U.P.B. Sci. Bull., Series D, Vol. 80, Iss. 2, 2018 ISSN 1454-2358

VIBRATION OF ROTOR BLADES WITH LARGE DEFORMATIONS IN A ROTATING NONINERTIAL REFERENCE FRAME

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This paper is filling an important gap in the scientific literature dealing with the vibrations of rotor blades which sustain large deformations during normal operation. Although the occurring phenomena are known, there is no succinct and complete approach in recent literature in order to clarify this aspect. Therefore, it will be shown that the traditional approach to natural vibration calculation is not sufficient in order to obtain accurate results.

Keywords: rotor, noninertial, large deformations.

1. Introduction

The problems regarding rotating machinery are related to flexural vibrations and precession motion, in which the apparent stiffening of the parts under centrifugal force [1], the apparent softening due to radial displacement in centrifugal force field [2] and the influence of large displacements with changing direction of the variable inertial load [3] make difficult any theoretical model and the results are not easy to obtain. The resulting vibration modes should account for the variable inertial loads applied during rotation. In order to find the vibration eigenshapes and frequencies in top-level industries such as aeronautics, the simple natural vibration calculation, using the manufactured geometry of the parts is unsatisfactory. This happens because the components of turbomachinery and in general all the parts which are heavily loaded in aerospace industry, are experiencing large modifications of the geometry during aircraft operation [4], when the vibration eigenshapes are considerably different from the manufacturing geometry, defined in an inertial reference frame. The originality of this paper resides in gathering all the aspects involved in the mechanical analysis of the vibration shapes and natural frequencies of the highly loaded and highly deformed parts subjected to rotation and analyzed in a non-inertial reference frame. This is especially useful for the unexperienced specialist who is confronted with the need to design and compute aeronautical moving parts as mentioned above.

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Nomenclature

$\{g\}$ - vector of displacements

$\{r\},\{\dot{r}\}$ - vectors of position and speed

Ω, $\{\Omega\}$ - rotational speed scalar, rotational speed vector

$[M]$ - mass matrix

$[D]$ - damping matrix

$[C]$ - the gyroscopic matrix

$[K]$ - the stiffness matrix

$[Z]$ - centrifugal matrix

$[K_G]$ - geometric or differential stiffness matrix

$[K_D]$ - structural damping stiffness matrix

$[K_T]$ - torque derived stiffness matrix

$\{F\}$ - dynamic forces

$[C]_L$ - equivalent damping matrix in the large deformation hypothesis

$[K]_L$ - equivalent stiffness matrix in the large deformation hypothesis

$\{R\}$ - reaction forces of the mechanical structure

$[T]$ - transformation matrix from local coordinate system to the global

$\{q_e\}$ - displacements in the local coordinate system of the finite element

$\{F_e\}$ - internal finite element force in element coordinate system

$[K]_{lin}$ - linear stiffness matrix

$[K]_{diff}$ - differential stiffness matrix

m - concentrated mass of a finite volume

m_r - mass of the rotor

e - eccentricity of the rotor center of mass at a certain time related to the axis of rotation

2. Theoretical aspects

Considering the force field of the rotating machinery in a non-inertial reference system, there are strict requirements in computing stresses, displacements and vibrations of aircraft engines fan blades, rocket engines turbopump turbine blades. Due to the very small safety factors [5], it is required an excellent knowledge of the problem and very accurate results [6]. Consequently, the principles defined in this paper are applied to the fields of airplanes design, space shuttles and rocket engines, concerning some of the critical parts involved in the safe operation limits of this machines.

The stress distribution and the acceleration field along the investigated rotating part lead to substantial change in vibration natural frequencies. A particular case is the combination between the two effects mentioned above, when the load application is modified interactively as a function of part deformation. An important example of these phenomena is materialized in the modification of the local direction of the inertial forces of centrifugal type during the large structure deformation [3]. Therefore, it is mandatory in a first stage, to calculate exactly the influence of these effects and add them in the final vibration calculation [7].

In order to understand the phenomena underlying the operation of a rotational machine one can start from the Newton second principle which states that in an inertial reference frame the sum of the forces acting on any physical body is equal with the mass of the body multiplied by its acceleration.

$$\{F\} = m \cdot \{a\} \tag{1}$$

Usually the rotors are designed and analyzed in a rotating reference frame linked with the rotor itself. By this approach all parts of the rotors can be calculated using the well known classical theory. Therefore the Newton second law is adapted in order to account for the acceleration observed in the rotational reference frame. For this the time derivation operator is expressed in the rotational frame regarding the position in space {r} of a material point of mass m [8].

$$\left(\frac{d\{r\}}{dt}\right)_{inertial} = \left[\left(\frac{d\{r\}}{dt}\right) + \{\Omega\} \times \{r\}\right]_{rotating} \tag{2}$$

$$\begin{aligned}\left(\frac{d^2\{r\}}{dt^2}\right)_{inertial} = &\left[\left(\frac{d^2\{r\}}{dt^2}\right) + 2 \cdot \{\Omega\} \times \left(\frac{d\{r\}}{dt}\right) + \dots\right. \\ &\left. + \{\Omega\} \times \left(\{\Omega\} \times \{r\}\right) + \frac{d\{\Omega\}}{dt} \times \{r\}\right]_{rotating}\end{aligned} \tag{3}$$

If the equation above is multiplied by the mass m of the studied physical body the expression of Newton second law in the rotating frame can be obtained.

$$m\left(\frac{d^2\{r\}}{dt^2}\right)_{inertial}=\left[m\left(\frac{d^2\{r\}}{dt^2}\right)+m\cdot 2\cdot\{\Omega\}\times\left(\frac{d\{r\}}{dt}\right)+\ldots\right.$$
$$\left.+m\cdot\{\Omega\}\times\left(\{\Omega\}\times\{r\}\right)+m\cdot\frac{d\{\Omega\}}{dt}\times\{r\}\right]_{rotating} \tag{4}$$

The above equation serves for the adaptation of the Newton second law in the non-inertial rotating reference frame. The trajectory of the material point of mass m in the rotating non-inertial reference frame can be determined using three fictitious forces: the centrifugal force, the Coriolis force and the Euler force. Concluding from the perspective of the perturbation forces one can observe that a perturbation force expressed in the inertial frame can be substituted by a perturbation force in the rotating non-inertial reference frame plus three additional fictitious forces depending on the rotation of the non-inertial frame. The three fictitious forces, Coriolis, centrifugal and Euler are the following:

$$-m\cdot 2\cdot\{\Omega\}\times\left(\frac{d\{r\}}{dt}\right)_{rot};\ -m\cdot\{\Omega\}\times\left(\{\Omega\}\times\{r\}\right);\ -m\cdot\frac{d\{\Omega\}}{dt}\times\{r\}. \tag{5}$$

In order to solve complex problems in the rotating machines the finite element technique is employed. Additionally, more advanced principles like Hamilton principle are used in order to account also for nonconservative forces like the damping forces. For the finite element discretized rotor, the small masses denoted above as m are organized in matrices like the mass matrix [M] or the centrifugal matrix [C] [9]. Therefore, the above fictitious forces take the more complex matrix shape of the finite element theory

$$2\Omega[C]\{\dot{r}\};\quad m\cdot\Omega^2\{r\};\quad \dot{\Omega}[C]\{r\}. \tag{6}$$

Therefore in case that the working regime of the analyzed rotor implies rapid rotational accelerations and decelerations one more term should be considered in the stiffness matrix assembly, the circulatory matrix $\dot{\Omega}[C]$ and one more term $m_r\cdot\dot{\Omega}\cdot e$ ads to the centrifugal inertial force as an additional inertial force, both terms counting for the Euler force.[9] The circulatory matrix accounts for the Euler force as in the axysymmetrical rotor or more generally formulated in the perfect balanced rotors and the second term depending of $m_r\cdot e$ is related to the eccentricity of that rotor at a time t in relation with the equilibrium axis. Considering the dynamic equilibrium equation, with the addition of the rotational acceleration terms, this becomes

$$[M]\{\ddot{g}\}+\left([D]+2\Omega[C]\right)\{\dot{g}\}+$$
$$+\left([K]-\Omega^2[Z]+\Omega^2[K_G]+\Omega[K_D]-[K_T]+\dot{\Omega}[C]\right)\{g\}-\{F\}=\{0\} \tag{7}$$

where the $m_r \cdot \dot{\Omega} \cdot e$ is encapsulated in the force term {F} together with the imbalance centrifugal force $m_r \cdot \Omega^2 \cdot e$ and the other forcing functions existent in the mechanical system. This equation have to be solved by direct integration with Runge-Kutta algorithm or something similar using real physical coordinates [10], [9]. The equation (7) is expressed in a noninertial reference system linked to the rotor like is depicted in the figure 1 with red color.

Theoretically the equation (7) is known to be the complete form of the dynamic equilibrium equation for the rotors [11] which condensed takes the form

$$[M]\{\ddot{g}\}+[C]_L\{\dot{g}\}+[K]_L\{g\}-\{F\}=\{0\}. \tag{8}$$

The complete list of stiffness matrices comprises the classical stiffness matrix K, the geometric or differential stiffness matrix K_G, the structural damping stiffness matrix K_D, the centrifugal stiffness (softening) matrix Z, the torque derived stiffness matrix K_T and the Euler force related circulatory matrix $\dot{\Omega}[C]$. All this stiffness matrices are included in the dynamic equilibrium equation (7) under the action of dynamic forces {F}, in order to solve for the dynamic equilibrium position of the rotating finite element.

In case that the rotational speed is constant or varies slowly a nonlinear implicit algorithm is used first to find the equilibrium position of the rotor and then to compute the vibrations using a linear algorithm. Regarding the equilibrium equation and considering only the effect of large displacements, large rotation, one gets

$$[K]_L\{g\}-\{F\}=\{0\} \ . \tag{9}$$

With L are denoted the condensed terms for large displacements. In order to establish the vibration shapes, the equilibrium position will be calculated before the vibration analysis. Therefore, is considered only the last term of this equation, which multiplies the displacement vector {g} in eq. (9). Generally, the large displacements problems can be solved using iterative algorithms which account for the modification of the geometry and for a complex geometry, the finite element discretization is used. Thus, the stiffness matrix is expressed in terms of each finite element. For the global matrix, a transformation matrix from the element local coordinate system to the global coordinate system is employed. For the small displacements problems, this matrix has a constant expression for every element. In the case of large displacements, the changing position of the finite elements require changing the element coordinate transformation matrix at every time step, in order to account for the new position of the finite element in the whole part assembly [12].

Using the notation {R} for the reaction forces of the mechanical structure, then the stiffness matrix can be expressed as

$$[K]_L = \frac{d\{R\}}{d\{g\}} \ , \tag{10}$$

or using the transformation T from global to local element coordinate system,

$$[K]_L = \frac{\partial\{[T]\cdot\{F_e\}\}}{\partial\{g\}} \ , \tag{11}$$

equation which can be developed as

$$[K]_L = [T]\cdot\frac{\partial\{F_e\}}{\partial\{g\}} + \frac{\partial[T]}{\partial\{g\}}\cdot\{F_e\} \ . \tag{12}$$

As a consequence, from relation (11) and (12) one can observe a new term appearing from the variation of the coordinate transformation matrix [T], which contributes to the stiffness of the structure. Practically, this is accounting for the geometry modification and usually contributes with additional stiffness: the geometry is adapting in order to sustain the load.

A typical example of this fact is a fishing rod [13],[3]. The tip of a fishing rod is very weak in bending but has a good capability in tension. When a fish is apprehended, the tip of the fishing rod changes the geometry by rotation of the tip sections in such a way as to work in tension, not in bending, thus supporting a much heavier load.

Developing this simple analogy in the field of high speed rotation machinery, the "fish" becomes the large inertial loads acting on the components optimized for decreased weight, with the lowest allowable safety factors. By the same mechanism, slender and thin parts which could be deformed with bare hands, can sustain huge inertial loads appearing during the operation at high speed rotation in non-inertial reference frames. But this remarkable design capability requires a carefully planned design effort, in order to use the big deformation accompanying different working regimes in the advantage of the application.

For example, the very flexible fan blades of the modern aircrafts are designed to deform in such manner, that the blade stagger angle is adapting to the working regime in order to maximize the efficiency [13], [10] at every usual rotating speed, thus obtaining an important fuel economy.

Considering the coordinates in the local coordinate system of the finite element $\{q_e\}$, equation (12) becomes

$$[K]_L = [T]\cdot\frac{\partial\{F_e\}}{\partial\{q_e\}}\cdot\frac{\partial\{q_e\}}{\partial\{g\}} + \frac{\partial[T]}{\partial\{g\}}\cdot\{F_e\} \ . \tag{13}$$

Using the notations $$\left[K\right]_{lin} = \left[T\right] \cdot \frac{\partial\left\{F_e\right\}}{\partial\left\{q_e\right\}} \cdot \frac{\partial\left\{q_e\right\}}{\partial\left\{g\right\}} \tag{14}$$

and $$\left[K\right]_{diff} = \frac{\partial\left[T\right]}{\partial\left\{g\right\}} \cdot \left\{F_e\right\}. \tag{15}$$

Finally, the equation (13) can be written

$$\left[K\right]_L = \left[K\right]_{lin} + \left[K\right]_{diff} \ . \tag{16}$$

An apparently curious fact is how it is possible for structural damping to generate a displacement stiffness matrix, when it is well known that damping is acting, when a part is moving with a certain speed like in a fluid. The answer to this apparent issue is that the equilibrium equation (7) is expressed in a non-inertial reference frame and all damping elements that are connected to a rotating part but are not rotating, appear to exert forces on the rotating part [11]. The differential stiffness [K_G] is arising from the fact that once the dynamic equilibrium position is achieved, every perturbation to the system has to act not only against the common stiffness of the mechanical system, but also against the load forces, such as the centrifugal force. Usually by a careful design this force acts to reestablish the intended equilibrium of the system. This in turn results in a change of the natural frequencies of the loaded system. The differential stiffness matrix is also named in the literature the "geometrical stiffness matrix".

Another curious phenomenon is related to the amount of torque carried by a slender rotating mechanical system, like a shaft. It was demonstrated that for an exceeding amount of torque, torsion will lead to lateral buckling [14]. In general, the torque contributes to the softening of the rotating machine by the matrix [K_T]. Thus, a perturbation force in the lateral direction regarding the rotating direction will produce an increased displacement proportional with the torque transmitted by the rotating machinery.

The centrifugal softening matrix [Z], accounts for the contribution of the centrifugal force as a force perturbation which acts in radial direction. Practically, when an elementary mass of the system is subjected to a force in radial direction, a small displacement occurs at this point. The small radial displacement makes the inertial load, known as centrifugal force, to increase with the increasing radial distance to the axis of rotation. Therefore, any radial force is accompanied with an increase of the centrifugal force, in that particular point. This effect produces an additional displacement, appearing as if the mechanical system becomes "softer" than its real stiffness. This centrifugal softening matrix is a particular type of the more general "follower force" [3] stiffness matrix which accounts for the modification of the load with the geometry deformation. Finally, the circulation matrix $\dot{\Omega}\left[C\right]$ related to the Euler force acts similar with the [K_T] matrix.

3. Numerical example

An example is proposed, to demonstrate the application of the above algorithm, including: stress stiffening, spin softening and large displacements influences.

The example is represented by a "blade" of rectangular shape, but curved about a circle of 560 mm radius (Figure 1). The thickness is 3mm and the width is 20mm. The material is steel, with Young modulus 217 GPa, density 7850 Kg/m^3 and Poisson coefficient 0.3. The blade is spinning mounted on a shaft of 150 mm radius. The rotation speed is 50 rot/s. To show the importance of considering the additional stiffness matrices, two cases were considered:

- First case, when the commercial solver MSC.NASTRAN is used with default settings and
- second case, when the additional stiffness matrices are used by actively selecting them during preprocessing with the same solver MSC.NASTRAN.

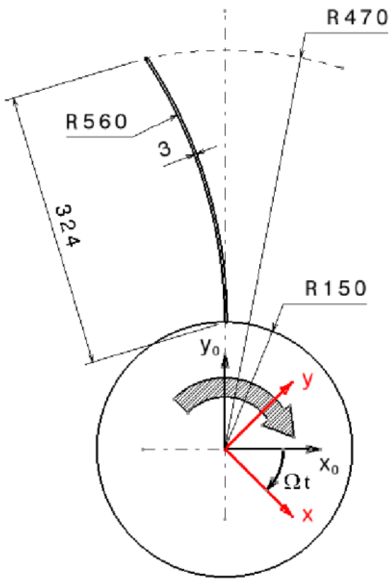


Fig. 1. Scheme of the proposed demonstrating rotor

As can be observed from table 1, first row, the error regarding the first vibration mode using just the [K] stiffness matrix is 54%, rendering the results completely wrong and useless in practice. Regarding the relative Von Mises stress, the error is of the same magnitude. This proves that using even a well-known commercial software like MSC.NASTRAN, but with classical default settings, without knowing and selecting the appropriate stiffness matrices for the problem, leads to wrong and completely inaccurate results.

Table 1

The results considering only K stiffness matrix and K+K_G+Z stiffness matrix

Nr.	Considered stiffness matrix	Displacement under inertial load [mm] (%error)	Stress value under inertial load [Mpa] (%error)	First natural frequency value [Hz] (%error)	Vibration relative stress [Mpa] (%error)
1	K	0 (100%)	0 (100%)	24 (54%)	11970 (43%)
2	K+K_G+Z	78	629	52	21130

On the second row in the table 1, are the correct results, obtained with the same commercial software (Figure 2), but by activating the large displacement and spin softening matrices and solving the problem in two stages:

- In the first stage, the NASTRAN solver is used to obtain the dynamic equilibrium position at the rotating working regime (50 rot/s) involving the [K], [Z] stiffness matrices.
- During the second stage in solving the problem, the natural frequencies of the mechanical system are calculated, accounting for the differential stiffness matrix [K_G] which includes the stress inside the blade, generated by the rotational speed during the first stage solver analysis and adding the centrifugal matrix [Z] which accounts for the non-inertial characteristics of the centrifugal field (the so-called spin softening). For this case, the damping is negligible and the shaft is not considered, so the matrices [K_D] and [K_T] are not involved in the example problem.

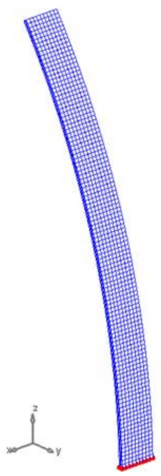


Fig. 2. FEM modeling of the blade

This is the new algorithm proposed, with two stage solving technique, illustrated also in the figure 3 and figure 4 which is the only correct one. The figure 3 shows the blade deforming from the initial curved shape, under the centrifugal force up to the equilibrium position. The blade shows a maximum deformation at tip of 78mm and a maximum Von Mises stress of 629 MPa.

In the figure 4, the blade subjected to small perturbation is freely vibrating around the equilibrium position, corresponding to a specific mode of the selected natural frequency, which is simple bending. The resulting modal displacements and modal stresses, corresponding to the fundamental vibration frequency are obtained using the Lanczos algorithm, with the option for maximum displacement normalization.

In order to find the allowable maximum dynamic response for this particular mode shape, the Goodman diagram is used [15]. Knowing the maximum static stress (629 MPa), the Goodman diagram provides the allowable dynamic stress. This allowable dynamic stress can be divided by the maximum stress obtained using the Lanczos algorithm (21130MPa in our example, see table_1), resulting the coefficient for scaling the normalized displacements.

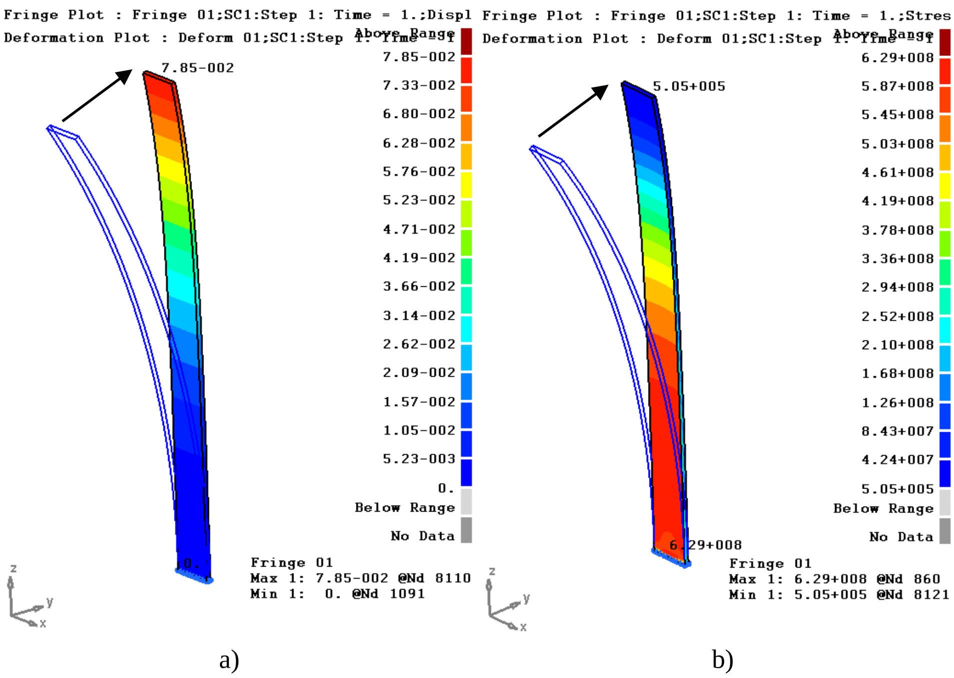
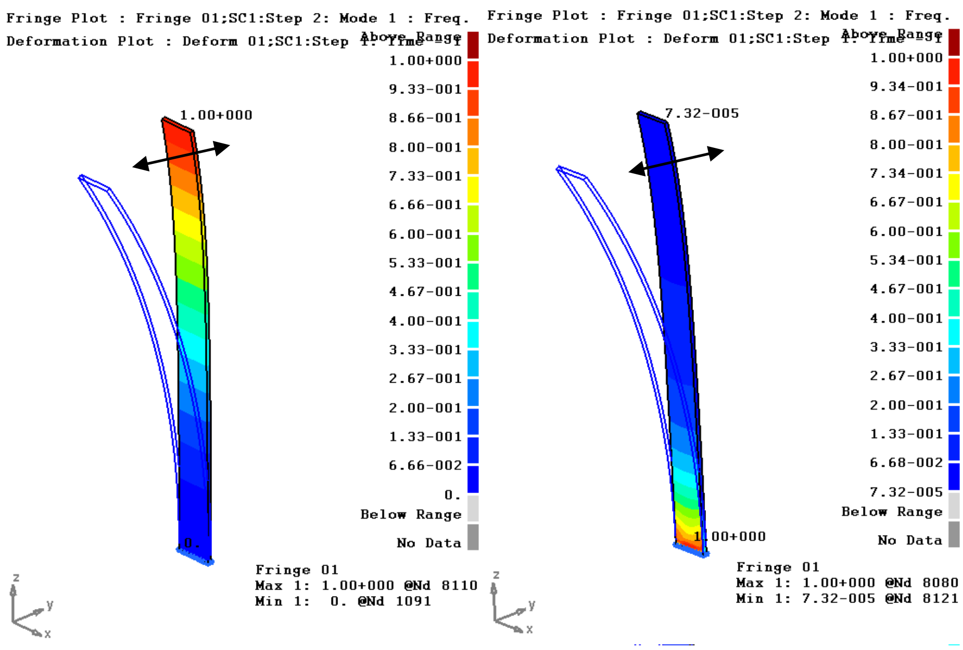


Fig. 3. Static equilibrium at the 50 rps
a) displacements [m] b) Von Mises stress [Pa]



a) b)

Fig. 4. First flexion vibration around the equilibrium position
a) normalized displacements. b) normalized Von Mises stress

Finally, scaling the normalized displacements obtained with the Lanczos algorithm, will result the eigenshapes, with displacements corresponding to the real maximal allowable dynamic response as shown in ref. [15].

6. Conclusions

The contribution of this paper to the present literature in the field of rotating machinery, analyzed in a non-inertial reference frame, is to emphasize the correct formulation of the dynamic equilibrium equation, leading to correct results. This formulation is neither implicit, nor theoretically described in any commercial software used today in the aerospace industry.

Moreover, the paper is presenting a complete image of the phenomena involved in the mechanical analysis of rotating machinery in a non-inertial rotating frame, including a succinct explanation for each phenomenon.

This is reflecting the state of the art in the present free vibration theory and should be considered in the author’s opinion, a mandatory and unique guiding algorithm for the researcher/designer in the general field of rotating machinery.

Usually, the torque effect is neglected in all recent specialized literature, even if it has considerable implication in the results precision, when dealing with slender, heavily loaded rotors. Such effects will be further investigated, and this represents a perspective for continuation of the present research.

Acknowledgement

The first author would like to thank Ministry of National Education for granting his research for a Ph.D. thesis at the University Politehnica of Bucharest and the guidance of the Ph.D. committee at the Faculty of Biotechnical Systems Engineering.

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