A NEW TIME-FREQUENCY COMBINED METHOD BASED ON IMPROVED COMPLETE ENSEMBLE EMD AND AFFINE SMOOTH PSEUDO WIGNER-VILLE DISTRIBUTION

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During the bilinear time-frequency analysis of complex non-stationary signals with multiple components, in order to delete or decrease the self-crossing terms caused by the nonlinear frequency modulation component and the cross-terms generated by the multi-components, a new time-frequency combined method is proposed, which is based on the improved complete integrated empirical mode decomposition with adaptive noise (ICEEMDAN) and Affine smooth pseudo Wigner-Ville distribution (ASPWVD). The signal is first decomposed into several single-component intrinsic mode functions (IMFs) by ICEEMDAN. Then, the false IMFs is removed by the correlation coefficient criterion. The left IMFs are analyzed by the ASPWVD to extract the frequency characteristic information of the signal. The effectiveness of the new proposed method is verified by comparing the results of a simulation signal with those of other time-frequency methods. The method is also used to extract the characteristic frequency components of the vibration signal sampled from a FotonBJ493ZQ3 type engine test-bed so as to provide data basis for identifying the vibration sources of the engine. This proves the practicability of the time-frequency combined method.

Keywords: time-frequency, Improved complete ensemble empirical mode decomposition, Affine smooth pseudo Wigner-Ville distribution, cross-term, intrinsic mode function

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1. Introduction

In the process of analyzing non-stationary signals, either time domain analysis or spectrum analysis cannot alone be used to extract the signal information in time domain, frequency domain and amplitude domain at the same time. Its resolution is poor[1]. Time-frequency analysis methods are now considered as the most effective methods to deal with non-stationary signals, which can extract the information in the above three dimensions simultaneously. The common methods include the short-time Fourier transform (STFT) and the continuous wavelet transform (CWT). However, STFT uses the fixed window functions[2]. CWT adopts flexible window functions, which can change the window length according to frequency changes. But it needs to select the wavelet base function, and the number of the decomposition layers in advance. This results in a certain previous knowledge of parameters and poor adaptability[3].

Hilbert transform (HHT) proposed by Huang E et al. [4] does not need to select these base functions. It can be used to directly decomposes the nonstationary signal into several intrinsic mode functions (IMFs) through empirical mode decomposition (EMD), and then conduct spectrum analysis for each IMF component, showing its high adaptability. However, problems such as modal aliasing and endpoint effect will inevitably occur in using EMD method[5].Wu Z et al. [6] proposed the ensemble Empirical Mode Decomposition (EEMD) to solve the problem of modal aliasing by adding the white noise signal. But due to the limited number of adding times, noise residues are prone to occur. Torres M.E et al. [7] proposed complete ensemble empirical mode decomposition with adaptive noise (CEEMDAN) which is able to solve the problem of noise residue in fewer average times by adding the adaptive white noise signal at each decomposition stage. But some false IMFs may produce after decomposition. By using the local mean of signals to extract IMFs, the improved adaptive ensemble empirical mode decomposition with adaptive noise (ICEEMDAN) proposed by Colominas[8] is able to effectively solve the problems of the modal aliasing , the noise residue and the false IMFs[9].

Wigner-Ville distribution (WVD) is a kind of bilinear time-frequency distribution to be very suitable for time-frequency analysis of non-stationary signals, which has the characteristics of high resolution. In process of analyzing multi-component signals, WVD inevitably produces cross-terms as it is a bilinear time-frequency distribution[10]. The generated cross-terms may be the cross-terms of multi-component signals or the self-crossing terms of Nonlinear

FM signals. These cross-terms seriously affect the extraction of signal features. There have been various attempts to eliminate or reduce the cross-terms of WVD. Cohen class time-frequency methods are used to smooth WVD by changing the kernel functions to reduce the cross-terms. Pseudo Wigner-Ville distribution (PWVD), smoothed pseudo Wigner-Ville distribution (SPWVD) and Choi-Williams distribution (CWD) etc.[10] are typical Cohen class time-frequency methods[11]. Affine time-frequency distribution is different from the Cohen time-frequency distribution in respect of the time and frequency shift invariance, which can meet the time and scale of the expansion shift invariance. By designing the Affine smoothing function, the cross-term is greatly reduced, and the frequency resolution is significantly improved [12]. Compared with Cohen time-frequency distribution, ASPWVD can analyze further Nonlinear FM signals more clearly and accurately.

As a single time-frequency analytic method cannot effectively extract the frequency characteristics of non-stationary signals, Cai Yanping et al. proposed an EMD-WVD method and applied it to the fault diagnosis of internal combustion engines [13], but the EMD-WVD method could not eliminate the self-crossing terms of Nonlinear FM signals. Yang Xiaoyu et al.[1] proposed an EEMD-CWD method and applied it to gear fault diagnosis, which can more effectively eliminate the influence of cross- terms, but the time-frequency resolution is still poor. In this paper, a time-frequency combined method based on ICEEMDAN and ASPWVD is proposed in order to get the better time-frequency resolution and delete the cross-terms. This Method first uses the ICEEMDAN to decompose the nonstationary signal to achieve some IMFs, then uses the each IMF to correlate with the original signal to get the own correlation coefficient and selects the effective IMFs according to the correlation coefficient rule, and finally analyzes the selected IMFs by using ASPWVD. Through analyzing a simulation signal, it is proved that this method can extract better the frequency characteristic components of the non-stationary signal.

2. Algorithm theory

2.1 ICEEMDAN algorithm

The ICEEMDAN algorithm [8] is based on the EMD decomposition principle, and adds a controllable White Gaussian noise to the signal during each decomposition process to generate multiple noisy signals and average them, so as to solve the defect of modal aliasing and enhance the anti-interference performance. Compared with CEEMDAN that takes the average of the IMFs of multiple noisy signals as the IMF generated by each iteration of the original signal, ICEEMDAN takes the residual of the previous iteration minus the average of the local means of the current iteration as the IMF generated by each iteration of the original signal. ICEEMDAN can further reduce the noise residue and aliasing in the IMFs and improve the decomposition effect.

In the ICEEMDAN algorithm, the algorithm operator $E_k(\cdot)$ is defined as the kth modal component obtained by EMD, and let $M(\cdot)$ be the algorithm operator which produces the local mean of the signal x that is applied to. It can be noticed that $E_1(x) = x - M(x)$. The coefficient β_k is defined as the signal-to-noise ratio (SNR) at each stage. $W^{(i)}$ is defined as a white noise with mean value of 0 and variance of 1. The algorithm steps are listed as follows.

（1）Add Gaussian white noise to the original signal, and perform EMD for each new signal $x^i = x + \beta_0 E_1 w^i$ (i=1,2,...,N) and achieve the local mean denoted as $M(xi)$. The first residue r_1 is obtained by averaging the $M(xi)$ in N times. The formula is as follows.

$$r_1 = \frac{1}{N}\sum_{i=1}^{N} M(x^i) \tag{1}$$

（2）When k=1, calculate the first IMF of ICEEMDAN.

$$\tilde{d}_1 = x - r_1 \tag{2}$$

（3）Estimate the second residue as the average of local means of the realizations. According to $r_1 + \beta_1 E_2(W^{(i)})$, get the second residue r_2.

$$r_2 = \frac{1}{N}\sum_{i=1}^{N} M\left(r_1 + \beta_1 E_2\left(W^{(i)}\right)\right) \tag{3}$$

（4）Calculate the second IMF of ICEEMDAN.

$$\tilde{d}_2 = r_1 - r_2 \tag{4}$$

（5）When k=3,...,K calculate the kth residue r_k.

$$r_k = \frac{1}{N}\sum_{i=1}^{N} M\left(r_{k-1} + \beta_{k-1} E_k\left(W^{(i)}\right)\right) \tag{5}$$

（6）Calculate the kth IMF.

$$\tilde{d}_k = r_{k-1} - r_k \tag{6}$$

（7）Let $k = k + 1$, return to the step 3 for next k until the residual meets the iteration termination condition. Judge whether the residual is monotonic, and adopt the Cauchy convergence criterion, that is the standard deviation between two adjacent IMFs is limited: $\sigma = \left\|\tilde{d}_{k+1} - \tilde{d}_k\right\|_2 / \left\|\tilde{d}_k\right\|_2$. When the standard deviation is less than a threshold value (in this paper, let $\sigma < 0.2$), the iteration is terminated.

2.2 ASPWVD theory

Cohen type bilinear time-frequency distribution is based on the invariance of time shift and frequency shift [14], while Affine bilinear time-frequency distribution is realized by time shift and stretching transformation. Affine time-frequency distribution is also a kind of time-frequency energy distribution, which is similar to Cohen's time-frequency distribution, but its transformation method is different. The main content of the ASPWVD is to design a smoothing function in order to reduce the cross-terms and improve the time-frequency resolution. It adopts a smoothing function which separates from time domain and frequency domain.

$$ASPW_x(t,a) = \frac{1}{a}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h\left(\frac{\tau}{a}\right) g\left(\frac{s-t}{a}\right) x\left(s+\frac{\tau}{2}\right) x^*\left(s-\frac{\tau}{2}\right) d_s d_\tau \qquad (7)$$

As shown in Formula 7, the time and scale resolution can be determined independently by selecting the g and h window functions. The ASPWVD has the two following advantages.

1) Compared with Cohen class, ASPWVD greatly improves time resolution and frequency resolution, and also inhibits the occurrence of cross-terms to a certain extent.

2) ASPWVD is able to analyze the nonlinear signal more precisely, especially the frequency resolution is greatly enhanced compared with other distributions.

2.3 Design of time-frequency combined method

According to the above theory, we can design a time-frequency combined method. First use ICEEMDAN to decompose the non-stationary signals to achieve a series of IMFs with different frequencies from high to low. Then, the IMFs are correlated with the original signal to obtain the their own correlation coefficients. Use the cross-correlation criterion to select the effective IMFs. Finally, ASPWVD is used to analyze these IMFs and extract the frequency characteristic components of the original non-stationary signal. The implementation flowchart of the time-frequency combined method is shown in Fig.2-1.

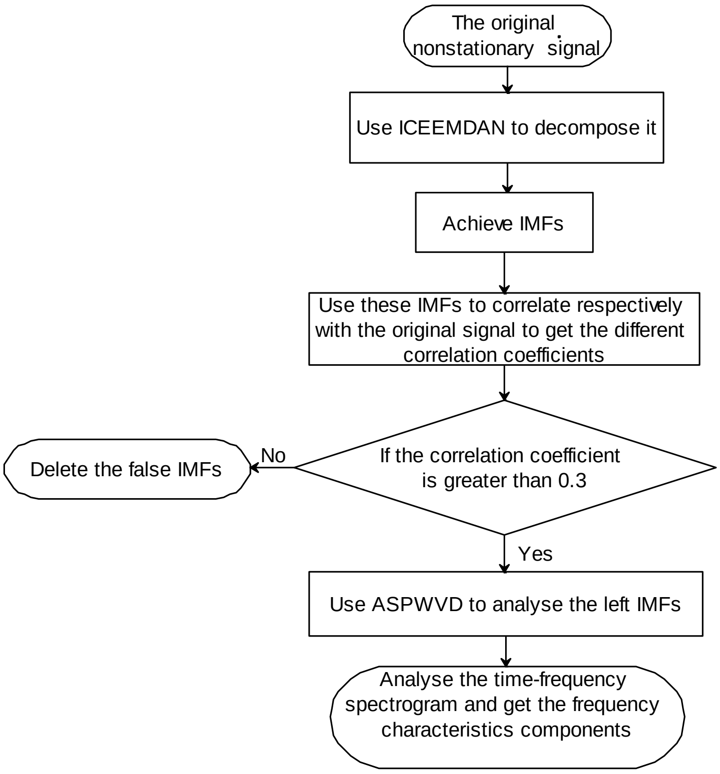


Fig. 2-1 The implementation flowchart of the time frequency combined method

3. Simulation signal analysis

3.1 Simulation signal

Design the expression of simulation signal as follows:

$$x(t) = \cos(100\pi \cdot t) + \cos(220\pi \cdot t) + \sin\big(2\pi \cdot (250t + 2\sin(30t))\big) + n(t) \tag{8}$$

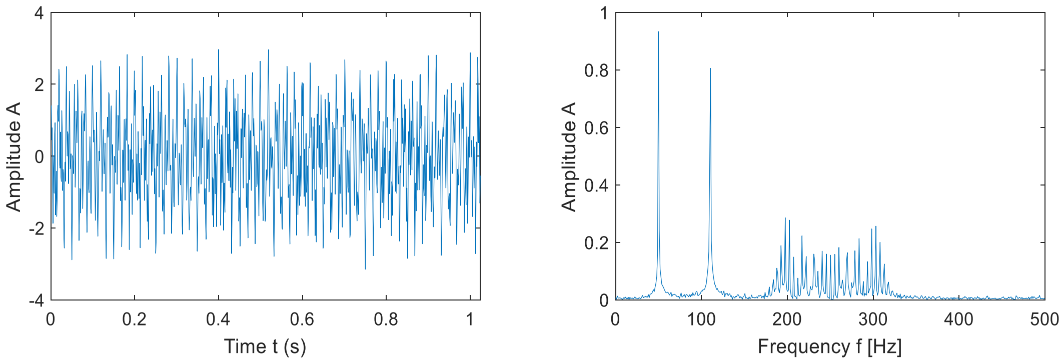
The simulation signal is composed respectively of two cosine waves with base frequency of 50 Hz and 110 Hz, a sine wave with base frequency of 250 Hz which is modulated by the frequencies $2\sin(30t)$ and a white noise signal. The modulated frequency of the simulation signal can be calculated as follows.

$$\omega(t) = \frac{d\theta(t)}{dt} = \frac{d\big(2\pi(250t+2\sin(30t))\big)}{dt} = 500\pi + 120\pi\cos(30t) \tag{9}$$

$$f(t) = \frac{\omega(t)}{2\pi} = 250 + 60\cos(30t) \tag{10}$$

According to the formula (10), the frequency range is 190 ~ 310hz, and the frequency varies with the rule of cosine wave. $n\,(t)$ is a white Gaussian noise with SNR = 20 to increase the signal complexity.

The sampling frequency is 1000 Hz, and the number of sampling points is 1024. The waveform and frequency spectrum of the simulated signal are shown in Fig. 3-1.



(a) Waveform (b) Spectrum

Fig. 3-1 waveform and spectrum of simulation signal

It can be seen from Fig. 3-1 (b) that the spectrum obtained by fast Fourier transform (FFT) analysis can't identify the information of FM components.

3.2 Analysis of the simulation signal using different time-frequency combined methods

3.2.1 ASPWVD

Use ASPWVD alone to analyze the simulation signal. The analytic result is shown in Fig. 3-2. It can be seen that there are 50Hz, 75Hz, 110Hz base frequency components and three sine wave frequency modulation components. Among them,75Hz base frequency component and two frequency modulation components of sinusoidal wave (blue waves in Fig. 3-2) are false components, so serious cross-terms appear. Because of the white noise, mistiness appear at both ends of the wave curves. But the all wave curves is clear to see, showing that the ASPWVD has a good time-frequency resolution.

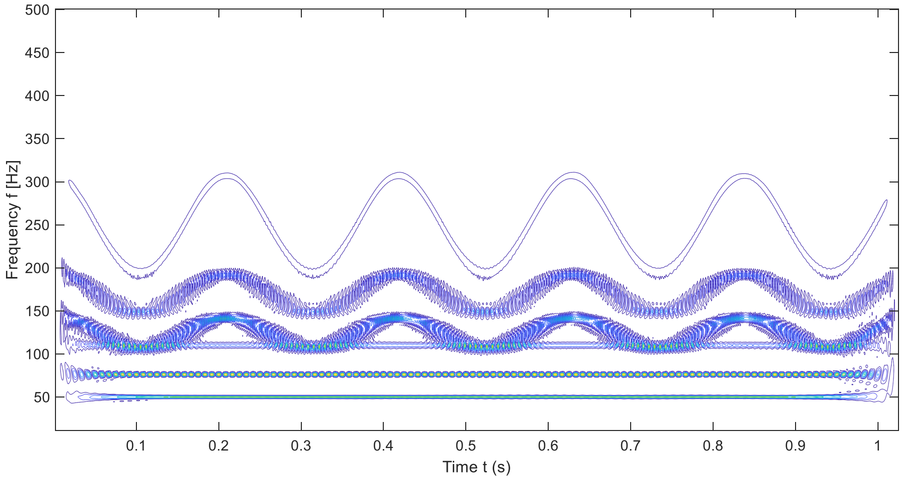


Fig. 3-2 ASPWVD analysis of simulation signal

3.2.2 ICEEMDAN

Use the ICEEMDAN to decompose the simulation signal to obtain nine IMFs seen in Fig. 3-3. It is clear that the top waveform in the figure is the original simulation signal waveform and the bottom IMF9 is the residue.

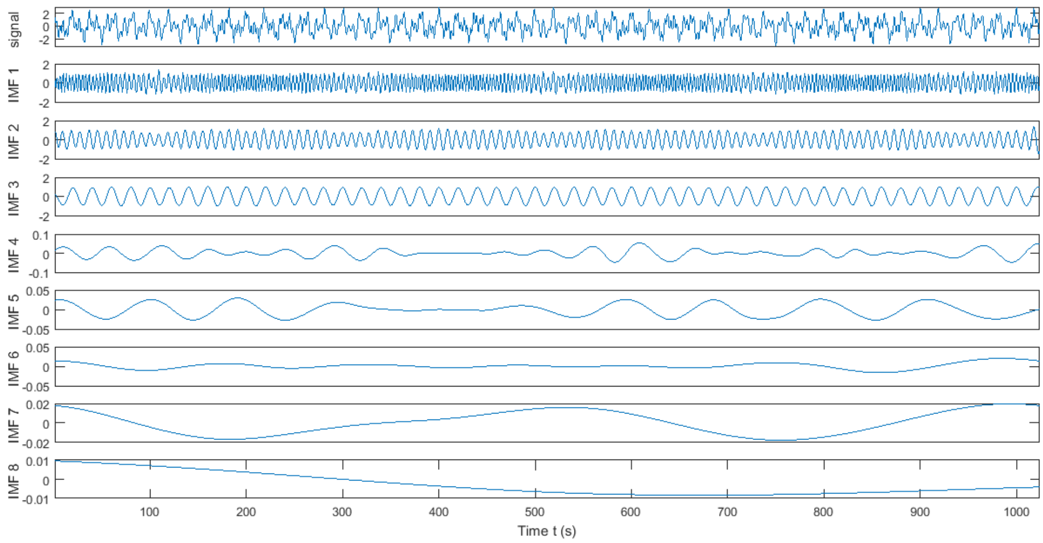


Fig. 3-3 ICEEMDAN decomposition of simulation signal

The IMF1 ~ IMF8 are correlated respectively with the original simulation signal to get their own cross-correlation coefficients as shown in Table 1.

Table 1

The correlation coefficients between each IMF and the simulation signal

Signal component	IMF1	IMF2	IMF3	IMF4	IMF5	IMF6	IMF7	IMF8
correlation coefficient	0.6177	0.6052	0.5705	0.0120	0.0130	0.0198	0.0107	0.0115

According to the correlation coefficient criterion[14], the IMF with a correlation coefficient higher than 0.3 is a true IMF, and the one lower than 0.3 is a false IMF. From the table, it is clear that the IMF1~IMF3 are true IMFs, and the others are false IMFs and will be eliminated.

3.2.3 Comparison of four time-frequency methods for the true IMFs

Four time-frequency methods including WVD, SPWVD, CWD and ASPWVD are used to analyze the IMF1~IMF3 respectively. The analytic results were shown in Fig. 3-4.

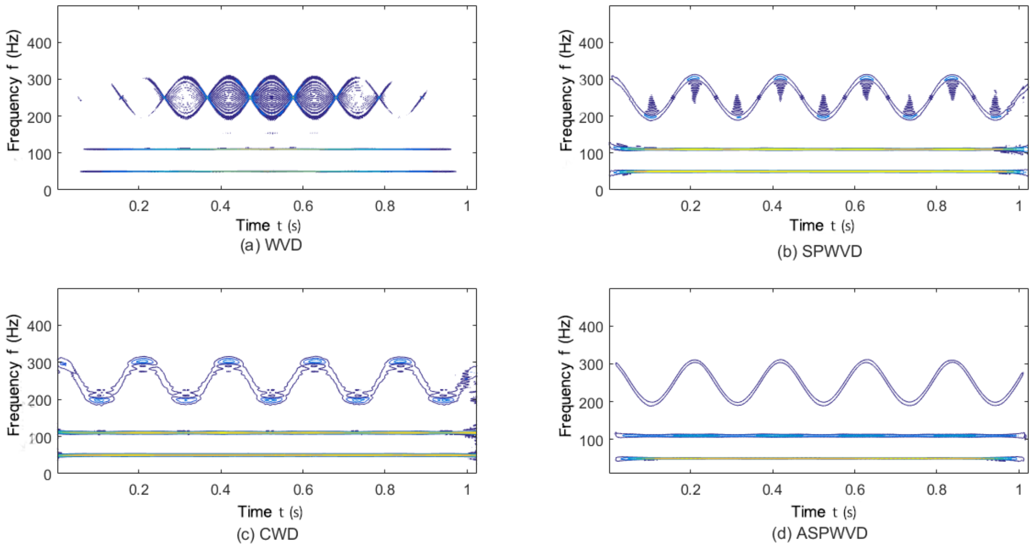


Fig. 3-4 Analytic results of four time-frequency methods for the IMF1-IMF3

In Fig.3-4, it can be seen that after the ICEEMDAN decomposition, each time-frequency method can effectively eliminate the cross-terms between each component and reduce the influence of the white noise.

In Fig.3-4 (a), the WVD has the best time-frequency aggregation [15-16], but has serious self-crossing terms and cannot distinguish the FM components.

In Fig.3-4 (b), the SPWVD has better time-frequency aggregation, but there is a certain self-crossing terms.

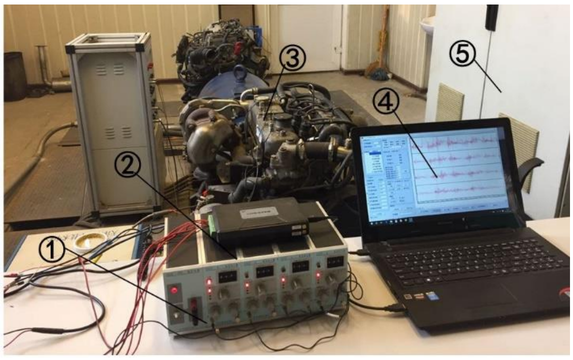
In Fig.3-4 (c), there is almost no self-crossing terms after the CWD, but the time-frequency aggregation is poor.

In Fig.3-4 (d), The ASPWVD has better time-frequency aggregation, and there is no self-crossing term. The overall performance is the best.

In summary, the time-frequency combined method based on ICEEMDAN and ASPWVD can eliminate the cross-terms of multi-component signals and the self-crossing terms of Nonlinear FM signals. At the same time, it has good time-frequency aggregation and is able to effectively extract the frequency characteristic components of non-stationary signals.

4. Application example

In order to further verify the effectiveness and practicability of the designed time-frequency combined method, the vibration signal of a FotonBJ493ZQ3 diesel engine was collected and analyzed. Fig. 4-1 shows the testing case for the engine vibration signal collection and the installation location of the four sensors. Four YD-12 type acceleration sensors are selected to collect the vibration signal of the head engine cylinder. The vibration signals are amplified by a DHF-4 type charge amplifier, and collected by a USB data acquisition card and sent to a notebook computer, and the signals are displayed and saved by an automobile testing and analytic software seen in Fig. 4-1.



(a) Field testing case

① Charge amplifier ② USB data acquisition card ③ piezoelectric acceleration sensor ④ Automobile multi-functional tester ⑤ Dynamometer

(b) Sensor installation location

①~④ sensor installation sequences

Fig. 4-1 Field testing case and sensor installation location

During the test, the engine speed is 2682r / min, the gain of DHF-4 charge amplifier is 10, and the sampling frequency is 10000 Hz. Fig. 4-2 shows the collected four-channel engine vibration signal waveform.

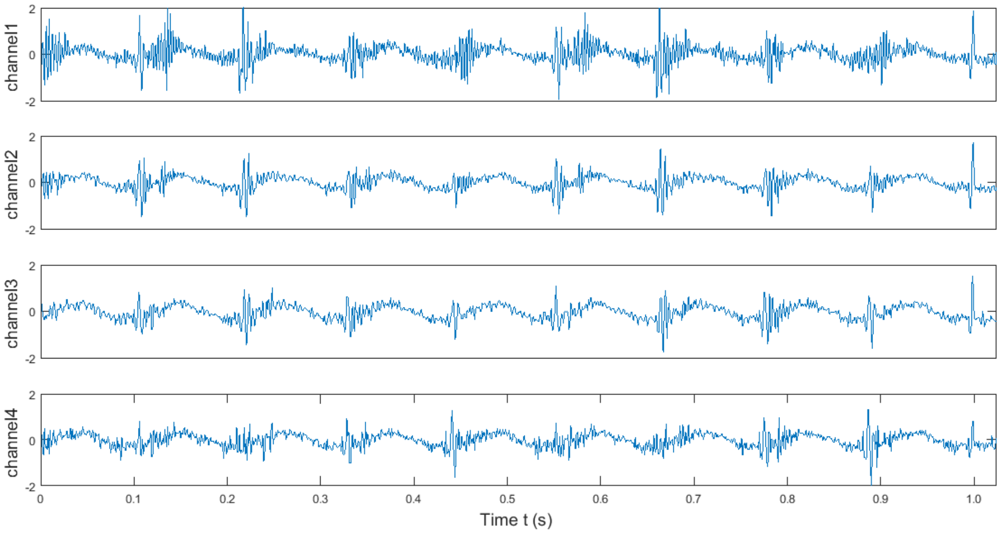


Fig. 4-2 Vibration signal waveform of four channels

It can be seen from Fig. 4-2 that at the same speed, the waveforms collected by sensors at different positions are very similar, have obvious periodic characteristics, and their periods are the same. The vibration amplitude of the first channel is larger than others.

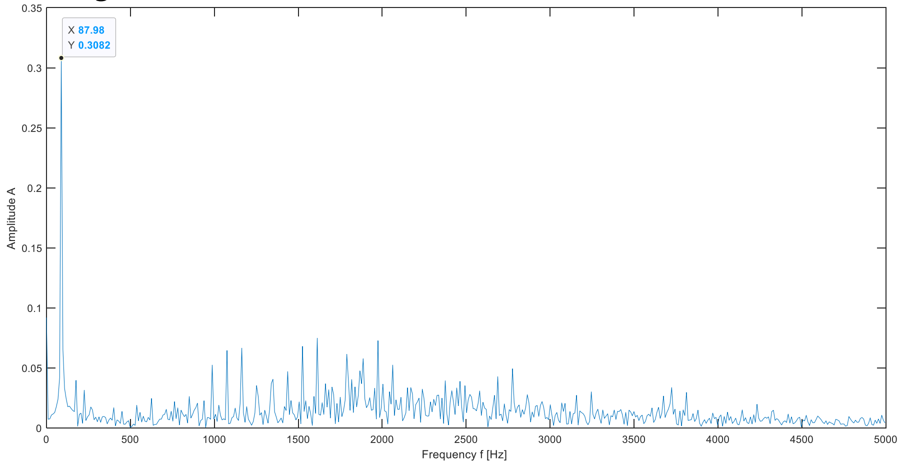


Fig. 4-3 Spectrum of the third channel signal

The third channel signal is selected for analysis. Fig. 4-3 shows the spectrum of the third channel signal. It can be seen from the figure that the signal energy is mainly concentrated at 87.98hz of the peak frequency point, and most of the remaining energy of the signal is concentrated in the frequency range of 1000 Hz to 4000 Hz.

The ICEEMDAN method is performed on the signal of the third channel. The decomposition result is shown in Fig. 4-4, in which IMF8 is the residue. The correlation coefficients between IMF1 ~ IMF7 and the original vibration signal are shown in Table 2. The correlation coefficients of IMF1, IMF2 and IMF5 are all greater than 0.3, which are the true IMFs, the others are false ones.

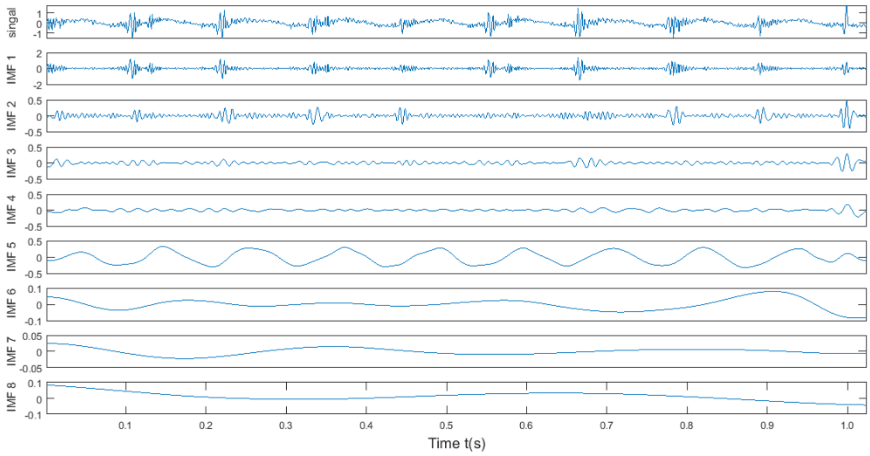


Fig. 4-4 ICEEMDAN decomposition results of the third channel signal

Table 2

The correlation coefficients between each IMF and the third channel signal

Signal component	IMF1	IMF2	IMF3	IMF4	IMF5	IMF6	IMF7
correlation coefficient	0.6793	0.4385	0.1581	0.1729	0.6215	0.1562	0.1006

Observed from these time-domain waveforms, IMF1 and IMF2 have obvious periodic shock characteristics, and IMF5 has a sinusoidal signal characteristics. Use ASPWVD to analyze respectively IMF1, IMF2, and IMF5, and superimpose each time-frequency spectrum. The result is shown in Fig. 4-5.

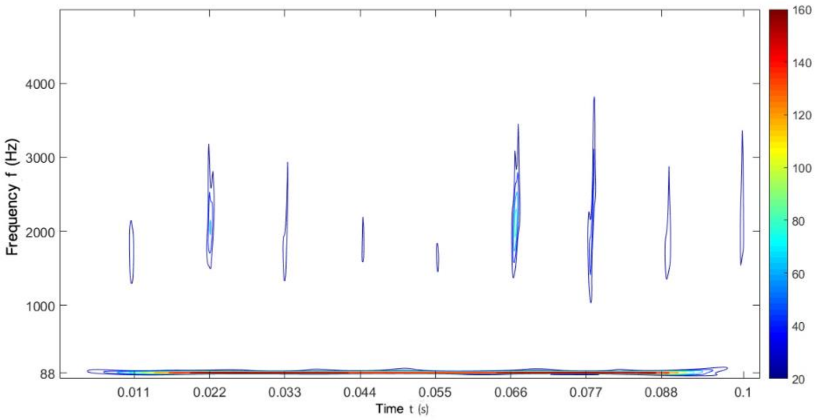


Fig. 4-5 Time-frequency diagram of third channel signal by using ASPWVD based on ICEEMDAN

According to the running mechanism of the engine, each cylinder of the four cylinder of four stroke engine produces a combustion pressure peak every 720° per rotation of the crankshaft. This combustion pressure will produce a vibration shock signal. Therefore, the four cylinders produce a combustion pressure peak every 180°. That is their frequencies are twice that of the crankshaft.

The reciprocating inertia forces of crankshaft connecting rod system of diesel engine can be expressed as follows

$$
\begin{aligned}
P_j &= m_j a \\
&= m_j R\omega^2(\cos\varphi + \lambda\cos\varphi) \\
&= m_j R\omega^2 \cos\varphi + m_j R\omega^2 \lambda \cos\varphi \\
&= P_{Ij} + P_{IIj} \qquad (11)
\end{aligned}
$$

P_{Ij} is the first-order inertial force, and the magnitude is $m_j R\omega^2 \cos\varphi$. P_{IIj} is the second-order inertial force, and the magnitude is $m_j R\omega^2 \lambda \cos\varphi$.

For an in-line four-cylinder diesel engine, the frequency of the second-order inertial force is twice the rotation frequency of the crankshaft. As the masses of the reciprocating parts of each cylinder are equal, it can be seen from the analysis of the ignition sequence that the first-order inertial forces of the four cylinders cancel each other, and the second-order inertial forces superimpose each other, and the amplitude of inertial forces is $4\lambda M_j r\omega^2$. When the force acts on the engine body, it will produce a violent vibration which is one of the main vibration sources of the engine.

The frequency of the second-order inertial force is written as

$$f = \frac{Qn}{60} \tag{12}$$

Q is the proportional coefficient, $Q = 1$ when the first-order inertia force is calculated, $Q = 2$ when the second-order inertia force is calculated, n is the crankshaft rotation speed.

When the crankshaft speed is 2682 r/min, the frequency of the second-order inertial force is 89.40 Hz. In Fig. 4-5, it can be seen that the maximum peak value marked from the spectrum is 87.89 Hz. Compared with 89.4Hz, the error is 1.69%. This shows that the two frequencies are consistent with each other. So it is explained that the one of the main vibration sources is caused by the unbalance modulation of the crankshaft rotation.

This unbalance modulation case can also be seen in Fig. 4-5. There are periodic intervals of 0.011s, which are caused by the impact when the combustion pressure reaches its peak. The impact frequency(90.91Hz) is nearly the same as that of the second-order inertia force. This result is consistent with the above theoretical analysis.

The designed time-frequency combined method based on ICEEMDAN and ASPWVD can be able to effectively extract the frequency characteristic information from the engine vibration signal so as to provide a data basis for the engine vibration source analysis.

5. Conclusion

A new time-frequency combined method based on ICEEMDAN and ASPWVD is realized. The process is that using ICEEMDAN to decompose the non-stationary signal to get a serial of IMFs, the decomposed IMFs are selected by the correlation coefficients between each IMF and the original simulation signal according to the correlation coefficient criterion, then ASPWVD is used to analyze the true IMFs. Use the designed method to analyze a simulation signal. The analytic result is compared with those of other time-frequency methods. It is proved that the time-frequency combined method based on ICEEMDAN and ASPWVD can be able to effectively eliminate the self-crossing terms produced by nonlinear FM signals and the cross-terms between different component signals, and obtain a good time-frequency aggregation. This time-frequency combined method is more effective and superior than other time-frequency methods or other time-frequency combined methods such as EMD-WVD, EEMD-CWD etc. in analysis of the bilinear nonstationary signals. In this paper, this designed method is also used to analyze the vibration signal of an engine. The result shows that it can be able to effectively extracts the frequency characteristic information of the engine vibration signal, and its practicability is verified. This combined method can be applied to analyze the non-stationary signals in other engineering fields, such as bearing signals and transmission signals etc.

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