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STOCHASTIC MODELING FOR THE FOLLOW-UP THE BEARINGS DEGRADATION

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Stochastic modeling is an approach very interesting for the follow-up time evolution of physical phenomena (state degradation, wear, shock), it is generally used for the description of a correlation between the process size values and those measured or calculated.

The objective is to discuss the feasibility of two follow-up models degradation installation (Wiener and Gamma), in this subject, we took into account the analytically calculated degradation probability of a bearing type 22309 SKF exploited under very severe conditions (iron and steel plant) like followed parameter.

In this context, we adjusted the values of our sample according to the normal and Gamma laws, then we estimated modeling parameters (μ, σ, α, and β) those which allowed to present the models.

The results presented inform us about the evolution of the probability of degradation relating to the real state of the bearing studied of with dimensions, as well as the advantage of the use and the choice of such a model with different dimensions. They are satisfactory considering the actual values very close and are well correlated with those modeled. However, these models consist in providing tendency evolution curves of the degradation parameters, which are useful for the improvement of the equipment reliability and to make a success of such a program of predictive maintenance.

Keywords: predictive maintenance, degradation bearings, stochastic modeling, Wiener Process, process Gamma

1. Introduction

The bearings are the components more sensitive of the revolving machines. To avoid expensive stops of production due to an unforeseen failure, they thus should permanently be supervised, and to detect the appearance of the least defect.

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Traditional methods of vibratory analysis[1] (detection of envelope, spectral analysis, kurtosis, peak factor, etc.) more specific techniques such as the shock wave, they make it possible to detect the defects at a more or less early stage, and to sometimes even determine the exact origin. Of course, there is not universal method; the best tool for monitoring and of diagnosis is that which associates several techniques of maintenance.

The choice and the installation of a maintenance policy for the dependability of equipment with an optimizing aim of its reliability and the knowledge of the residual lifespan of a part, or a body became paramount. Indeed, if the application of a systematic preventive maintenance (anticipated change) causes additional significant costs, thus, waiting of the failure of the machine in the case of a curative maintenance.

It is seen that the most widespread techniques of maintenance currently are based on the monitoring of the machine in service which, starting from a statement of indicators (conditional maintenance) and using the models and algorithms adequate, allowing to establish a diagnosis on the real state of the machine like its future evolution (predictive maintenance). In all the cases, the condition necessary for the decision-making in many cases is the analysis, as well as the characterization of the evolution in time of the parameter or the indicator followed (reached thresholds by the analysis of the tendency) such: the vibratory level [2], propagation of a crack, the deterioration of the quality of a lubricant, evolution of a wear.

The basic requirement for the choice of a degradation model is to recounts, as accurately as possible, the characteristics related to the lifespan of the machine. In this context, one sees various models which were implemented in optics to approach as well as possible in a real state of a body. Among these models: The Markovian model [3] applied for the follow-up of the change of state and the models stochastic Brownian [4,5] (Model of Wiener, Gamma) for the follow-up and the estimate of future degradations and even for the improvement of equipment estimated reliability. This work will be organized in the following way:

- Methodology of work.
- Presentation of the models Wiener and Gamma.
- Results and discussions.

2. Stochastic modeling of the degradation process

2.1 Model of Markov

In mathematics, a process of Markov is a stochastic process for which only information present concerning a variable is useful and is enough to anticipate its future propagation. In other words, a chain of Markov is a random process [6] of evaluated discrete variables, which implies a number of states. These states are bound by possible transitions, each one with an associated probability (P_{ij}: probability of transition from a state i to one moment (*t*) in a state j to one moment (*t+1*)) and each state has a joined observation. The transition from a state to another is only dependent on the current state and not on the last states, (it is called also model without memory). The real sequence of the states is not observable.

In this type of structure, each state can be reached or connected to the other states (figure 1), i.e. all the states communicate between them, according to a chart, all the states are thus connected by arrows indicating the direction of transition and the probability corresponds handle [7].

In this context, the law of "probability of transition" from a state i towards a state j by is defined by:

$$P_{ij} = \mathrm{P}(X_n = j | X_{n-1} = i) \tag{1}$$

With which the model associated is given by:

$$X^{(n)} = X^{(0)} = M^n \tag{2}$$

I.e., the probability of degradation of a bearing for a given age group (n) equalizes with the product of the probability of degradation with *t=0* and power n (corresponding to the age group) of the matrix of transition.

With:

P_{ij}: probability of transition from the state (i) to the state (j).

M: it is the matrix of transition.

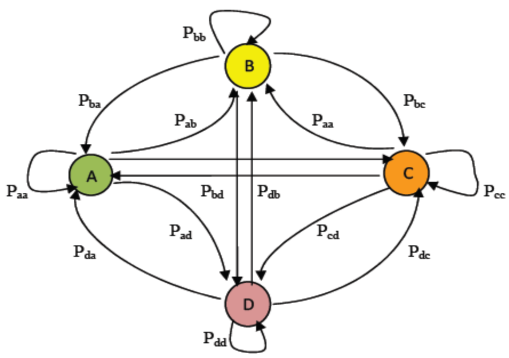

Fig. 1. Model of Markov in graphic form [7].

2.2 Model of a process Gamma

The modeling of a Gamma process describes a trajectory of degradations to positive independent increases which can be stationary or no stationary [5,8].

It is said that the *X(t)* process is a stationary Gamma process, *G(α, β)* if:
• *X(0)* = 0, initial condition.
• *X(t)* is a process with independent and positive increases
• for all *t>0* and Δ*t>0*, the law of evolution is given by:

$$X(t+\Delta t)-X(t)=G(\alpha\Delta t,\beta) \quad (3)$$

With; $G(\alpha\Delta t,\beta)$ is a law gamma whose α and β are respectively the parameters to be of form and scale, this law has as a function of density:

$$f(x)=\frac{\beta^{-\alpha\Delta t}}{\alpha\Delta t}x^{\alpha h\Delta t-1}e^{\frac{x}{\beta}} \quad (4)$$

The process Gamma defined above represented linear degradations on average; generally, it has respectively as a hope and variance the following expressions:

$$E(X(t))=\alpha\beta t \quad (5)$$

$$Var(X(t))=\alpha\beta^2 t \quad (6)$$

2.3 Model of a process Wiener

A Wiener process can be described by a trajectory of degradations to independent increases which are not necessarily positive. One distinguishes the Wiener processes of linear trend and non-linear.

Such a *W(t)* process is known as Wiener process of parameters *m* and σ^2 respectively the linear trend (the average) and the variance if:
• *W(0)* = 0, initial condition.
• *W(t)* is a stochastic process with independent increases with continuous trajectories.
• For all *t*>0 and Δ*t>0*, the law of evolution of this process is given by:

$$W(t+\Delta t)-W(t)=N(m\Delta t,\sigma^2\Delta t) \quad (7)$$

With, $N(m\Delta t,\sigma^2\Delta t)$ is a normal law and which has for all *t*>0 a function of density:

$$f(x)=\frac{1}{\sigma\sqrt{2\pi\Delta t}}e^{\frac{(x-m\Delta t)^2}{(2\sigma^2\Delta t)}} \quad (8)$$

3. Assumptions and hypothesis

- The A bearing which reaches a level of degradation (vibratory) inadmissible (according to the standard ISO 2372), will be followed directly by an operation of change.
- Vibratory measurements (tool for evaluation of degradation) are taken under the same conditions.

In the initial state (t=0), all the bearings are in a not wearing state (degradation $X_0 = 0$).

4. Methodology of work

Our objective is to make adapt stochastic models (Wiener and Gamma) to the statistical results (probabilities of degradation), calculated at the base of the data obtained at the time of a vibratory operation of follow-up of a batch of 60 bearings.

Table 1

Results of the follow-up of the degradation of the bearings

Age group	Duration (month)	Nb bearing	Nb bearing used	Nb used cumulated	Probability of degradation (%)	Cumulative probability of degradation (%)
1	2	60	0	0	**0,00**	0,00
2	4	60	0	0	**0,00**	0,00
3	6	57	3	3	**0,05**	0,05
4	8	51	6	9	**0,10**	0,15
5	10	50	1	10	**0,02**	0,17
6	12	47	3	13	**0,05**	0,22
7	14	43	4	17	**0,06**	0,28
8	16	37	6	23	**0,10**	0,38
9	18	29	8	31	**0,14**	0,52
10	20	22	7	38	**0,11**	0,63
11	22	15	7	45	**0,12**	0,75
12	24	9	6	51	**0,10**	0,85
13	26	9	0	51	**0,00**	0,85
14	28	4	5	56	**0,08**	0,93
15	30	0	4	60	**0,07**	1,00

I.e. the modeling of the evolution of the phenomenon wearing of a bearing, and this for the prediction of its failure, knowing that a phenomenon of degradation can be modeled by a process gamma (increasing degradations), or a process of Wiener (increasing or decreasing degradations).

Let us recall that this type of bearing is installed on centrifugal fans which work under conditions very severe (temperatures, dust, gives rhythm variable production), and this during 30 months a vibratory partner of measurement (t = 0, ….., 30) at intervals of two months measurement, therefore one is in front of 15 vibratory measurement of which the data are represented by table 1.

4.1 Analyze normality of the data

This analysis of normality makes it possible to check if the distribution of the real data (probability of degradation calculated by the analytical method), follows a normal law in the case of a Wiener process or a law Gamma if it is about a Gamma process.

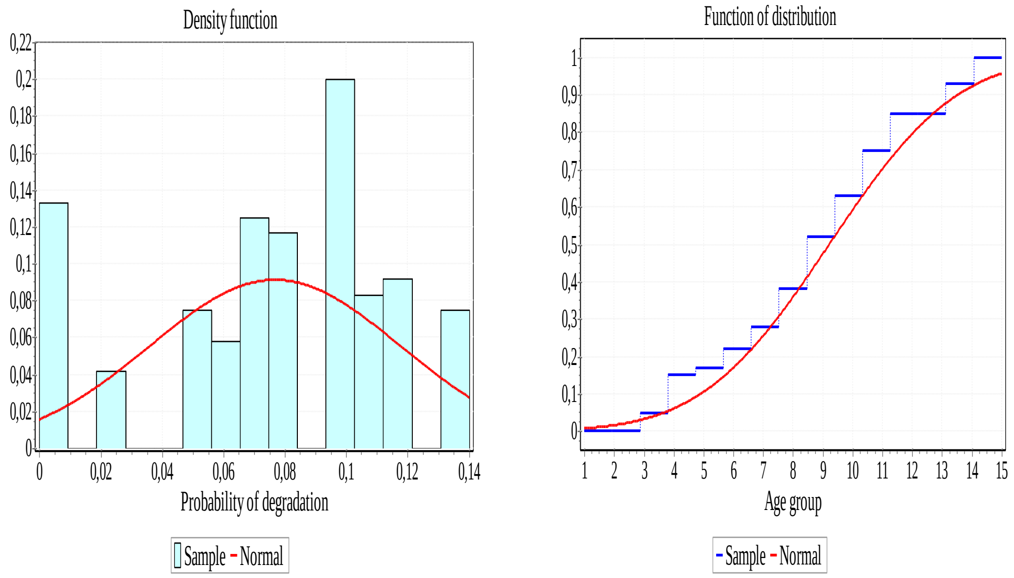

Fig. 2. The density and distribution functions of density according to the normal law

4.2 Checking of the adjustment compared to the law Gamma

In the same manner, we proceeded to one to check data if they follow a law Gamma or not, the results of adjustment are represented by figure 3.

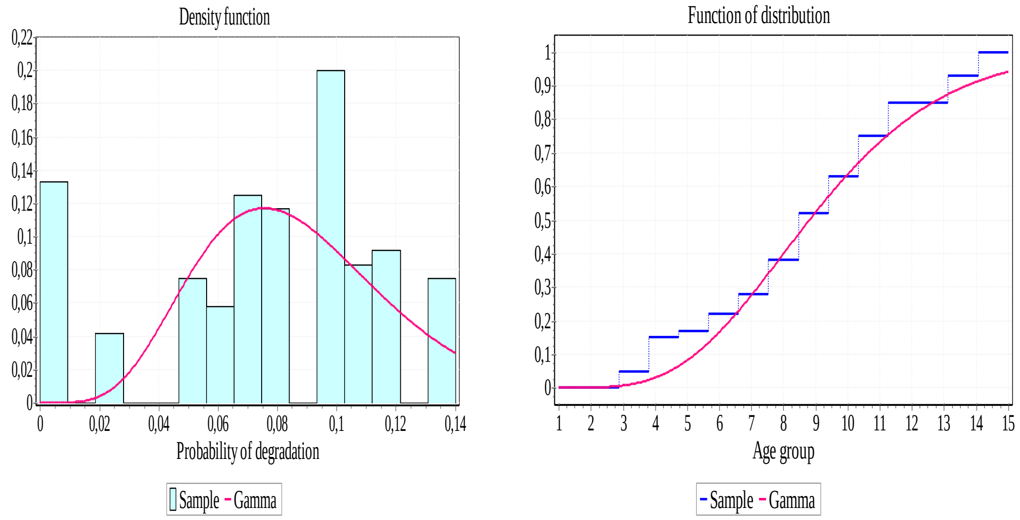

Fig. 3. The density and distribution functions of Gamma law

4.3 Statistical test

From another side to confirm if the distribution of the data can adequately be modeled by a law Gamma or not, one thus will carry out a test of Kolmogorov-Smirnov (figure 4), this last compares the function of uniform distribution with the function of distribution of the analyzed sample. The idea is to calculate the maximum distance between the theoretical and empirical functions. If this distance exceeds a certain value, that one will read in a table, one will say that the sample is bad or to reject.

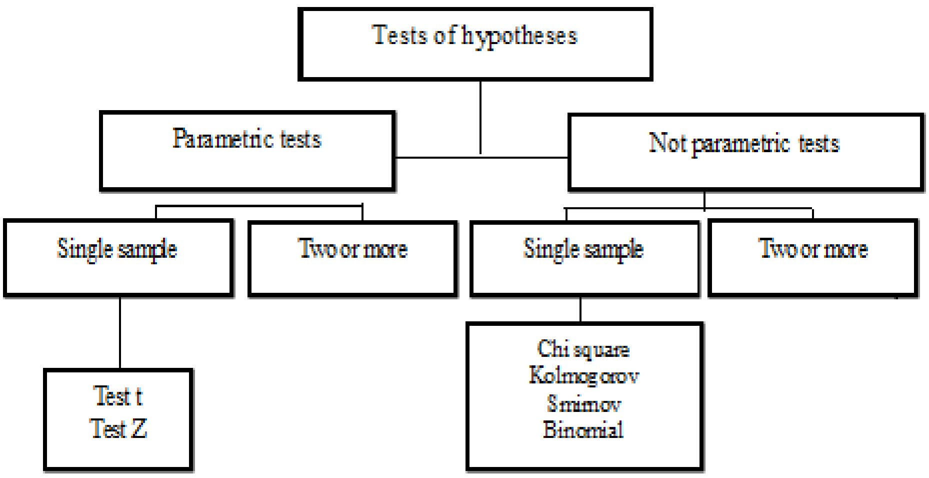

Fig.4. Tests of assumptions of the statistical data [9]

The parameters of the statistical test are represented in the table 2.

One clearly sees according to these results which one cannot reject the assumption that our sample respectively follows a normal law and a law Gamma, and this for a probability of confidence of 95% (risk 0.05).

Table 2

Results of the statistical test for the normal law

	Test of Kolmogorov-Smirnov									
	law Normal (σ = 0,0408 μ = 0,0768)					law Gamma (α = 6,8088 β = 0,0130)				
Sample size	**15**					**15**				
Statistics	0.16471					0.17365				
Value of P	0.75211					0.63452				
α (Risk)	0.2	0.1	0.05	0.02	0.01	0.2	0.1	0.05	0.02	0.01
Critical value	0.266	0.304	0.338	0.377	0.404	0.266	0.304	0.338	0.377	0.404
Reject ?	Non	Non	**Non**	Non	Non	Non	Non	**Non**	Non	Non

5. Presentation of models Wiener and Gamma

The models suggested for follow-up of the evolution of the degradation of the bearings are given by the differential equation below; it results from the equations (3) and (7),

$$dx = \mu . dt + \sigma . dw \qquad (9)$$

With; μ the average, σ the standard deviation.
dw: is the variation (the value of the increment) according to the law of distribution (normal or Gamma), it is obtained starting from the function of corresponding distribution.

In addition, between two consecutive age groups ($\Delta t = h$), the increment of degradation can be given by:

$$dx = X(t + h) - X(t) \qquad (10)$$

The parameters of modeling are considered and given by the software MathWave Easy-Fit 5.4 used already for the adjustment of our sample and which are as follows:

- In the case of a Wiener process: μ = 0.0768; σ = 0.0408.
- In the case of a process Gamma: α = 6.8088; β = 0.0130.

In end, following a formulation under Excel, we arrived to present the trajectories of the processes of degradation of the bearings studied according to models' quoted above (Wiener and Gamma) and which are represented by figure 5.

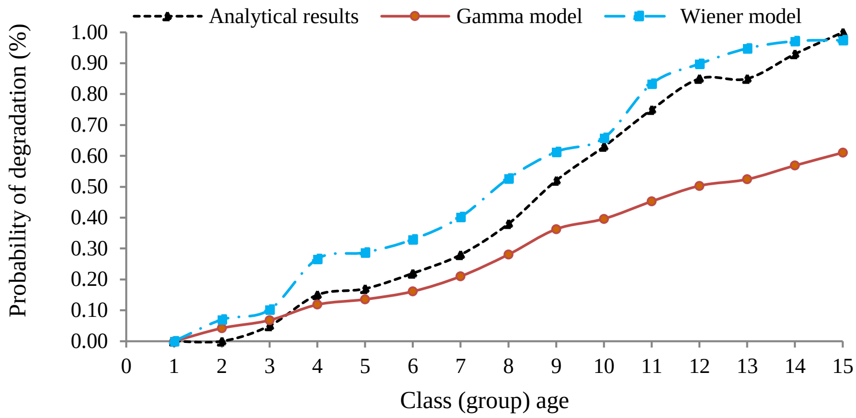

Fig.5. Trajectories according to models

6. Results and discussion

The results obtained according to the figure 5 show clearly that the models suggested (Wiener and Gamma) bring a good estimate of the probability of degradation of the bearings, and that the distribution of these values follows the same pace as those calculated by the analytical model. Based on the coefficient correlation, the values of the results obtained starting from the stochastic models suggested compared to those calculated analytical, which are better and acceptable (R_W=0.96 for Wiener model and R_G=0.98 for Gamma model), with an advantage when one uses the Gamma model to estimate degradation at minimal operation lives, but when this duration becomes significant, one sees clearly that one must choose a Wiener model.

7. Conclusion

In predictive maintenance, the follow-up of the evolution of the degradation of the state of a body with an aim of predicting significant breakdowns requires a good command of the techniques and tools of decision-making at end to avoid the going beyond of the thresholds of danger predetermined by standards or the specialists.

For example, the analysis of the tendency during the use of the vibratory analysis systematically makes it possible to make an evaluation on the real state of the behavior of any equipment. In case of the defect, it is a priceless help to identify and evaluate the gravity of the anomaly of it.

Presented study that we consider supplements watch that stochastic modeling is also an invaluable mathematical tool being able to provide a good prediction of the possible breakdowns with the analysis of a weak history. For the analysis, we considered a sample of probabilities of degradation of the bearings calculated by an analytical method, which us a makes it possible to propose two stochastic models of estimate known (Wiener and Gamma), one also sees that the models suggested can also contribute to the planning of a predictive maintenance in order to avoid the unforeseen ones and of which the goal to improve the reliability of the production equipment.

A future study appears to be necessary to generalize this modeling with the other type of bearings, by holding account the various accelerated factors which can the phenomenon of degradation such as; the quality of lubrication, the loading of the bearing and its environment.

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