DYNAMIC PREISACH MODEL USED FOR ESTIMATION OF SILICON IRON ALLOY HYSTERESIS CYCLE

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The paper describes a dynamic Preisach hysteresis model applicable in the case of thin non-oriented electrical steels. The model identification procedure is divided into two steps. The first step uses the Everett function and, as experimental data, the measured concentric minor loops, introduced in a scalar Preisach model. The second step consist of putting in evidence the magnetization process, through the Preisach dynamic model, by using fitted measured data. The last developed model takes into account the eddy currents and the domain wall displacement. The hysteresis cycles, obtained through simulations, in the case of different measuring frequencies, are compared with the experimental ones.

Keywords: dynamic Preisach model, Everett integral, closed form evaluation, magnetization process, non-oriented electrical steel.

1. Introduction

Magnetic materials present a hysteresis dependence, between other physical properties. The mathematical implementation of this type of dependence is more necessary nowadays, because of the increased need of special electrical devices; therefor a rate dependent *B-H* cyclic dependence must be taken into account. The magnetic hysteresis has been numerically implemented through different approximations, such as Prandtl-Ishlinskii, Preisach, Duhem, Bouc-Hodgon, Jiles-Atherton, and Stoner-Wohlfarth models. A few models take into account the whole range of the magnetization processes from demagnetization to saturation and the most used is the Preisach model.

The classical Preisach model [1-6] explains the static hysteresis phenomenon in ferromagnetic materials. It is based on the mathematical assumption that the major hysteresis cycle is composed through a high number of square hysteresis loops (hysterons) and it is independent on mechanical stress, temperature and frequency. The static model is inadequate for the real magnetic materials' description, because the congruency property of minor loops is not applicable in all the cases, so general models have to be developed. Also, the

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accommodation or reptation phenomenon [7, 8], which consists of the affirmation that "it takes many cycles, before a minor loop to close" is not properly put in evidence.

It is well known that the Preisach density is very important and the magnetization can be determined through Everett functions, computed from experimental measurements, based on first order reversal curves or concentric minor loops. Numerically computed magnetization through experimentally identified Everett functions can increase the measurement errors, because it can lead to numerical instabilities. Many papers [5, 9-20] report Preisach functions that can fit experimental data such as: Gaussian, Lorentzian, Super Lorentzian, and Log Normal. In [21] it was put in evidence that a Lorentzian distribution is a very suitable tool to estimate the hysteresis cycle of the electrical steels.

In the case of moving Preisach model [8] the properties of non-congruency and accommodation are well integrated. The input data are in this case a set of effective magnetic field strengths H_e, which are function of the magnetic field strength H and of the magnetization M as it follows:

$$H_e = H + kM. \tag{1}$$

A remarkable property of this model is the so called "linear skew congruency", consisting in a property that the congruent minor hysteresis cycles are connected trough a line with a slope of $1/k$ [8, 13]. The factor k is defined as moving parameter and it is related to the longitudinal magnetostatic interactions that are associated to the Barkhausen effects in ferromagnetic materials [22]. In the case of the moving Preisach model, the rectangular hysteresis loop of the hysterons is replaced by a non-rectangular shaped cycle.

Usually it is considered that an elementary hysteresis cycle is related to a magnetic particle, whereas the Preisach distribution is linked to a statistical distribution function [1-20].

In the paper there is presented a mathematical Preisach distribution based on a closed formalism of the Everett integral [18, 23, 24]. In the second section the formulation of the model with the closed form of the Everett function is discussed. An algorithm [12, 18, 23-25], which is based on the Fixed-Point method linked to the closed-form of the Everett function, is applied in order to compute a rate dependent hysteresis model, which incorporates a frequency dependence of the magnetization phenomenon. The dynamic Preisach model is applied in the case of high quality non-oriented electrical steel NO20, based on experimental measurements done at a peak magnetic polarization J_p = 1.5 T and four measuring frequencies f = 20, 50, 100, and 500 Hz, respectively. The simulated hysteresis loops are compared with the experimental ones.

2. Everett function in closed-form expression used in Preisach distribution computation

As presented in [24, 26, 27] for the scalar Preisach model, in which the ferromagnetic alloy behavior is considered to be a superposition of elementary hysteresis cycles with rectangular shape, characterized by two values of switching up h_1 and switching down h_2 fields, the hysteresis operator at t_k moment could be expressed as it follows:

$$\gamma\left(h_1, h_2, H\left(t_k\right)\right) = \begin{cases} -1 & \text{if } H\left(t_k\right) \le h_1, \\ +1 & \text{if } H\left(t_k\right) \ge h_2, \\ \gamma\left(h_1, h_2, H\left(t_{k-1}\right)\right) & \text{if } h_1 \le H\left(t_k\right) \le h_1. \end{cases} \tag{2}$$

Introducing $\mu(h_1, h_2)$ as Preisach function, the magnetic polarization J, which almost equals the magnetic flux density in the case of soft magnetic materials [24, 26], is given by (3):

$$J\left(t\right) = \iint_T \mu\left(h_1, h_2\right)\gamma\left(h_1, h_2, H\left(t\right)\right) dh_1 dh_2, \tag{3}$$

where T is the Preisach triangle [26, 27] and $H(t)$ is the magnetic field strength considered at a moment t. When a magnetic field is applied the Preisach triangle T is divided into T_1, in which the operators switch up to +1, and T_2 that is the domain of the switching down (at –1) operators. These two domains are separated through the "staircase line", which has to be memorized during computations. Starting from staircase line points, the magnetic polarization can be calculated at each moment t, using the Everett function (5). Considering, that the Preisach density is a product of two values, computed in the case of h_1 and h_2 from a unidimensional function φ_i (4), as follows [9]:

$$\mu\left(h_1, h_2\right) = \sum_{i=1}^{n} \varphi_i\left(h_1\right)\varphi_i\left(-h_2\right), \tag{4}$$

we get

$$E\left(x, y\right) = \int_x^y \int_x^{h_2} \mu\left(h_1, h_2\right) dh_1 dh_2, \tag{5}$$

where x and y are staircase line vertices' coordinates. Figure 1 shows the Preisach triangle with the staircase line. Here one can notice the values of the Everett function E_i and the positive and negative saturation magnetic field strengths $\pm H_S$.

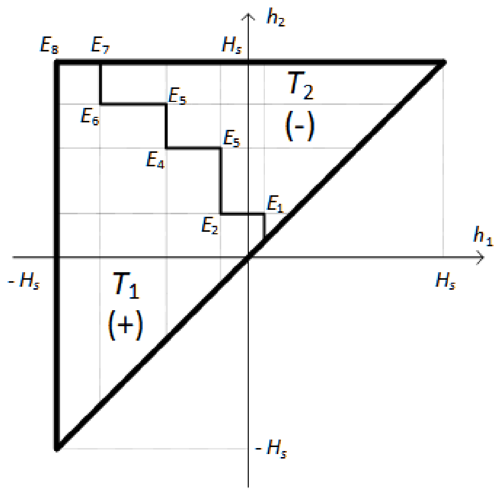


Fig. 1. The Preisach triangle and the staircase line [24, 28].

In literature are considered different expressions for the unidimensional Preisach distributions such as Lorentz, Gauss, and Normal Logarithm. In this paper expression (6) was chosen [24, 28], allowing to evaluate a closed-form of the Everett function. Its computation only contains basic mathematical operators such as: subtraction, addition, division, multiplication, and computation of exponential and logarithmic functions, constituting thus a closed-form, fact that generates a faster determination of the magnetic polarizations [29].

$$\varphi_i(x) = \frac{a_i e^{-\frac{x-b_i}{c_i}}}{\left(1+e^{-\frac{x-b_i}{c_i}}\right)^2}, \tag{6}$$

where a_i, b_i and c_i are fitting parameters, which correspond to the magnitude, mean value and square value of the variance, respectively.

The integral presented in (5) may be evaluated, resulting in a closed-form expression [28, 29]:

$$E(x,y)=\sum_{i=1}^{n}\left(a_i c_i e^{\frac{b_i}{c_i}}\right)^2 \frac{\left(e^{\frac{x}{c_i}}-e^{\frac{y}{c_i}}\right)+\dfrac{\left(e^{\frac{b_i}{c_i}}+e^{\frac{y}{c_i}}\right)\left(1+e^{\frac{b_i}{c_i}}e^{\frac{x}{c_i}}\right)}{1-\left(e^{\frac{b_i}{c_i}}\right)^2}\log\dfrac{\left(1+e^{\frac{b_i}{c_i}}e^{\frac{x}{c_i}}\right)\left(e^{\frac{b_i}{c_i}}+e^{\frac{x}{c_i}}\right)}{\left(1+e^{\frac{b_i}{c_i}}e^{\frac{x}{c_i}}\right)\left(e^{\frac{b_i}{c_i}}+e^{\frac{y}{c_i}}\right)}}{\left(1-\left(e^{\frac{b_i}{c_i}}\right)^2\right)^2\left(e^{\frac{b_i}{c_i}}+e^{\frac{y}{c_i}}\right)\left(1+e^{\frac{b_i}{c_i}}e^{\frac{x}{c_i}}\right)}, \tag{7}$$

In order to compute the magnetic polarization, the staircase line has to be memorized and implemented as a “last in first out” structure. The Everett function could be calculated as presented in Fig. 1, using as input data the concentric minor hysteresis cycles, introduced in the static Preisach model [24, 26-29]. In the analytical case, the relationship (7) is used directly, without being necessary to store the values of the Everett function, making the magnetic polarization computation procedure to become very fast.

3. Rate dependent dynamic Preisach model that includes the influence of the measuring frequency on the magnetization process

The inclusion of the measuring frequency in the Preisach model is a very important step, because it takes into account the eddy current phenomenon, which consists of the “widening” of the hysteresis cycle, when the frequency increases. As presented in [24, 28-31] a supplementary variable H_m will be used as input data in the classical formulation of the Preisach model, based on Everett integrals. This variable will be delayed with the value of the considered magnetic field strength H. In the rate dependent model three parameters a_m, b_m, c_m will be used (8), as follows

$$\frac{dH_m}{dt}=a_m\left(H-H_m\right)-b_m\frac{dB}{dt}+c_m\frac{dH}{dt}, \tag{8}$$

where H is input magnetic field strength vector magnitude, dB/dt is the first derivative of the magnetic flux density.

The flux density is given by the constitutive relation for the magnetic field:

$$\mathbf{B}=\mu_0\mathbf{H}+\mathbf{J}, \tag{9}$$

where μ_0 is the vacuum magnetic permeability and the magnetic polarization J is a function of H_m. The dependence $J(H_m)$ is given as a result of the Preisach scalar model.

The relationship (6) permits the evaluation in closed-form of the Everett function [29]:

$$\frac{dJ}{dH_m} = 2\mu_0 \int_{H_m^r}^{H_m} \mu\left(h_1, H_m\right) dh_1, \tag{10}$$

where H_m^r is the extreme value of the magnetic field strengths vector magnitude. The integral expression (10) could be evaluated, by using the three fitting parameters a_i, b_i and c_i, presented in Chapter 2 [29]:

$$\frac{dJ}{dH_m} = \sum_{i=1}^{n} 2\mu_0 \left(a_i e^{\frac{b_i}{c_i}} \right)^2 \frac{c_i}{e^{\frac{b_i}{c_i}}} \frac{e^{-\frac{H_m}{c_i}}}{\left(1+e^{-\frac{H_m}{c_i}}\right)^2} \left(\frac{1}{1+e^{\frac{b_i+H_m^r}{c_i}}} - \frac{1}{1+e^{\frac{b_i+H_m}{c_i}}} \right). \tag{11}$$

Using

$$J_{rev}\left(t\right) = k_1 H\left(t\right) + k_2 \tanh\left(\frac{H\left(t\right)}{k_3} \right), \tag{12}$$

which simulates a reversible component J_{rev}, the total $\frac{dJ}{dH_m}$, due to reversible and irreversible magnetization processes, may be obtained by summing up relationship (11) and $\frac{dJ_{rev}}{dH_m}$ [29], where the parameters k_1, k_2 and k_3 are identified as in [32].

4. Results and Discussions

In order to apply the dynamic Preisach model to estimate the hysteresis cycle for non-oriented electrical thin steel NO20, a complicated identification procedure was applied. Firstly, using the concentric hysteresis loops measured at 10 Hz, introduced as input data in the classical scalar Preisach, the parameters of the relationships (6) and (12) are computed. In the case of the scalar Preisach model [5-10, 23-30] the magnetic polarization can be considered as follows [29]:

$$J=-J_l+2\sum_{i=1}^{n}\left(a_i e^{\frac{b_i}{c_i}}c_i\right)^2 \frac{\left(e^{\frac{H}{c_i}}-e^{-\frac{H_l}{c_i}}\right)-\frac{\left(e^{\frac{b_i}{c_i}}+e^{\frac{H}{c_i}}\right)\left(1+e^{\frac{-b_iH_l}{c_i^2}}\right)}{\left(e^{\frac{b_i}{c_i}}\right)^2-1}\log\frac{\left(1+e^{\frac{b_iH}{c_i^2}}\right)\left(e^{\frac{b_i}{c_i}}+e^{\frac{-H_l}{c_i}}\right)}{\left(1+e^{\frac{-b_iH_l}{c_i^2}}\right)\left(e^{\frac{b_i}{c_i}}+e^{\frac{H}{c_i}}\right)}}{\left(\left(e^{\frac{b_i}{c_i}}\right)^2-1\right)\left(e^{\frac{b_i}{c_i}}+e^{\frac{H}{c_i}}\right)\left(1+e^{\frac{-b_iH_l}{c_i^2}}\right)}, \tag{13}$$

$$J=J_l-2\sum_{i=1}^{n}\left(a_i e^{\frac{b_i}{c_i}}c_i\right)^2 \frac{\left(e^{\frac{H_l}{c_i}}-e^{\frac{H}{c_i}}\right)-\frac{\left(e^{\frac{b_i}{c_i}}+e^{\frac{H_l}{c_i}}\right)\left(1+e^{\frac{b_iH}{c_i^2}}\right)}{\left(e^{\frac{b_i}{c_i}}\right)^2-1}\log\frac{\left(1+e^{\frac{b_iH}{c_i^2}}\right)\left(e^{\frac{b_i}{c_i}}+e^{\frac{H_l}{c_i}}\right)}{\left(1+e^{\frac{b_iH_l}{c_i^2}}\right)\left(e^{\frac{b_i}{c_i}}+e^{\frac{H}{c_i}}\right)}}{\left(\left(e^{\frac{b_i}{c_i}}\right)^2-1\right)\left(e^{\frac{b_i}{c_i}}+e^{\frac{H_l}{c_i}}\right)\left(1+e^{\frac{b_iH}{c_i^2}}\right)}, \tag{14}$$

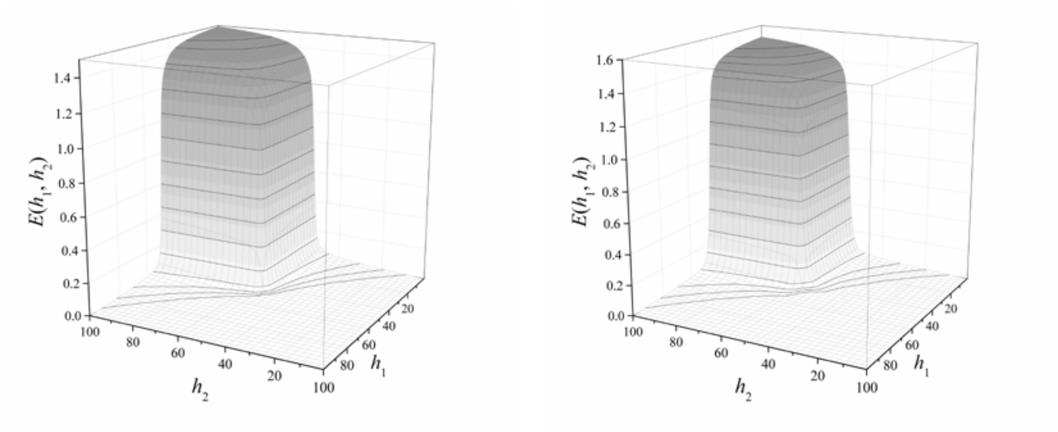
where the expression (13) is for the descending branch and the relationship (14) is used for the ascending branch of the hysteresis cycle [12, 18, 23-30]. The quantities H_l and J_l represent the coordinates of the closing point of the concentric minor loops and they can be determined with (15) [28, 30-33].

The second step of the identification procedure compute the parameters of the equation (8) and, finally, the hysteresis cycles for 20, 50, 100 and 500 Hz are estimated.

$$J_l = \sum_{i=1}^{n}\left(a_i e^{\frac{b_i}{c_i}} c_i\right)^2 \frac{\left(1-e^{-2\frac{H_l}{c_i}}\right)\left(\left(e^{\frac{b_i}{c_i}}\right)^2-1\right)-\left(1+e^{\frac{-b_i H_l}{c_i^2}}\right)\log\dfrac{\left(1+e^{\frac{b_i H_l}{c_i^2}}\right)\left(e^{\frac{b_i}{c_i}}+e^{\frac{-H_l}{c_i}}\right)}{\left(1+e^{\frac{-b_i H_l}{c_i^2}}\right)\left(e^{\frac{b_i}{c_i}}+e^{\frac{H_l}{c_i}}\right)}}{\left(\left(e^{\frac{b_i}{c_i}}\right)^2-1\right)^2\left(1+e^{\frac{-b_i H_l}{c_i^2}}\right)^2}, \quad (15)$$

The experimental measurements were done on a laboratory digital wattmeter, which is based on a standardized measuring procedure in total accordance with IEC 60404-3 [34]. It was used a double C laminated yoke with a magnetic path length of 240 mm; a 288 turns coil generates the magnetic field and the magnetic flux density is measured with a 250 turns coil. For the acquisition of the signal it was used a 12 bit encoding 500 MHz HDO4054 LeCroy oscilloscope. The measurements were done at a fixed value of the peak magnetic polarization J_p of 1.5 T and four different frequencies f = 20, 50, 100, 500 Hz.

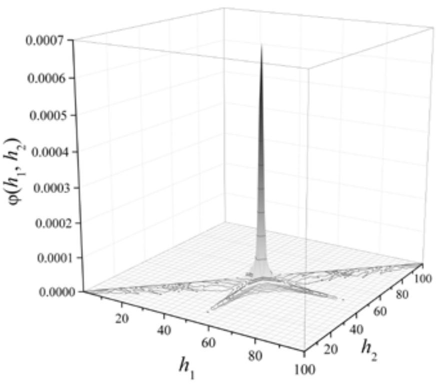
Figure 2 shows the numerically computed Everett function variations $E(h_1, h_2)$, obtained in the case of two values of the frequency. It can be observed that the Everett function values increase with frequency and in the case of 500 Hz has a maximum value of 1.6.



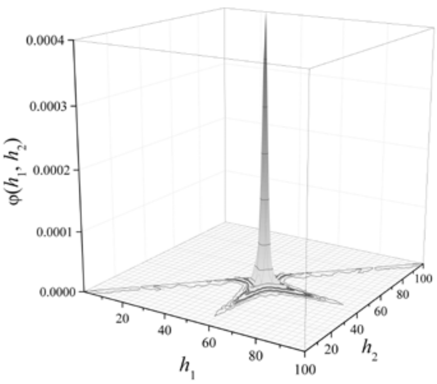
a. f = 20 Hz. b. f = 500 Hz.

Fig. 2. Everett integral surfaces in the case of NO20 high quality non-oriented steel for two values of the frequency.

The Preisach distribution for two values of the measuring frequencies are presented in Fig. 3. This distribution is usually result from a proper simulation of the magnetization processes and it is a function of two unknown values. The maximum value of the Preisach density is obtained at the middle point of the line $h_1 = h_2$. This mono peak structure indicates the presence of a predominant magnetization process, which is the domain wall displacement in the case of non-oriented steel, probably related to some metallurgical transition phases of the alloy. The main contribution of the Preisach density is concentrated along the $h_1 = h_2$ line and it is associated with the reversible domain wall motion [35].



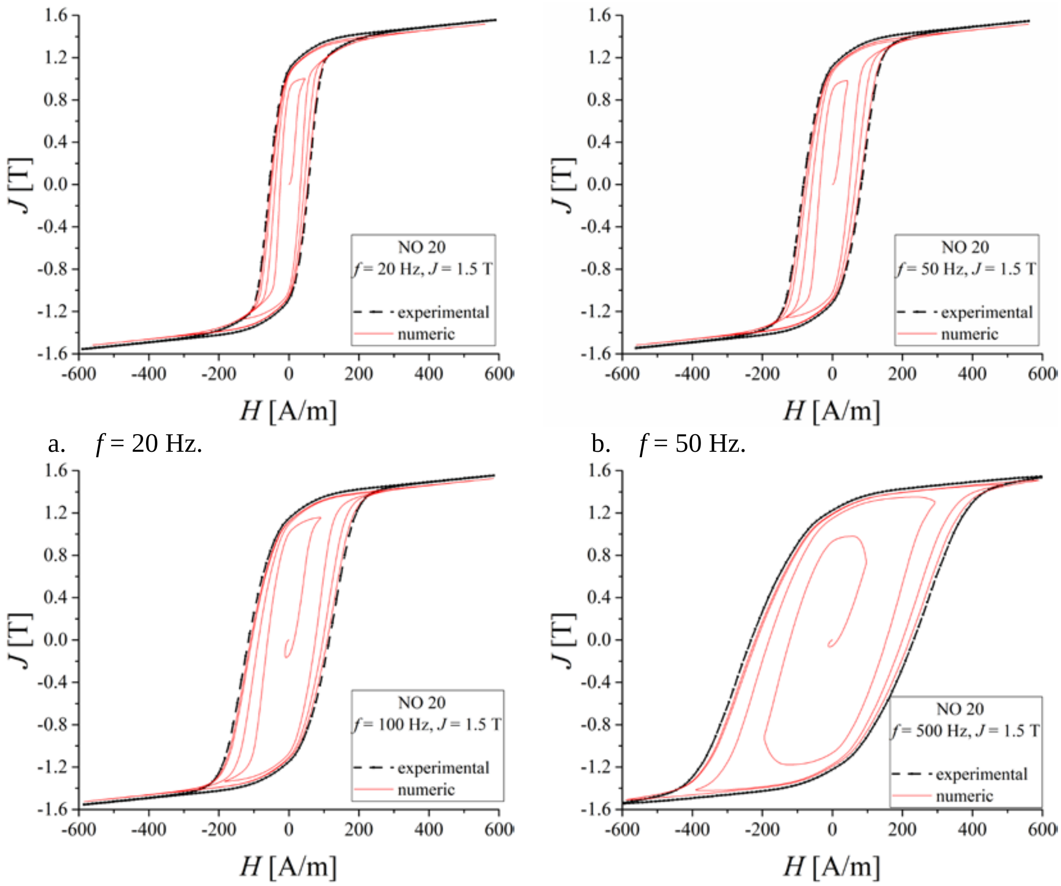
a. f = 20 Hz.



b. f = 500 Hz.

Fig. 3. Preisach density in the case of NO20 high quality non-oriented steel for two values of the frequency.

The minor differences between computed and experimental data observed in the saturation region (Fig. 4) are due to some small errors, made in the hysteresis cycle reconstruction at very high magnetic polarizations. Therefore, a low variation of J produces a high variation of H and the Preisach plane discretization could become important. However, the mathematical Preisach model, presented in this paper, is a very adequate tool to evaluate the hysteresis cycle of non-oriented electrical steel and it provides accurate results in the case of sinusoidal excitations. The magnetization process of the non-oriented steel is also shown in Fig. 4 and a clockwise hysteresis phenomenon is put in evidence with the dynamic Preisach model.



a. *f* = 20 Hz.

b. *f* = 50 Hz.

c. *f* = 100 Hz.

d. *f* = 500 Hz.

Fig. 4. Experimentally determined and computed hysteresis cycles using the dynamic Preisach model.

5. Conclusions

In this paper an important dynamic behavior reported in [29] was considered to the classical Preisach model, by including a supplementary differential equation. Consequently, a closed-form Everett function was used. This approximation leads to a fast computation because it only consists of elementary mathematical operations. The identification procedure is a very complex one, and it was done in two steps as presented in Chapter 4. The rate dependent hysteresis model takes into account the eddy current effect, by using three parameters. The hysteresis cycle is estimated with a high precision in all the analyzed cases, as compared to measured experimental data.

The study presented in the paper was performed on NO20 non-oriented steel, using a digital laboratory wattmeter. Examining this type of alloy, we put in evidence a sharp spike variation for the Preisach density. This fact demonstrates a direct link with a more important magnetization process, namely domain wall displacement. Also, this approach allowed us to conclude that the minor differences, existing in the case of the saturation zone, are due to minor errors made in the process of reconstructing the hysteresis cycle.

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