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WEAK CONVERGENCE OF A NEW ITERATE FOR SOLVING SPLIT FIXED POINT PROBLEMS

Yufei Zhao[1], Yonghong Yao[2]

In this paper, we study a split fixed point problem involving three nonlinear operators in Hilbert spaces. To solve this split fixed point problem, we propose a new iterative algorithm by applying fixed point technique. Under plain conditions, we show that the sequence generated by the suggested algorithm converges weakly to a solution of the split fixed point problem.

Keywords: split fixed point, demicontractive operator, pseudocontractive operator, weak convergence.

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1. Introduction

Let H_1 be a real Hilbert space equipped with the inner produce $\langle \cdot, \cdot \rangle$ and the induced norm $\|\cdot\|$, φ be a self-mapping of H_1 and $\{x_n\}$ be a sequence in H_1. Throughout, we employ the following expressions: (i) $\mathrm{Fix}(\varphi)$ stands for the fixed point set of φ, i.e., $\mathrm{Fix}(\varphi) := \{u \in H_1 : u = \varphi(u)\}$; (ii) $x_n \rightharpoonup x^\dagger$ means x_n weakly convergent to $x^\dagger$ as $n \to \infty$; (iii) $x_n \to x^\dagger$ indicates x_n strongly convergent to $x^\dagger$ as $n \to \infty$; (iv) $\omega_w(x_n)$ denotes the set of the weak cluster points of $\{x_n\}$ in H_1, i.e., $\omega_w(x_n) := \{v \in H_1 :$ there exists a subsequence $\{x_{n_i}\}$ of $\{x_n\}$ such that $x_{n_i} \rightharpoonup v(i \to \infty)\}$.

Let H_1 and H_2 be two real Hilbert spaces. Let $\varphi, \phi : H_1 \to H_1$ and $\psi : H_2 \to H_2$ be three nonlinear operators. Let $A : H_1 \to H_2$ be a bounded linear operator. In this paper, we focus on the split fixed point problem which is to find a point $p^\dagger \in H_1$ such that

$$p^\dagger \in \mathrm{Fix}(\varphi) \cap \mathrm{Fix}(\phi) \text{ and } Ap^\dagger \in \mathrm{Fix}(\psi). \tag{1}$$

If $\varphi \equiv I$, then (1) reduces to the following two-sets split fixed point problem of finding a point $p^\dagger \in H_1$ such that

$$p^\dagger \in \mathrm{Fix}(\phi) \text{ and } Ap^\dagger \in \mathrm{Fix}(\psi). \tag{2}$$

The prototype of the two-sets split fixed point problem is the split feasibility problem which aims to seek a point $p^\dagger \in H_1$ such that

$$p^\dagger \in C \text{ and } Ap^\dagger \in Q, \tag{3}$$

where $C \subset H_1$ and $Q \subset H_2$ are two closed convex sets.

The mathematical model (3) was refined by Censor et. al. ([3]) from the intensity-modulated radiation therapy and was investigated extensively by many scholars ([1–3, 18]). An important method to solve (2) is fixed point method with the help of the equivalent

[1]School of Mathematical Sciences, Tiangong University, Tianjin 300387, China, e-mail: zyf@163.com

[2]Corresponding author. School of Mathematical Sciences, Tiangong University, Tianjin 300387, China and Center for Advanced Information Technology, Kyung Hee University, Seoul 02447, Republic of Korea, e-mail: yyhtgu@hotmail.com

relation: $p^\dagger$ solves (2) if and only if $p^\dagger = \phi(p^\dagger - \varsigma A^*(I-\psi)Ap^\dagger), \varsigma > 0$. Based on this fact, Censor and Segal [4] introduced an iterative algorithm defined by: $z_0 \in H_1$,

$$z_{n+1} = \phi(z_n - \varsigma A^*(I-\psi)Az_n), \quad n \geq 0, \tag{4}$$

where A^* is the adjoint of A and $\varsigma \in (0, 2/\|A\|^2)$.

The above fixed point method has been further extended and improved by many scholars, see [5–17, 19–26]. At the same time, we note that solving (2) is equivalent to solve the fixed point equation $x = \phi(x) - \varsigma A^*(I-\psi)Ax(\varsigma > 0)$. By using this equivalent relation, Zheng et. al. [27] suggested an iterative algorithm defined by: $y_0 \in H_1$,

$$y_{n+1} = (1-\mu)y_n + \mu[\phi(y_n) - \varsigma A^*(I-\psi)Ay_n],\ n \geq 0. \tag{5}$$

Motivated and inspired by the work in this field, in this paper, we construct an iterative method to solve (1) by utilizing the fixed point technique. Under plain conditions, we show that the sequence generated by the suggested method converges weakly to a solution of the split fixed point problem (1).

2. **Preliminaries**

In this section, we collect several notations and lemmas. Let H be a real Hilbert space. First, we have the following two well-known equalities in H:

$$\|x+y\|^2 = \|x\|^2 + 2\langle x, y\rangle + \|y\|^2,\ \forall x, y \in H, \tag{6}$$

and

$$\|(1-\mu_1)x + \mu_1 y\|^2 = (1-\mu_1)\|x\|^2 + \mu_1\|y\|^2 - (1-\mu_1)\mu_1\|x-y\|^2 \tag{7}$$

for all $x, y \in H$ and $\forall \mu_1 \in \mathbb{R}$.

An operator $f : H \to H$ is said to be δ-demicontractive if $\mathrm{Fix}(f) \neq \emptyset$ and there is a constant $\delta \in [0,1)$ such that, for each $\hat{x} \in \mathrm{Fix}(f)$,

$$\|f(x) - \hat{x}\|^2 \leq \|x - \hat{x}\|^2 + \delta\|x - f(x)\|^2,\ \forall x \in H, \tag{8}$$

equivalently,

$$\langle x - f(x), x - \hat{x}\rangle \geq \frac{1-\delta}{2}\|x - f(x)\|^2,\ \forall x \in H. \tag{9}$$

Recall that an operator $g : H \to H$ is said to be

(i) δ-Lipschitz if

$$\|g(x) - g(y)\| \leq \delta\|x-y\|,\ \forall x, y \in H,$$

where $\delta \geq 0$ is a constant.

(ii) pseudocontractive if

$$\langle g(x) - g(y), x - y\rangle \leq \|x-y\|^2, \forall x, y \in H,$$

equivalently,

$$\|g(x) - g(y)\|^2 \leq \|x-y\|^2 + \|(I-g)x - (I-g)y\|^2,\ \forall x, y \in H.$$

Lemma 2.1 ([27]). *Let H_1 and H_2 be two real Hilbert spaces. Let $A : H_1 \to H_2$ be a bounded linear operator and let A^* be the adjoint operator of A. Let $\phi : H_1 \to H_1$ and $\psi : H_2 \to H_2$ be two demicontractive operators. Then,*

$$x \in \mathrm{Fix}(\phi) \text{ and } Ax \in \mathrm{Fix}(\psi) \Leftrightarrow x \in \mathrm{Fix}(\phi - \varsigma A^*(I-\psi)A),$$

for all $\varsigma > 0$.

Lemma 2.2 ([28, 29]). *Let H_1 be a real Hilbert space. Let $\varphi : H_1 \to H_1$ be a δ_2-Lipschitz pseudocontractive operator. Then,*

(i) *φ is demi-closedness, namely, $x_n \rightharpoonup p(n \to \infty)$ and $\varphi(x_n) \to q(n \to \infty)$ imply that $\varphi(p) = q$.*

(ii) *for each* $\hat{z} \in \mathrm{Fix}(\varphi)$*, we have*

$$\|\varphi((1-\mu_2)x+\mu_2\varphi(x))-\hat{z}\|^2 \le \|x-\hat{z}\|^2+(1-\mu_2)\|x-\varphi((1-\mu_2)x+\mu_2\varphi(x))\|^2,$$

for all $x \in H_1$*, where* $0<\mu_2<\frac{1}{\sqrt{1+\delta_2^2}+1}$.

Lemma 2.3. *(Opial) Let* Γ *be a nonempty closed convex subset of a real Hilbert space* H_1*. Let* $\{u_n\}$ *be a sequence in* H_1*. Suppose that*

(i) $\forall \hat{z} \in \Gamma$, $\lim_{n\to\infty}\|u_n-\hat{z}\|$ *exists;*
(ii) $\omega_w(u_n) \subset \Gamma$.

Then the sequence $\{u_n\}$ *converges weakly to some point in* Γ.

3. Main results

In this section, to solve the split fixed point problem (1), we construct a new iterate algorithm and show that the sequence generated by the algorithm converges weakly to a solution of (1) under some mild conditions.

Let H_1 and H_2 be two real Hilbert spaces. Let $A: H_1 \to H_2$ be a bounded linear operator and let A^* be the adjoint operator of A. Let $\varphi: H_1 \to H_1$ be a δ_2-Lipschitz pseudocontractive operator. Let $\phi: H_1 \to H_1$ be a δ_1-demicontractive operator and $\psi: H_2 \to H_2$ be a δ_3-demicontractive operator.

The solution set of the split fixed point problem (1) is denoted by Γ, namely,

$$\Gamma := \{x \in H_1 : x \in \mathrm{Fix}(\phi)\cap\mathrm{Fix}(\varphi) \text{ and } Ax \in \mathrm{Fix}(\psi)\}.$$

Now, we present an iterative algorithm to solve the split fixed point problem (1).

Algorithm 3.1. *Let* α*,* β*,* γ*,* μ_1 *and* μ_2 *be five constants. For an initial guess* $u_0 \in H_1$*, define the sequence* $\{u_n\}$ *iteratively by*

$$\begin{cases} y_n = (1-\gamma)u_n+\gamma\phi(u_n)-\alpha A^*(I-\psi)Au_n, & (10)\\ v_n = u_n-\beta(u_n-y_n), & (11)\\ z_n = (1-\mu_2)v_n+\mu_2\varphi(v_n), & (12)\\ u_{n+1} = (1-\mu_1)v_n+\mu_1\varphi(z_n),\ n\ge 0. & (13)\end{cases}$$

Next, we analyse the convergence of the sequence $\{u_n\}$ generated by Algorithm 3.1.

Theorem 3.1. *Suppose the following conditions are satisfied:*

(i) $\|A\| \ne 0$ *and* $\Gamma \ne \emptyset$*;*
(ii) $I-\phi$ *and* $I-\psi$ *are demiclosed at zero;*
(iii) $\delta_1 \in [0,1)$, $\delta_2 \ge 1$, $\delta_3 \in [0,1)$, $\gamma \in (0,1]$, $\beta \in (0,\frac{1-\delta_1}{2\gamma})$, $\alpha \in (0,\frac{1-\delta_3}{2\beta\|A\|^2})$ *and* $0<\mu_1<\mu_2<\frac{1}{1+\sqrt{1+\delta_2^2}}$.

Then the sequence $\{u_n\}$ *defined by Algorithm 3.1 converges weakly to some point in* Γ.

Proof. Let $\hat{z}$ be any point in Γ. Observe that $\hat{z}=\phi(\hat{z})=\varphi(\hat{z})$ and $A\hat{z}=\psi(A\hat{z})$. Further, by Lemma 2.1, we have $\hat{z}=(1-\gamma)\hat{z}+\gamma\phi(\hat{z})-\alpha A^*(I-\psi)A\hat{z}$.

By (7) and (13), we have

$$\begin{aligned}\|u_{n+1}-\hat{z}\|^2 &= \|(1-\mu_1)(v_n-\hat{z})+\mu_1(\varphi(z_n)-\hat{z})\|^2\\ &= (1-\mu_1)\|v_n-\hat{z}\|^2+\mu_1\|\varphi(z_n)-\hat{z}\|^2-\mu_1(1-\mu_1)\|v_n-\varphi(z_n)\|^2.\end{aligned} \tag{14}$$

Applying Lemma 2.2, we obtain

$$\begin{aligned}\|\varphi(z_n)-\hat{z}\|^2 &= \|\varphi((1-\mu_2)v_n+\mu_2\varphi(v_n))-\hat{z}\|^2\\ &\le \|v_n-\hat{z}\|^2+(1-\mu_2)\|v_n-\varphi(z_n)\|^2.\end{aligned} \tag{15}$$

Combining (14) and (15), we have

$$\begin{aligned}\|u_{n+1}-\hat{z}\|^2 &\le \|v_n-\hat{z}\|^2+\mu_1(\mu_1-\mu_2)\|v_n-\varphi(z_n)\|^2\\ &\le \|v_n-\hat{z}\|^2.\end{aligned} \tag{16}$$

Next, we estimate $\|v_n-\hat{z}\|$. According to (6) and (11), we obtain

$$\begin{aligned}\|v_n-\hat{z}\|^2 &= \|u_n-\hat{z}-\beta(u_n-y_n)\|^2\\ &= \|u_n-\hat{z}\|^2-2\beta\langle u_n-\hat{z},u_n-y_n\rangle+\beta^2\|u_n-y_n\|^2.\end{aligned} \tag{17}$$

Taking into account (10), we have $u_n-y_n=\gamma(u_n-\phi(u_n))+\alpha A^*(I-\psi)Au_n$. Then,

$$\begin{aligned}\langle u_n-\hat{z},u_n-y_n\rangle &= \langle u_n-\hat{z},\gamma(u_n-\phi(u_n))\rangle+\langle u_n-\hat{z},\alpha A^*(I-\psi)Au_n\rangle\\ &= \gamma\langle u_n-\hat{z},u_n-\phi(u_n)\rangle+\alpha\langle Au_n-A\hat{z},(I-\psi)Au_n\rangle.\end{aligned} \tag{18}$$

and

$$\begin{aligned}\|u_n-y_n\|^2 &= \|\gamma(u_n-\phi(u_n))+\alpha A^*(I-\psi)Au_n\|^2\\ &\le (\gamma\|u_n-\phi(u_n)\|+\alpha\|A\|\|(I-\psi)Au_n\|)^2\\ &\le 2\gamma^2\|u_n-\phi(u_n)\|^2+2\alpha^2\|A\|^2\|(I-\psi)Au_n\|^2.\end{aligned} \tag{19}$$

Since ϕ and ψ are δ_1-demicontractive and δ_3-demicontractive, respectively, utilizing (9), we obtain

$$\langle u_n-\hat{z},u_n-\phi(u_n)\rangle \ge \frac{1-\delta_1}{2}\|u_n-\phi(u_n)\|^2, \tag{20}$$

and

$$\langle Au_n-A\hat{z},(I-\psi)Au_n\rangle \ge \frac{1-\delta_3}{2}\|(I-\psi)Au_n\|^2. \tag{21}$$

Taking into account (18), (20) and (21), we have

$$\langle u_n-\hat{z},u_n-y_n\rangle \ge \frac{\gamma(1-\delta_1)}{2}\|u_n-\phi(u_n)\|^2+\frac{\alpha(1-\delta_3)}{2}\|(I-\psi)Au_n\|^2. \tag{22}$$

Substituting (19) and (22) into (17), we attain

$$\begin{aligned}\|v_n-\hat{z}\|^2 \le\ & \|u_n-\hat{z}\|^2-\beta\gamma(1-\delta_1)\|u_n-\phi(u_n)\|^2-\beta\alpha(1-\delta_3)\|(I-\psi)Au_n\|^2\\ &+2\beta^2\gamma^2\|u_n-\phi(u_n)\|^2+2\beta^2\alpha^2\|A\|^2\|(I-\psi)Au_n\|^2\\ =\ & \|u_n-\hat{z}\|^2-\beta\gamma(1-\delta_1-2\beta\gamma)\|u_n-\phi(u_n)\|^2\\ &-\beta\alpha(1-\delta_3-2\beta\alpha\|A\|^2)\|(I-\psi)Au_n\|^2\\ \le\ & \|u_n-\hat{z}\|^2.\end{aligned} \tag{23}$$

By virtue of (16) and (23), we have

$$\begin{aligned}\|u_{n+1}-\hat{z}\|^2 &\le \|u_n-\hat{z}\|^2+\mu_1(\mu_1-\mu_2)\|v_n-\varphi(z_n)\|^2\\ &\le \|u_n-\hat{z}\|^2.\end{aligned} \tag{24}$$

Then, $\lim_{n\to\infty}\|u_n-\hat{z}\|$ exists, denoted by l^*.

Thanks to (24), we have

$$\mu_1(\mu_2-\mu_1)\|v_n-\varphi(z_n)\|^2 \le \|u_n-\hat{z}\|^2-\|u_{n+1}-\hat{z}\|^2 \to 0,$$

which yields that

$$\lim_{n\to\infty}\|v_n-\varphi(z_n)\|=0. \tag{25}$$

Furthermore, based on (16) and (23), we conclude that

$$\|u_{n+1}-\hat{z}\| \le \|v_n-\hat{z}\| \le \|u_n-\hat{z}\|$$

which implies that $\lim_{n\to\infty}\|v_n - \hat{z}\| = l^*$.

Owing to δ_2-Lipschitz continuity of φ, we have

$$\begin{aligned}\|v_n - \varphi(v_n)\| &\le \|v_n - \varphi(z_n)\| + \|\varphi(z_n) - \varphi(v_n)\|\\ &\le \|v_n - \varphi(z_n)\| + \delta_2\|z_n - v_n\|\\ &\le \|v_n - \varphi(z_n)\| + \delta_2\mu_2\|v_n - \varphi(v_n)\|.\end{aligned}$$

Thus, $\|v_n - \varphi(v_n)\| \le \frac{1}{1-\delta_2\mu_2}\|v_n - \varphi(z_n)\|$ which together with (25) implies that

$$\lim_{n\to\infty}\|v_n - \varphi(v_n)\| = 0. \tag{26}$$

Moreover, from (12), we obtain

$$\|z_n - v_n\| \le \mu_2\|v_n - \varphi(v_n)\| \to 0. \tag{27}$$

In view of (23), we acquire

$$\begin{aligned}&\beta\gamma(1-\delta_1-2\beta\gamma)\|u_n - \phi(u_n)\|^2 + \beta\alpha(1-\delta_3-2\beta\alpha\|A\|^2)\|(I-\psi)Au_n\|^2\\ &\quad\le \|u_n - \hat{z}\|^2 - \|v_n - \hat{z}\|^2 \to 0,\end{aligned}$$

which leads to that

$$\lim_{n\to\infty}\|u_n - \phi(u_n)\| = 0 \tag{28}$$

and

$$\lim_{n\to\infty}\|(I-\psi)Au_n\| = 0. \tag{29}$$

Note that $\|y_n - u_n\| \le \gamma\|\phi(u_n) - u_n\| + \alpha\|A\|\|(I-\psi)Au_n\|$. Combining with (28) and (29), we deduce

$$\lim_{n\to\infty}\|y_n - u_n\| = 0$$

and hence

$$\|v_n - u_n\| \le \beta\|y_n - u_n\| \to 0.$$

Therefore, $\|u_{n+1} - u_n\| \le \|v_n - u_n\| + \mu_1\|v_n - \varphi(z_n)\| \to 0$.

Next, we show $\omega_w(u_n) \subset \Gamma$. Choosing any $z^\dagger \in \omega_w(u_n)$, there is a subsequence $\{u_{n_j}\}$ of $\{u_n\}$ such that $u_{n_j} \rightharpoonup z^\dagger (j \to \infty)$. It is easy to seen that $v_{n_j} \rightharpoonup z^\dagger$, $z_{n_j} \rightharpoonup z^\dagger$ and $Au_{n_j} \rightharpoonup Az^\dagger$. Thus,

$$\left.\begin{array}{r} I-\varphi \text{ is demiclosed at zero}\\ v_{n_j} \rightharpoonup z^\dagger\\ \|v_{n_j} - \varphi(v_{n_j})\| \to 0\end{array}\right\} \Rightarrow z^\dagger \in \mathrm{Fix}(\varphi),$$

$$\left.\begin{array}{r} I-\phi \text{ is demiclosed at zero}\\ u_{n_j} \rightharpoonup z^\dagger\\ \|u_{n_j} - \phi(u_{n_j})\| \to 0\end{array}\right\} \Rightarrow z^\dagger \in \mathrm{Fix}(\phi),$$

and

$$\left.\begin{array}{r} I-\psi \text{ is demiclosed at zero}\\ Au_{n_j} \rightharpoonup Az^\dagger\\ \|(I-\psi)Au_{n_j}\| \to 0\end{array}\right\} \Rightarrow Az^\dagger \in \mathrm{Fix}(\psi).$$

So, $z^\dagger \in \Gamma$ and $\omega_w(u_n) \subset \Gamma$.

Finally, we show that $\{u_n\}$ is convergent. In fact, we have (i) $\forall \hat{z} \in \Gamma$, $\lim_{n\to\infty}\|u_n - \hat{z}\|$ exists; (ii) $\omega_w(u_n) \subset \Gamma$. Consequently, according to Lemma 2.3, we conclude that the sequence $\{u_n\}$ converges weakly to some point in Γ. □

According to Algorithm 3.1 and Theorem 3.1, we can obtain the following algorithms and corollaries.

Algorithm 3.2. *Let α, β and γ be three constants. For an initial guess $u_0 \in H_1$, define the sequence $\{u_n\}$ iteratively by*

$$\begin{cases} y_n = (1-\gamma)u_n + \gamma\phi(u_n) - \alpha A^*(I-\psi)Au_n, \\ u_{n+1} = u_n - \beta(u_n - y_n),\ n \geq 0. \end{cases}$$

Corollary 3.1. *Suppose the following conditions are satisfied:*

(i) $\|A\| \neq 0$ *and* $\Gamma_1 \neq \emptyset$;
(ii) $I-\phi$ *and* $I-\psi$ *are demiclosed at zero;*
(iii) $\delta_1 \in [0,1)$, $\delta_3 \in [0,1)$, $\gamma \in (0,1]$, $\beta \in (0, \frac{1-\delta_1}{2\gamma})$ *and* $\alpha \in (0, \frac{1-\delta_3}{2\beta\|A\|^2})$.

Then, the sequence $\{u_n\}$ generated by Algorithm 3.2 converges weakly to some point in Γ_1 where Γ_1 is the solution set of the split problem (2).

Algorithm 3.3. *Let α, β, μ_1 and μ_2 be four constants. For an initial guess $u_0 \in H_1$, define the sequence $\{u_n\}$ iteratively by*

$$\begin{cases} y_n = u_n - \alpha A^*(I-\psi)Au_n, \\ v_n = u_n - \beta(u_n - y_n), \\ z_n = (1-\mu_2)v_n + \mu_2\varphi(v_n), \\ u_{n+1} = (1-\mu_1)v_n + \mu_1\varphi(z_n),\ n \geq 0. \end{cases}$$

Corollary 3.2. *Suppose the following conditions are satisfied:*

(i) $\|A\| \neq 0$ *and* $\Gamma \neq \emptyset$;
(ii) $I-\psi$ *is demiclosed at zero;*
(iii) $\delta_2 \geq 1$, $\delta_3 \in [0,1)$, $\beta \in (0,1)$, $\alpha \in (0, \frac{1-\delta_3}{2\beta\|A\|^2})$ *and* $0 < \mu_1 < \mu_2 < \frac{1}{1+\sqrt{1+\delta_2^2}}$.

Then, the sequence $\{u_n\}$ generated by Algorithm 3.3 converges weakly to some point in Γ_2, where $\Gamma_2 := \{x \in H_1 : x \in \mathrm{Fix}(\varphi)$ and $Ax \in \mathrm{Fix}(\psi)\}$.

4. Conclusion

In this paper, we investigate the split fixed point problem (1) in which two operators are demicontractive and another one is pseudocontractive. Based on fixed point technique, we construct an iterative algorithm to solve the split fixed point problem (1). We prove that the sequence $\{u_n\}$ defined by the algorithm 3.1 converges weakly to some solution of (1) under some plain conditions imposed on the operators and the parameters.

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