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STUDY ON THE CENTERLINE EXTRACTION METHOD OF WELD IMAGE LINE STRUCTURED LIGHT STRIPE

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In practice, using vision measurement technology based on structured light to reconstruct the complex weld is a feasible way for providing the essential parameters for the welding polishing robot and guiding it to grind the weld by force control. In this paper, we represent a method to extract the centerline of structured light stripe in weld image. First, obtains the high-quality light stripe image using an industrial camera outfitted with a filter and preprocessed to isolate the entire structured light stripe from the background image. Second, using the weighted gray centroid method, roughly extract the original stripe center to obtain the initial point of the stripe center. Then, using principal component analysis, find the normal direction of the light stripe and calculate the gray distribution function of the stripe using second-order Taylor expansion along the normal direction to obtain the correct position of the stripe center. Finally, use cubic spline interpolation to create a smooth centerline. The testing results suggest that this method outperforms the Steger algorithm in terms of speed and extraction accuracy when compared to the classic gray centroid method. These works set the foundation for real-time 3D reconstruction of welds.

Keywords: line structured light, centerline extraction, Steger algorithm, principal component analysis, gray centroid method, spline interpolation

1. Introduction

In today's manufacturing process, we widely use automatic and intelligent equipment, especially robot welding equipment. However, there are few automatic pieces of equipment for weld grinding and polishing. In processing operations such as grinding and polishing of weld allowance or automobile parts, the contact surfaces are usually complex, or there are high requirements for the surface quality. Therefore, the application of industrial robots in grinding is the future manufacturing trend [1].

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At present, most robots used in industrial manufacturing mainly work with the help of teaching aids [2]. In order to enable the welding-polishing robot carries out force control to grind the welds on complex surfaces in real-time smoothly. It is necessary to use machine vision technology to obtain the surface morphology characteristics of welds and provide the necessary parameters for the welding polishing robot. As a typical active vision measurement method [3], line structured light technology has been widely used in non-contact precision measurement of geometric parameters with its convenience and robustness. The line structured light system for 3D reconstruction comprises a one-dimensional or multi-dimensional mobile platform, industrial camera, and line laser transmitter fixed with the industrial camera [4]. It has the advantages of simple structure, low cost, fast measurement speed, and easy integration with other motion coordinates. Therefore, it is an ideal choice to realize the complete measurement of complex surfaces [5].

The principle of 3D reconstruction using line structured light is laser triangulation [6]. The laser stripe is projected onto the object to obtain the geometry of the laser stripe in the image. After obtaining the position relationship between the line structured light projection plane and the camera through coordinate transformation, we can calculate the 3D information of the object shape. This measurement mainly includes two steps: first, extract the center of the laser stripe in the collected image, and then triangulate the extracted center point to recover the distance data. Because the extraction error of the stripe center determines the reconstruction accuracy after system calibration, fast and accurate extraction of laser stripe center from the image is one of the most critical steps to realize 3D reconstruction of weld [7].

In recent years, many scholars have done much research on efficiently extracting the centerline of the linear structured light stripe. Traditional algorithms include the extreme value method, direction template method, and gray centroid method [8]. The extreme value method [9] selects the maximum pixel on the section as the center point, which has low accuracy and is sensitive to noise. Huang et al. [10] proposed an extraction algorithm combining threshold iterative extreme value method and weighted gray centroid method, which improves the extraction efficiency and can be used to scan and measure different objects. Hu [11] et al. proposed the direction template method, which can reduce the impact of noise on centerline extraction, but the amount of calculation is large and can’t meet real-time requirements. Lei et al. [12] proposed an algorithm to extract the light stripe center based on the threshold method and direction template. The algorithm has fast extraction speed, strong stability, anti-interference ability, and broken line repairability. Although these traditional centerline extraction algorithms meet the requirement of real-time, the accuracy can only reach the pixel level. According to the research of Steger et al. [13], taking an image with a

12mm focal length lens at a position 50cm away from the workpiece, every 0.1-pixel deviation in the image will cause a 50um movement in the actual situation. Therefore, the extraction of the pixel-level centerline is challenging to meet the requirements of high-precision weld grinding.

The gray centroid method is a sub-pixel centerline extraction algorithm. Li et al. [14] proposed a gray centroid method based on moving least square (MLS) and the optimized algorithm has better robustness than the original algorithm. Although the gray centroid method meets the requirements of fast extraction of sub-pixel accuracy, it is difficult to obtain a high-precision centerline due to significant environmental interference factors.

In order to improve the accuracy of centerline extraction, Steger [15] et al. proposed a centerline stripe extraction algorithm based on the Hessian matrix. Firstly, the structured light stripe image is subjected to five times two-dimensional Gaussian convolution to obtain the normal direction. The gray distribution function on the cross-section of the structured light stripe is expanded by second-order Taylor expansion. The centerline obtained by this algorithm has high sub-pixel accuracy and robustness, but the computational complexity is enormous, challenging to meet the requirements of real-time 3D reconstruction.

Based on the algorithm proposed by Steger, this paper presents an improved algorithm. First, obtains the high-quality light stripe image by using the filter and appropriate image preprocessing means. Then roughly extracts the light stripe center by using the weighted gray centroid method. The principle component analysis (PCA) method is then used to replace the Hessian matrix in order to determine the normal direction of the light stripe. After receiving the exact position of the center point by using the second-order Taylor expansion, repairs the breakpoint by cubic spline interpolation to obtain a smooth centerline. It improves the extraction speed and lays a foundation for real-time acquisition of the 3D point cloud in the later stage.

2. Algorithm Description

2.1 Principle

The beams from standard laser generators have a Gaussian distribution. Since the light intensity in the Gaussian laser stripe has the property of uneven distribution, the actual distribution will deviate from the Gaussian distribution when the laser is not perpendicular to the projection plane, which prevents the system from obtaining stable measurement accuracy during the measurement process [16].

In addition, in the actual working environment, the intense arc light generated during welding will cause some interference to the line structured light. In order to eliminate the disturbance on image acquisition, select the wavelength with weak electric arc intensity as the laser wavelength of the structured light sensor. Add a filter [17] that only allows specific laser wavelengths to pass through to reduce the interference of high-intensity arc light on the structured light image significantly.

Steger's method solves the Hessian matrix of the image, uses the eigenvector corresponding to the eigenvalue with the largest absolute value of the Hessian matrix to give the normal direction of each point. To obtain the center of the stripe, it also carries out the second-order Taylor expansion of the gray distribution function of each pixel of the stripe along the normal direction. Therefore, the algorithm has high accuracy and robustness [18]. However, this algorithm needs to carry out 5 times' two-dimensional Gaussian convolution on the image, which cannot meet the requirements of real-time [14]. In order to reduce the computational complexity caused by two-dimensional Gaussian convolution, Cai et al. [19] proposed an algorithm using principal component analysis (PCA) instead of a Hessian matrix to calculate the normal direction of each point. As a classical data dimensionality reduction method, PCA was proposed by Pearson [20] in 1901. It is often used to analyze data and establish mathematical models to reduce the dimension of data sets while retaining the characteristics of the largest contribution of data to each other's differences. By decomposing the covariance matrix can obtain the principal components of the data and their weights. The principal components are the eigenvectors of the covariance matrix, and the weights are the eigenvalues of the covariance matrix. When extracting the centerline stripes, PCA is used to decompose the covariance matrix of the image gradient vector. The eigenvector corresponding to the eigenvalue with the largest absolute value is the normal direction of the structured light stripes. Compared with Steger's algorithm using the Hessian matrix of the image to solve the normal vector of the stripe, PCA does not need to carry out two-dimensional Gaussian convolution many times. It can reduce the amount of calculation [21] and significantly save time extracting the center of structured light.

In the actual welding process, due to the interference of isolated points, splashes, and other factors, the extracted centerline will have error points, discontinuities, and abnormal points, resulting in the fracture of the central line pattern. Cubic spline interpolation algorithm can be used to repair the discontinuities considering the real-time performance of centerline extraction. Cubic spline interpolation is a curve fitting method that uses the values of known points and mathematical functions to estimate the approximate values of other points, with less calculation and less time-consuming.

2.2 Algorithm Procedure

Based on the above theory, the central stripe extraction algorithm of structured light proposed in this paper mainly consists of the following steps:

(1) High-quality structured light images are obtained using an industrial camera equipped with filters;

(2) Preprocess the image by using median filter and OTSU algorithm to separate the light bar from the background image;

(3) Use the weighted gray centroid method to roughly extract the centerline and obtain the initial point of the stripe center;

(4) Obtain normal direction of light stripe by principal component analysis;

(5) Expand the gray distribution function of the stripe by second-order Taylor expansion along the normal direction to obtain the exact position of the stripe center;

(6) Use cubic spline interpolation to repair the discontinuity and obtain a smooth centerline;

The flow chart of the algorithm is as follows:

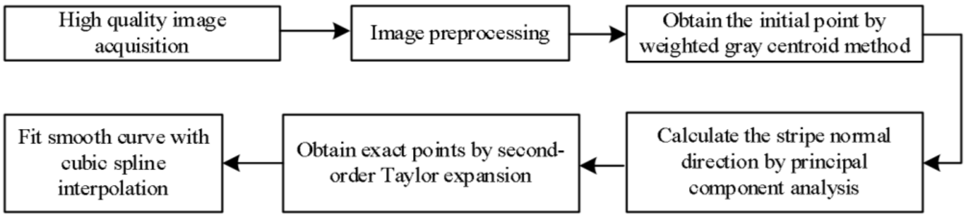


Fig.1 Flowchart of algorithm

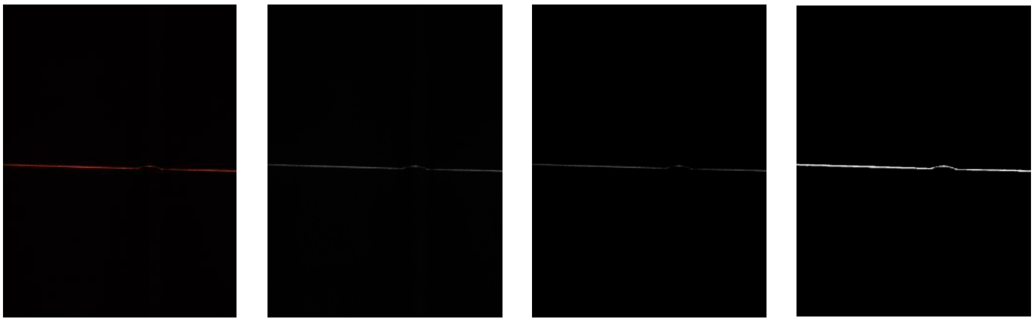
2.2.1Structure Light Image Acquisition and Preprocessing

When acquiring images, the test platform built in this paper parallel industrial camera and 650nm line laser transmitters and select 610-700nm band as the wavelength of line laser transmitters in the relatively weak bands of 440-480, 610-700, and 850-950nm welding arc spectra.

(a) (b) (c)

Fig. 2 Comparison of image acquisition quality with line structured light: (a) Industrial camera with filter added and line laser transmitter; (b) Filter installed; (c) Filter not installed

Then, a 615-665nm filter is placed on the camera to take a high-quality structured light image under actual welding and grinding conditions. The structured light image obtained by the filter is an image with bright stripes and dark background. They are apparent contrast. However, since most of the materials used for welding are rough metal, there are still some problems in the image, such as random noise distribution outside the light strip area, which seriously affects the subsequent processing. Therefore, the median filter is selected to convolute the collected image to eliminate the interference of pepper and salt random noise on the centerline extraction. Then, in order to separate the structured light stripe from the background and extract the region of interest (ROI) from the image, the Otsu method [22], with strong adaptability and no manual intervention, can be used for threshold segmentation by taking advantage of the high contrast of the structured light stripe image.



(a) (b) (c) (d)

Fig 3 (a) Initial image; (b) Grayscale image; (c) Image after median filtering; (d) ROI after threshold segmentation.

2.2.2 Extraction of Initial Points

After obtaining the ROI of the image, use the gray centroid method to roughly extract the sub-pixel stripe centers along the normal section. Because the light intensity of the stripe is presented as Gaussian distribution, the pixels in the center make a significant contribution to the calculation of the centerline when calculating the stripe center. Therefore, in this algorithm, the square weighted gray centroid method is used instead of the traditional gray centroid method, increase the weights of the pixels close to the centerline, reduce the weights of the pixels far from the centerline, making the extracted centerline closer to the actual one.

The mathematical expression is as follows:

$$Y_c = \frac{\sum_{y=1}^{n} I^2(x,y)y}{\sum_{y=1}^{n} I^2(x,y)} \tag{1}$$

In function (1), Y_c is the longitudinal coordinate of the center point of the structural light stripe on the section; I(x, y) is the gray value corresponding to the (x, y) coordinates on the image section, n is the number of pixels in the cross-section.

2.2.3 Acquisition of Stripe Normal Direction

After obtaining the initial point of the structured light stripe, the normal direction of the stripe needs to be further obtained by PCA to obtain the accurate centerline. The Sobel operator is a discrete first-order difference operator that can be used to convolute an image to obtain its gradient. Set the gradient vector of the image to $\begin{bmatrix} I_x & I_Y \end{bmatrix}^T$, the mathematical expression is as follows:

$$\begin{bmatrix} I_x & I_y \end{bmatrix}^T = \begin{bmatrix} \dfrac{\delta I(x,y)}{\delta x} \\ \dfrac{\delta I(x,y)}{\delta y} \end{bmatrix} \tag{2}$$

In function (2), I(x, y) represents the gray value of the image. Then, establish the covariance matrix of the gradient vector.

$$C = \begin{pmatrix} Cov(I_x \ \ I_x) & Cov(I_x \ \ I_y) \\ Cov(I_y \ \ I_x) & Cov(I_y \ \ I_y) \end{pmatrix} \tag{3}$$

In function (3), Cov(·) represents convolution operation, M is the selected area, with the extracted initial point as the center, the pixels on both sides of the initial point are basically symmetrically distributed. Therefore, the expected values $E(I_x)=0$ and $E(I_y)=0$ of Ix and I_y are approximately valid.

$$C = \begin{pmatrix} Cov(I_x \ \ I_x) & Cov(I_x \ \ I_y) \\ Cov(I_y \ \ I_x) & Cov(I_y \ \ I_y) \end{pmatrix} = \begin{pmatrix} E(I_x^2) - \left[E(I_x)\right]^2 & E\{[I_x - E(I_x)][I_y - E(I_x)]\} \\ E\{[I_x - E(I_x)][I_y - E(I_x)]\} & E(I_Y^2) - \left[E(I_Y)\right]^2 \end{pmatrix} = \sum_M \begin{pmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_x^2 \end{pmatrix} \tag{4}$$

According to function (4), eigenvalues and corresponding eigenvectors are as follow:

$$\lambda_1 = \frac{1}{2}(\sum_M I_x^2 + \sum_M I_y^2) + \frac{1}{2}\sqrt{(\sum_M I_x^2 - \sum_M I_y^2) + 4(\sum_W I_x I_y)^2} \tag{5}$$

$$\lambda_2 = \frac{1}{2}(\sum_M I_x^2 + \sum_M I_y^2) - \frac{1}{2}\sqrt{(\sum_M I_x^2 - \sum_M I_y^2) + 4(\sum_M I_x I_y)^2} \tag{6}$$

$$\gamma_1 = \left[\frac{1}{2}(\sum_M I_x^2 + \sum_M I_y^2) + \frac{1}{2}\sqrt{(\sum_M I_x^2 - \sum_M I_y^2) + 4(\sum_M I_x I_y)^2} \quad \sum_M I_x I_y\right]^T$$

(7)

$$\gamma_2 = \left[\frac{1}{2}(\sum_M I_x^2 + \sum_M I_y^2) - \frac{1}{2}\sqrt{(\sum_M I_x^2 - \sum_M I_y^2) + 4(\sum_M I_x I_y)^2} \quad \sum_M I_x I_y \right]^T \tag{8}$$

In function (5-8), λ_1 and λ_2 represent eigenvalue respectively, γ_1、γ_2 are corresponding eigenvectors. According to the physical meaning of the covariance matrix of the gradient vector, the eigenvector corresponding to the eigenvalue with the largest absolute value is the normal direction of the stripe. Because it is easy to deduce that $\lambda_1 > \lambda_2$, therefore, γ_1 is the normal direction of the stripe corresponding to λ_1. Finally, the unit vector $\mathbf{n} = \begin{bmatrix} \eta_x & \eta_y \end{bmatrix}^T$ in the normal direction of the stripe is obtained by normalizing γ_1.

2.2.4 Acquisition and Repair of Sub-pixel Stripe Center

After obtaining the normal direction of the light stripe, the pixel points with the abscissa X_C of the image and the ordinate Y_C calculated by the weighted gray centroid method are used as the initial points and expand by second-order Taylor expansion along the normal direction.

$$I(x_c + t\eta_x, y_c + t\eta_y) = I(x_C, y_c) + t\mathbf{n}^T \bullet \begin{bmatrix} I_x \\ I_y \end{bmatrix} + \frac{t^2}{2!}\mathbf{n}^T \bullet \begin{pmatrix} I_{xx} & I_{xy} \\ I_{yx} & I_{yy} \end{pmatrix} \bullet \mathbf{n} \tag{9}$$

To ensure that within one pixel, the condition satisfied by the stripe center point is $\begin{bmatrix} t\eta_x, t\eta_y \end{bmatrix} \in \left[-\frac{1}{2}, \frac{1}{2}\right] \times \left[-\frac{1}{2}, \frac{1}{2}\right]$, according to the characteristic that the light stripes satisfy Gaussian distribution, and $\frac{\partial I}{\partial t} = 0$, the specific formula is as follows:

$$t = -\frac{\eta_x I_x + \eta_y I_y}{\eta_x^2 G_{xx} + 2\eta_x \eta_y I_{xy} + \eta_y^2 I_{yy}} \tag{10}$$

Thus, the exact position of the stripe center is $(x_c + t\eta_x, y_c + t\eta_y)$.

3. Experiment and result analysis

3.1. Light strip center extraction experiment

This paper uses the line structured light vision system platform built in Fig. 4 to collect the line structured light stripe image. The laser wavelength is

650nm; the model of industrial camera is DAHENG MER2-160-75GM CMOS industrial camera, resolution ratio is 1440×1080 pixels; the camera lens model is Computar M0814-MP2, the focal length is 8mm; the camera lens is equipped with an AZURE BP635+-30nm filter. The line laser generates a line laser projected onto the object to be measured to form a line structured light strip. Combined with the one-dimensional mobile platform composed of EG515CA-800 linear guide slide and D57CME31 servo motor, industrial camera and fixed focus lens can realize the continuous acquisition of weld structured light image at a specific frame rate.

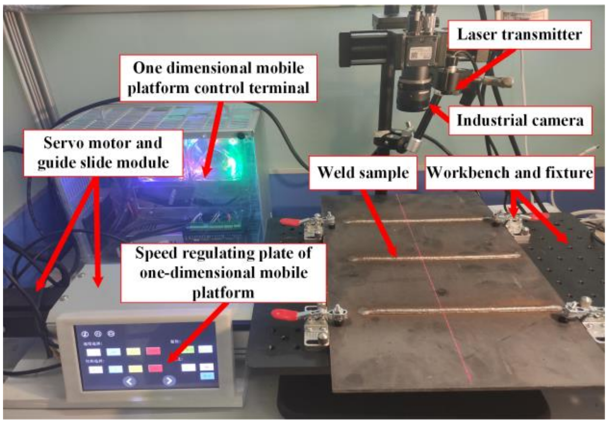
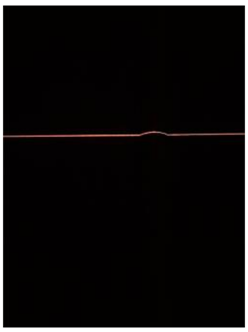
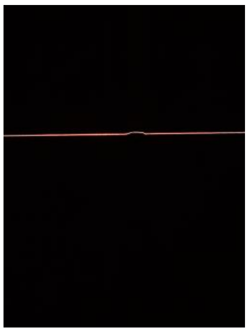


Fig 4 Line structured light vision test platform.

Table 1 shows the results of extracting the center of multiple weld structured light strips at different positions collected by the vision system platform shown in Fig. 4.

Table 1

Effect of structured light strip center extraction at different positions

(1) The structured light doesn't touch the weld	(2) The structured light is at the front of the weld	(3) The structured light is in the middle of the weld	(4) The structured light is at the end of the weld
			

From the Table 1 (1)-(4), it can be seen that the algorithm in this paper can extract the centerline of structured light at different positions. The centerline direction is the same as the light stripe, which is in line with the expected effect of the experiment.

3.2 Accuracy analysis of centerline extraction

In order to compare the centerline extraction accuracy of each algorithm, the centerline of the line structured light strip shown in Table 1 (1) is extracted by using the method in this paper, the gray centroid method and the Steger method, respectively. Fig. 4 is an enlarged result of centerline extraction using different algorithms. It can be seen from the figure that compared with the traditional gray centroid method, the light strip center extracted by this algorithm is closer to the Steger method with the highest accuracy.

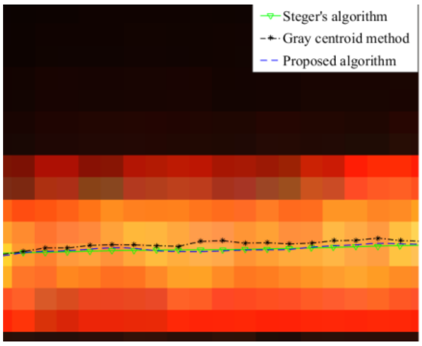


Fig. 4. Comparison of centerline extraction results with different algorithms

Taking the Steger algorithm with high precision as a reference, the Euclidean distance between the center points of the light strip extracted by the proposed algorithm, the traditional gray centroid method, and the Steger algorithm is calculated. Take this distance as the deviation, record the root mean square deviation (RMSD). Table 2 shows the results. Although the extraction accuracy of different images is different, this algorithm can generally improve the center extraction accuracy of the traditional gray centroid method.

Table 2

Comparison of light strip center extraction deviation

		Shape of light stripe			
		Table 1 (1)	Table 1 (2)	Table 1 (3)	Table 1 (4)
RMSD	gray centroid method /pixel	0.1141	0.1036	0.1134	0.0997
	proposed algorithm / pixel	0.1039	0.0982	0.1067	0.0934
	Error reduction rate /%	8.9	5.2	5.9	6.3

3.3 Program run time analysis

The computer used for image processing is a Dell precision t5820 tower graphics workstation; The CPU is Intel Xeon w-2245, 3.90GHz; the RAM is 64G; the image processing software used is MATLAB 2016a. Table 3 shows the comparison results of running time. As it can be seen from Table 3, the overall operation speed of this algorithm can be increased by eight times compared with the Steger algorithm, and the processing time of a single picture can be reduced to about 700ms. Therefore, it can better meet the requirements of real-time.

Table 3

The proceeding time of each algorithm

Shape of light stripe	Steger algorithm	gray centroid method	proposed algorithm
Table 1（1）	5.9775s	0.0586s	0.7325s
Table 1（2）	5.8308s	0.0599s	0.7160s
Table 1（3）	5.9136s	0.0662s	0.7098s
Table 1（4）	5.8880s	0.0549s	0.7050s

4. Conclusion

In this paper, we intensely studied the extraction method of the centerline of line structured light stripe, and analyzed the current situation and problems of centerline extraction technology. Proposed an improved centerline extraction method based on the Steger algorithm using the self-built line structured light vision platform and weld samples. The core of this method is to realize the accurate sub-pixel accuracy extraction of the preprocessed weld structured light stripe image by using the combination of the weighted Gray centroid method, principal component analysis (PCA) approach and Cubic spline interpolation. Experiments show that this method has higher accuracy than the classical gray centroid method. The extraction speed is eight times faster than the Steger algorithm, so it can better meet the needs of industrial real-time detection and contribute to the real-time 3D reconstruction of weld in the future.

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