_{i-1}}^{t} g(\boldsymbol{Q}(t),\boldsymbol{u}(t),t)dt = g(\boldsymbol{Q}(t_\xi),\boldsymbol{u}(t_\xi),t_\xi)*(t-t_{i-1}) \quad , \quad t_\xi \in [t_{i-1},t] \qquad (14)$$

At least one t_ξ exists, and there may be several t_ξ.

Substituting Formula (14) into Formula (12)，we can get:

$$dI(\boldsymbol{u}_i(t))/dt = \frac{g(\boldsymbol{Q}(t),\boldsymbol{u}(t),t) - g(\boldsymbol{Q}(t_\xi),\boldsymbol{u}(t_\xi),t_\xi)}{(t - t_{i-1})} \tag{15}$$

If $g(\boldsymbol{Q}(t),\boldsymbol{u}(t),t) - g(\boldsymbol{Q}(t_\xi),\boldsymbol{u}(t_\xi),t_\xi) > 0$, then $dI(\boldsymbol{u}_i(t))/dt > 0$. This means that $I(\boldsymbol{u}_i(t))$ increases over time, and we should end current phase as soon as possible.

If $g(\boldsymbol{Q}(t),\boldsymbol{u}(t),t) - g(\boldsymbol{Q}(t_\xi),\boldsymbol{u}(t_\xi),t_\xi) < 0$,then $dI(\boldsymbol{u}_i(t))/dt < 0$. This means that $I(\boldsymbol{u}_i(t))$ decreases over time, and we should keep current phase in green states as long as possible.

If $g(\boldsymbol{Q}(t),\boldsymbol{u}(t),t) - g(\boldsymbol{Q}(t_\xi),\boldsymbol{u}(t_\xi),t_\xi) = 0$,then $dI(\boldsymbol{u}_i(t))/dt = 0$. t_ξ may be the best point to end current phase.

The value of $g(\boldsymbol{Q}(t),\boldsymbol{u}(t),t)$ represents intersection traffic demand. Fig. 2 shows a typical case in which the traffic demand of an intersection changes within a signal phase.

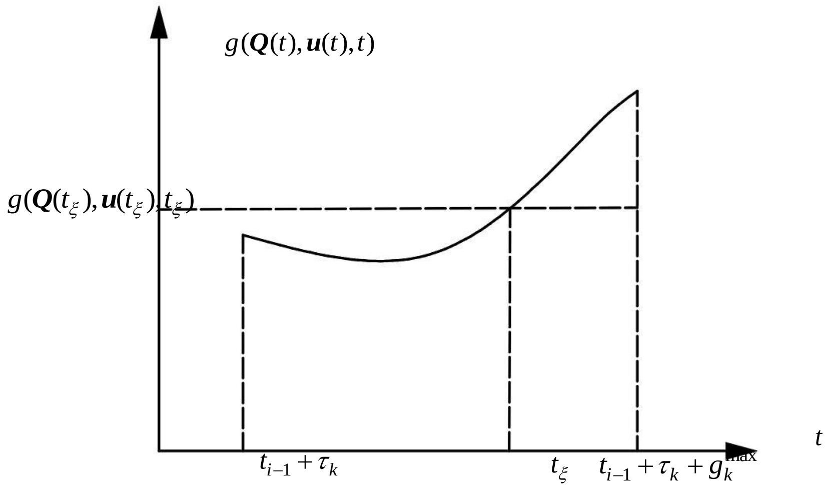

Fig. 2. Changes of traffic demand within a signal phase

There are queuing vehicles at the phase initial moment. In the queuing vehicles dissipation stage, the queuing vehicles pass through the intersection at saturation flow rate from the phase green light start moment $t_{i-1} + \tau_k$. Normally, the arriving flow rate of conflicting phases is less than the saturation flow rate and the traffic demand will decrease. When the queuing vehicles of the current phase have been released, the departing flow rate decreases to the arriving flow rate and the green light efficiency begins to decrease. If $g(\boldsymbol{Q}(t),\boldsymbol{u}(t),t) - g(\boldsymbol{Q}(t_\xi),\boldsymbol{u}(t_\xi),t_\xi) < 0$, current phase should maintain green light to keep continuous traffic flow ("green wave"). When the arriving flow rate of the conflicting phases larger than the arriving flow rate of current phase, the traffic demand will increase. If $g(\boldsymbol{Q}(t),\boldsymbol{u}(t),t)$ rises to $g(\boldsymbol{Q}(t_\xi),\boldsymbol{u}(t_\xi),t_\xi)$, current phase should be ended. Otherwise, the utilization efficiency of intersection space will be reduced. t_ξ is the balance point between green light efficiency and continuous traffic flow. For the phase duration is relatively short, the optimal phase duration is easy to obtain within reasonable calculation cost. The control period $t \in [t_0, t_n]$ is divided into continuous time periods according to $t_i - t_{i-1} = \tau_k + g_k$ (i=1, 2, 3, …) with traffic signal continuously circulating. It is assumed that at every traffic signal phase initial moment the optimal phase duration can be obtain by an appropriate global search algorithm, the optimal traffic signal state vector $\boldsymbol{c}^*$ is obtained online iteratively:

$$\boldsymbol{c}^*(t) = \{\boldsymbol{u}(t) \mid \min I(\boldsymbol{u}_i(t))\}, t \in [t_{i-1}, t_i], (i = 1, 2, 3, ...)$$

It is easy to prove:

$$\boldsymbol{c}^*(t) = \boldsymbol{u}^*(t)$$

By online iteratively solving the optimal phase duration, the approximate $\boldsymbol{u}^*(t)$ can be obtained. By online solving $\boldsymbol{c}^*(t)$, the efficient allocation of urban intersection space resources is realized.

4.3 Minimum expected traffic flow delay control rule

The objective of urban traffic signal control is to optimize the efficiency of green light (space utilization) at all intersections and minimize the overall traffic delay of the road network. Formula (11) is the microscopic control (decision) objective of urban traffic signal under global optimal control objective. According to Formula (11), a traffic signal distributed control model can be obtained. Firstly, a large number of intersections in the road network are separated into interconnected traffic signal control basic units by spatial discretization and the control equations of the traffic signal control basic unit are established. Then, the continuous time traffic signal optimal control is transformed into the single-phase green light duration optimal decision by time discretization. Through the above discrete process, the complexity of urban traffic signal control model is greatly reduced, which can be solved online in real time. Finally, the real-time control of urban traffic signal is realized through multi-agent architecture.

Formula (11) is used as the traffic signal control (phase switching) rule. Namely, the optimal green light duration of the current phase is calculated in real time at the initial of the phase. When the green light time ends, the next phase is switched, and the optimal green light duration of the next phase is calculated successively. All intersections of the road network adopt the same traffic signal control rule. The traffic signal control rule is the microscopic expression of urban traffic signal global optimal control objective. Each intersection real-time decides traffic signal according to the control rule. All intersections negotiate and collaborate with each other to approach the global goal. The system can be called a self-organizing system, if subsystems continuously cooperate and dynamically approach to the global goal. The traffic signal phase control rule in this paper is called self-organizing control rule.

5. Rule-based traffic signal control

5.1 Calculation of optimal phase duration

The local intersection traffic signal controller solves the optimal green light duration of current phase at the phase initial time. Taking intersection *m* as an example, the current traffic signal phase is assumed as *k*, $\boldsymbol{u}(t)$ is *y*-dimension vector, and τ_k is a constant. Only the green light duration of the *k* component needs to be calculated, and the states of the other *y*-1 components are red, which do not need to be calculated. In other words, the optimal green light duration of *k* phase $g_k^* \in [g_k^{\min}, g_k^{\max}]$ is solved according to Equation (11).

t_{i-1} is the initial moment of phase *k*, and the number of queuing vehicles in each direction N_j (t_{i-1}) at the initial moment of phase *k* can be obtained by detectors in each lane. In Formula (7), $q_{ij}^{\text{out}}(t)$ is short time forecasted based on adjacent intersections information communication; α_{ij} is obtain by statistics. The traffic signal phase duration is short, letting $E\{\int_{t_{i-1}}^{t_i} A_j(t)dt\} = 0$.

g_k^* can be sought through an appropriate search algorithm at each phase initial time. Generally, traffic signal control takes time step as the basic unit of signal decision and time step is usually an integer multiple of seconds. Suppose the time step is δ, a search algorithm can be expressed as:

$$g_k^* = \min\{I(\boldsymbol{c}_i)\}, \boldsymbol{c}_i = k(g_k^{\min} + i * \delta), (i = 0, 1, 2, ..., N)$$

$$N = \frac{g_k^{\max} - g_k^{\min}}{\delta}$$

$I(\boldsymbol{c}_i)$ the evaluation value according to Formula (10); $\boldsymbol{c}_i$ is the traffic signal state vector determined by phase *k* green light duration. The optimal green light duration can be obtained within *N* times numerical calculations.

5.2 Base phase green time constraint

The phase duration of traffic signal should meet the following constraints: under the constraint of the maximum green light duration, the phase duration should ensure that the queuing vehicles are released at one time. Otherwise, under the minimum delay target, the green light time allocated to some phases with low traffic efficiency may be insufficient, resulting in queuing vehicle multiple stops at the intersection. The green light duration g_k^b when the queuing vehicles just have been released is the phase basic green light duration, namely:

$$t_b = \{t \mid E\{Q_k(t) = 0\}\},\ t_b - t_{i-1} = g_k^b + \tau_k$$

$Q_k(t)$ is the number of vehicles queuing in the key lane of current phase and t_b is the moment when the vehicles queuing in key lane just have been released. Taking the basic green time duration as the constraint, the optimal green light duration g^*_k is solved according to the minimum expected traffic flow delay control rule:

$$\begin{cases} g_k^* \geq g_k^b \\ g_k^* \in [g_k^{\min}, g_k^{\max}] \end{cases} \tag{16}$$

If $g_k^b > g_k^{\max}$, the set of Formula (16) is empty, let $g_k^* = g_k^{\max}$.

6. Simulation verification

In order to verify the effectiveness, the self-organizing control is compared with the SCATS (Sydney Coordinated Adaptive Traffic System) under the same road network and the same traffic flow inputs. Fig. 3 shows a part of road network near National Convention and Exhibition Center in Qingpu District, Shanghai, China. The road network contains 5 intersections, and the traffic signal control system used here is SCATS. The red circles are the traffic flow inputs. The intersection number is marked in the figure.

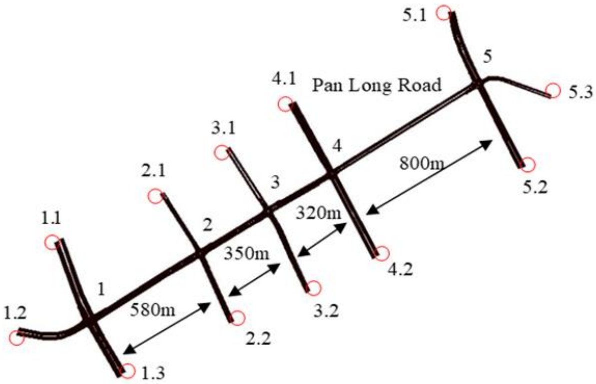

Fig. 3. Road network model

Fig. 4 shows the traffic flows of the road network on a working day (24 hours) in October 2019. The data are read from the SCATS system database and sorted out (from 0 AM to 1AM is time period 0, and from 1AM to 2 AM is time period 1, and so on). According to the figure, the road network traffic flow has significant fluctuations.

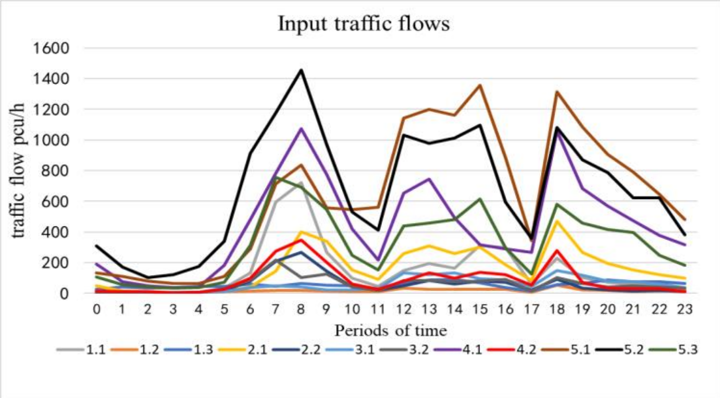

Fig. 4. Input traffic flows

The road network model shown in Fig. 3 is established on the PTV VISSIM software platform according to the actual road network parameters. Taking 3600s as a period, the traffic flow inputs are set (Poisson distribution) according to the data in Fig. 4, and the traffic flow distribution ratio of each section is set according to the actual detection data of the road network. SCATS traffic signal (history) schemes and traffic signal self-organizing control model are respectively used to control traffic signal of the 5 intersections in Fig. 3, and the control performances are compared.

The traffic signal schemes of SACTS system are read from the SCATS system database and loaded into the corresponding intersection through the software interface. These five intersections belong to the same control sub-area of the SCATS system and use the public traffic signal cycle. The control center constantly optimizes traffic signal parameters such as public cycles, green ratios and phase offsets to reduce traffic flow delay. The self-organizing control model calculates the local intersection traffic signal in real time according to the self-organizing control rule. Based on the actual traffic flow data and SCATS traffic signal schemes, the performance of self-organizing control model is evaluated by the above method. In the self-organizing control model, g_{min}=18s and g_{max} =87s are uniformly set at all intersections. According to traffic flow statistics, the saturation flow rate of the straight lane is 1650 pcu/h; the saturation flow rate of the left-turn lane is 1450 pcu/h. The average vehicle speed is 45 km/h and the standard deviation of the vehicle speeds distribution is 5. The time step is 1 second.

In order to evaluate the performance, two evaluation indexes are defined: average vehicle delay and average queuing length. Average vehicle delay means the average delay time of vehicles crossing the intersection and the unit is pcu/s. Average queuing length means the average queuing length in all directions at the intersection and the unit is m. Vehicle delay reflects the smoothness of vehicles passing the intersection. The average queuing length reflects the congestion degree at the intersection, and two indexes are used to evaluate the performance of traffic signal control. The performances of SCATS and self-organizing control (SOC) system from 7 to 22 time periods (from 7 AM to 23 PM) are researched. The time period is taken as the horizontal axis, the average vehicle delay and the average queuing length of the road network are shown in Fig. 5 and Fig. 6.

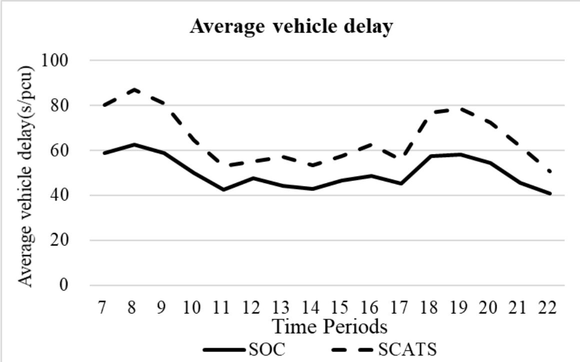

Fig. 5. Average vehicle delay for all intersects

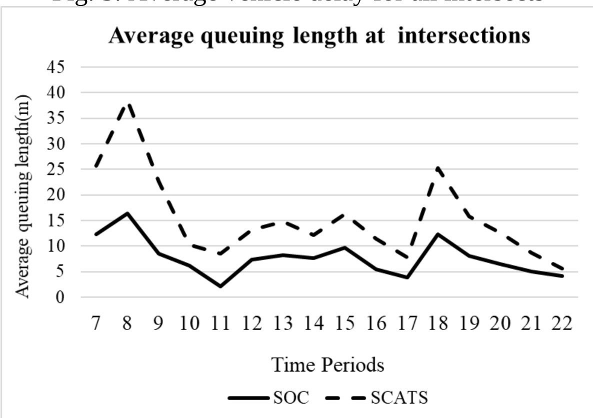

Fig. 6. Average queuing length for all intersections

The average vehicle delay and the average queuing length at intersections show that the self-organizing control is significantly better than SCATS. Analyzing Fig. 5 and Fig. 6, it can be seen that the performance of SCTAS system is close to self-organized control in stable and small traffic flow periods (11~17). In the morning and evening peak traffic periods (7~9, 18~20), when the traffic flow is high and the flow fluctuation is more obvious, the advantage of self-organizing control is more prominent. The main reason is that the self-organizing control considers the unsteady random characteristics of urban traffic flow and solves the current phase optimal green time online, which greatly reduces the random delay of traffic flow.

SCATS is a typical traffic signal centralized coordinated control system. The traditional centralized coordinated control model mainly has the following shortcomings:

(a) It is constrained by traffic signal public cycle. The public cycle *T* is determined by the traffic conditions at critical intersection. For non-critical intersections with small traffic flow, the value of public cycle *T* is far greater than their actual demand, resulting in the increase of average vehicle delay and average vehicle queuing length.

(b) In order to achieve the "green wave" control effect, the green light duration of coordinated phase is often longer than its actual demand, resulting in a significant increase of uncoordinated phase traffic delay.

(c) The offline traffic signal optimization model based on the average traffic flow state is difficult to accurately match real-time traffic state. Traffic flow has significant unsteady characteristics, and traffic flow state changes with every traffic signal circulation. The traffic signal scheme based on average state optimization is difficult to accurately match the real-time traffic demand.

When the traffic flow on the road network is small and relatively stable, the SCATS performance is good. Because the traffic signal public cycle and the traffic signal schemes based on the optimization of the traffic flow average state can well match the actual demand. However, when the traffic flow fluctuation is large and complex, such as in the morning and evening peak traffic hours, the traffic flow difference between intersections increases, and the traffic demand fluctuation of each traffic signal cycle increases. The deviation between the traffic flow average state and the actual traffic flow state increases. The traffic signal public cycle and the traffic signal schemes cannot well match the actual traffic demand, which leads to a significant increase of traffic delay. Self-organizing control bases on distributed architecture and short-term prediction of traffic demand. It is not restricted by traffic signal cycle and solves the current phase optimal duration online in real time. It can more accurately match the traffic demand under the condition of unsteady random characteristics, more efficiently allocate intersection space resources and reduce the traffic flow random delay.

In order to further demonstrate the effect of traffic signal self-organizing control, the traffic signal schemes are analyzed. Fig. 7 shows the statistical average values of traffic signal cycle length *T* under SCATS and self-organizing control. It can be seen that the cycle length of SCATS traffic signal scheme is much larger than that of self-organizing control. Under self-organizing control, the traffic signal is not constrained by the fixed public cycle length *T*. The phase time of every intersection is dynamically adjusted according to the actual demand of traffic flow, and the spatial resources of the intersection are configured efficiently and accurately. Self-organized control can configure traffic signal more flexibly and efficiently, improving green light efficiency and intersection space utilization rate.

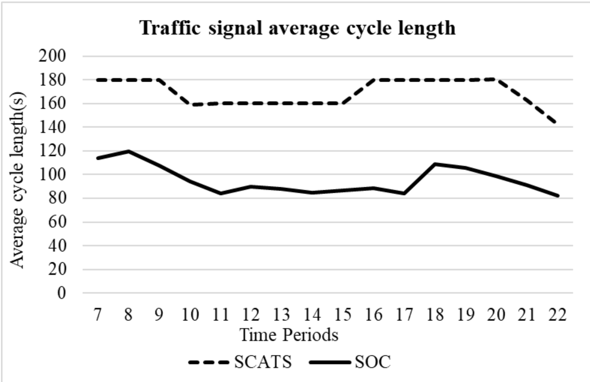

Fig. 7. Traffic signal average cycle length

6. Conclusions

The complexity of urban traffic signal control lies in the unsteady random characteristics of urban traffic flow and the rapid changes of traffic flow state. Traffic signal control needs to consider the unsteady traffic demand and respond quickly. Traditional traffic signal control model usually optimizes traffic signal parameters according to the average state of traffic flow, ignoring the random characteristics of traffic flow, which makes it difficult to accurately match the random traffic demand. Due to the constraints of public cycles, green ratios, phase offsets and other parameters, traditional traffic signal control models cannot configure traffic signal in real time according to random traffic demands.

In this paper, the traffic signal real-time decision rule is established. Based on equations of traffic signal control basic unit, the balance point between green light efficiency and continuous traffic flow ("green wave") is determined and the optimal phase duration can be solved at every phase initial time according to random traffic demands. The traffic signal phase decision-making rule is used as the self-organizing control rule. Simulation shows that the proposed traffic signal self-organizing control method has good performance in complex and unstable traffic conditions.

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