GEAR FAULTS DETECTION USING WAVELET-FILTERED CEEMDAN MODES: VARIABLE SPEED CASE

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The paper represents a contribution to the field of machine surveillance and fault detection. The study tests the applicability of a hybrid technique for gear faults detection under variable speed condition. Vibration signals have been captured from a gearbox containing a faulty spur gear. The decomposition of these signals with CEEMDAN has allowed the isolation of gears vibration response from other signal components. The filtration of the optimal intrinsic modes envelopes through a wavelet multiresolution analysis has successfully separated the shocks induced by faults from the meshing components and permitted the performance of envelope order spectrums that has effectively confirmed the gear fault presence.

Keywords: Vibration analysis, Gear faults, CEEMDAN, WMRA, Order analysis

1. Introduction

Gears represent an essential component in a wide variety of mechanisms that could be found in many applications, vitally when there is a need for speed and torque variation, or direction changing of a rotating power source. Like every other mechanical parts, gears may breakdown due to wear and tear, causing the malfunction or the complete failure of the system on which they are mounted. Therefore, enough surveillance must be provided for these components in order to prevent such consequences.

In the last few years, vibration analysis has been proven to be the best choice that guarantees detection, diagnosis, separation and even prediction of different faults related to various machine components [1], especially with the fast technological advancement that gave researchers and engineers access to high accuracy and resolution sensors, fast electronic modules and powerful computers that facilitate calculations, all of this has made the processing of vibration signals possible, and led to the appearance of many signal processing techniques that made easier the extraction of fault symptoms, even those related to gears, where a good technique must extract fault signs embedded in complex vibration signals ensued

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from a combination of multiple signals generated by different machine parts, due to the fact that gears often operates in rough and noisy environments, as in the case of gearboxes.

When the purpose is restricted solely to the detection, a time domain analysis may be satisfactory; a group of simple statistical parameters composed of a number of time domain signal metrics such as RMS, Crest factor, Kurtosis and others, can give some useful information concerning the gears condition [2 – 4], and may be used as good features for the tracking of faults progress [5]. The main problem with time domain techniques is that, the majority of the previous parameters are, in most cases, unable to differentiate faults occurred in different machinery components, as in the case of bearings and gears where defects in both parts appear in the same way in the time domain signal, as a succession of impulses generated each time a rolling element makes contact with a defect or a faulty tooth makes contact with another, which causes a rise in the overall vibration level, affecting the values of these statistical parameters. In such a case, the use of time domain analysis is constrained for detecting abnormal machinery behavior only or as a preprocessing technique that clears the way for other types of analysis. Therefore, a better technique would be the one that helps associating each symptom with its particular machine component; a frequency domain technique, a technique like Fourier Transform which is capable of decomposing a time signal into a set of sinusoidal components sorted by frequency and represented in form of a spectrum that shows the periodicities hidden in the signal, and allows the detection of defects by comparing the frequencies of apparent peaks with the characteristic frequencies of faults. However, for gearboxes, the raw vibration signal is often dominated by the meshing components, the shaft rotational speed components and the components of other parts such as bearings, plus some random components often described as noise. The isolation of impulses associated with gears faults, applying only a FT on the raw signal is then difficult, which explains the appearance of some techniques that aim at first separating the different modes hidden in the original signal. One of the most successful ones is the so-called Empirical Mode Decomposition (EMD), a technique that derives the natural oscillatory modes embedded in a signal from its extrema. The previous literature contains a plentiful of works that used this technique for successfully extracting the transient caused by gear cracks [6 – 8]. Newer versions of EMD appeared later to eliminate one of major shortcomings of the original technique; the mode mixing issue. The Ensemble Empirical Mode Decomposition (EEMD) used Gaussian white noise and averaging to solve the problem, and has been used to extract gear fault signature [9]. The Complete Ensemble Empirical Mode Decomposition with Adaptive Noise (CEEMDAN) has been developed later to minimize the computation time and give better decomposition [10], this latter version has been proved to be effective in gears fault detection in many works [11 – 14]. Another decomposition technique that has attracted researchers' attention is the Wavelet Transform (WT), a technique that decomposes a given signal on the basis of a changing-width special function called Wavelet, which can be chosen from a wide list depending on the application. The technique has found its application in gear fault detection as showed in many papers [15 – 17]. One of the useful applications is the utilization of Wavelet Multi-Resolution Analysis (WMRA) in order to separate the high-frequency signal components from the low-frequency ones, which can serve as a good filtering and detecting solution for gears defects as proven in [18] [22] [25 – 29].

For the above-mentioned techniques and others, the diagnostic procedure is almost the same; the machine is put into operation at a specific speed and then a vibration signal is captured and processed with one or more techniques to figure out the presence of defects. A special case that is rarely discussed in the literature, is the detection of gear faults under variable speed, due to the incapability of processing signals that have been measured under non-stationary conditions, with techniques that have been designed in the first place to be used with machines working under stable speed. Few are the works that have presented a solution for this issue, in [19] authors have used Hilbert-Huang transform and instantaneous dimensionless frequency normalization features to feed a support vector machine that classifies spur gears faults under variable speed. Order tracking have been used with Bi-spectrum, Cyclostationarity and synchronous averaging in the works [20], [21] and [22] respectively, to show the power of order analysis in redressing the variable speed problems and detect spur and planetary gear faults.

Aiming at benefiting from the power of the above-mentioned tools, this study proposes the combination of CEEMDAN, WMRA and Order tracking with the purpose of constructing a hybrid tool able to detect gear faults under variable speed (Fig.1). The vibration signal will be first decomposed by CEEMDAN in order to isolate the gears dynamic response from other signal components. For that, an optimal mode will be chosen from the gotten IMFs according to two proposed criteria which are the meshing order coverage and high Kurtosis values, the picked mode envelope will be then passed through two wavelet filters with the aim of extracting the fault signature as a reconstructed vector that maximally isolate the impulse train generated by the defect. After that, the envelope order spectrum will be applied on the obtained signal for fault presence confirmation. The hybrid tool has been verified with a faulty gearbox and found to be effective in variable speed condition.

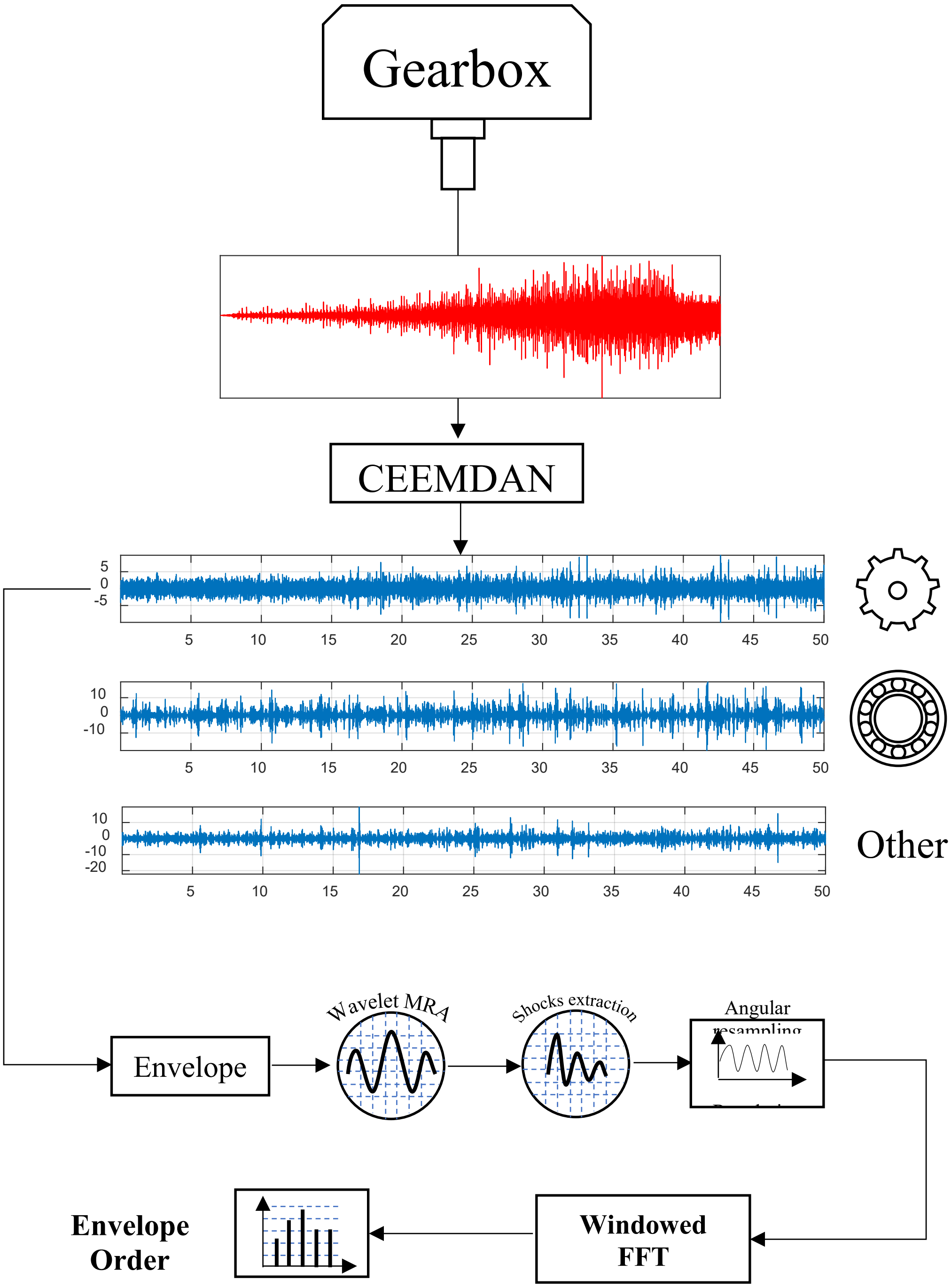


Fig. 1 The proposed technique algorithm

2. Tools and methodology

The study proposes the performance of a sequence of operations that have been found required for the non-stationary vibration signal analysis, in a way where the purpose is to reveal the presence of gear defects under variable speed. The role of different signal processing varies between decomposition, filtration, optimal selection, resampling and fault presence confirmation. The following section describes the executed procedures with the presentation of the tools that have been used at each stage.

Decomposition

For gearboxes, the raw vibration signal is often dominated by the meshing components, the shaft rotational speed components and the components of other parts such as bearings, plus some random components often described as noise. The isolation of gears related vibration components is necessary for a correct diagnostic procedure. For this step, the study proposes the use of CEEMDAN which is a tool based on EEMD, proven to give better results with impulse-generator faults such as bearings and gears faults [11 – 14] [19 – 22]. For the decomposition of a given signal $x(t)$ with CEEMDAN, the below steps are followed [10]:

1. Decompose I realizations of $x(t) + \varepsilon_0\, n^i(t)$ by EMD to obtain the first $\overline{IMF_1}$ by averaging:

$$\overline{IMF_1(t)} = \frac{1}{I}\sum_{i=1}^{I} IMF_1^i(t) \tag{1}$$

ε_0 is the signal-to-noise ratio, $n^i(t)$ is a white Gaussian noise, IMF is the obtained intrinsic mode.

2. Calculate the first residue as:

$$r_1(t) = x(t) - \overline{IMF_1(t)} \tag{2}$$

3. Decompose I realizations of $r_1(t) + \varepsilon_1\, E_1(n^i(t))$ until their first EMD mode and calculate the second mode:

$$\overline{IMF_2(t)} = \frac{1}{I}\sum_{i=1}^{I} E_1(r_1(t) + \varepsilon_1\, E_1(n^i(t))) \tag{3}$$

Where E_j is an operator that produces the j^{th} mode obtained by EMD.

4. For $k = 2 \ldots.K$, calculate the k-th residue:

$$r_k(t) = r_{k-1}(t) - \overline{IMF_k(t)} \tag{4}$$

5. For $k = 2 \ldots. K$, define the $(k + 1)$-th mode as:

$$\overline{IMF_{k+1}(t)} = \frac{1}{I} \sum_{i=1}^{I} E_1(r_k(t) + \varepsilon_k E_k(n^i(t))) \tag{5}$$

6. Go for step 4 for next k

Steps from 4 to 6 are repeated until the obtained residue in no longer feasible to be decomposed and satisfies:

$$R(t) = x(t) - \sum_{k=1}^{K} \overline{IMF_k}(t) \tag{6}$$

With K the total number of modes. The original signal $x(t)$ can be expressed in the end as:

$$x(t) = \sum_{k=1}^{K} \overline{IMF_k}(t) + R(t) \tag{7}$$

Optimal mode selection

After successfully decomposing the vibration signal into IMFs, the modes that are linked to gears must be recognized. For this purpose, two criteria are proposed so that only gears components are preserved:

- The meshing order coverage.
- The highest Kurtosis value.

The variation of the meshing stiffness is considered to be the source of gears vibration in many models that describes the dynamic response of toothed gear pairs [23, 24], the meshing frequency/order (f_m/O_m) which can be simply derived from the teeth number (Z) and the rotation frequency (f_r) as shown in equation 9, is then an efficient criterion for gears vibration response recognition, therefore, the first criterion guarantees that the chosen mode is definitely related to gears and maximally isolated from other components.

$$f_m = f_r \times \mathrm{Z} \tag{8}$$

$$O_m = Z \tag{9}$$

On the other hand, high kurtosis values are generally linked to impulses generated by defects, due to the fact that the latter habitually causes a rise in the peakedness of the signal distribution which can be mathematically sensed and measured by the fourth standardized moment. The second criterion is then proposed to make sure not only that the selected modes are related to gears, but also susceptibly contain the signs of gear defect. Signal Kurtosis is generally calculated as below:

$$Kurtosis = \frac{\int_{-\infty}^{+\infty}|x|^4\, p(x)dx}{(\int_{-\infty}^{+\infty}|x|^2\, p(x)dx)^2} = \frac{\mu_4}{\delta_2^2} = \frac{the\ 4th\ central\ moment}{the\ standard\ deviation} \quad (10)$$

Wavelet based filtration

The previous operation may have a principal role in the detection procedure, but still not lead to a final decision; the two proposed criteria promise an optimal selection of the modes that may contain gear fault signs, but does not confirm it. It is also noticed that CEEMDAN may provide IMFs that are partially mixed with noise, shaft speed components or other machine parts vibration response residues. The aim of this study is to completely isolate the impulses generated by gear faults for a certain decision. For this purpose, WMRA has been used as a filtration tool that separates the impulses from the shaft and meshing components by the mean of cross-correlation concept. Two filters have been constructed on the base of the Daubechis wavelet as a wavelet mother. The choice has been made according to the results obtained by previous works [22] [25 – 29] in which it has been proved that this wavelet is the best adapted in WMRA of shock signals. The convolution of the previously selected IMF with the first filter would give a vector cA_i containing its low-frequency components, applying the same operation with the second filter would provide us with a second vector cD_j containing the high-frequency remaining components. After that, the two obtained vectors will be passed through two reconstruction filters in order to fix the down-sampling suffered during the previous decomposition and obtain the final result that would be two vectors representing the $approximations$ (A_j) and $details$ (D_j) of the chosen mode with:

$$A_{j-1} = A_j + D_j \quad (11)$$

$$x = A_j + \sum_{i\leq j}^{n} D_i \quad (12)$$

An optimal vector which only contains the impulses train related to the defect, is finally nominated for the next step on the base of Kurtosis values.

Angular resampling and order spectrum performing

Once the fault time-domain signature is extracted by the previous procedures, a frequency domain analysis is needed to confirm the connection between the train of impacts and gears. This is normally done by a Fourier Transform which is capable of decomposing the time signal into a set of sinusoidal components sorted by frequency and represented in form of a spectrum that shows the periodicities found in the signal, allowing the linkage of defects with gears by comparing the frequencies of apparent peaks with the characteristic frequencies of faults. However, when the speed is variable, FT loses its proficiency due to the fact that it is only capable of extracting constant periodicities. The solution of this issue would be the application of FT in the angular domain rather than the time domain, in a procedure commonly known as order analysis (Fig.2). This is usually done by resampling the time domain signal into angular domain through the estimation of the angular position of the rotating shaft from the arrival times of a key-phasor or a tachometer pulses, assuming that the shaft has a constant angular acceleration and using a quadratic equation as below [30]:

$$\theta(t) = b_0 + b_1 t + b_2 t^2 \tag{13}$$

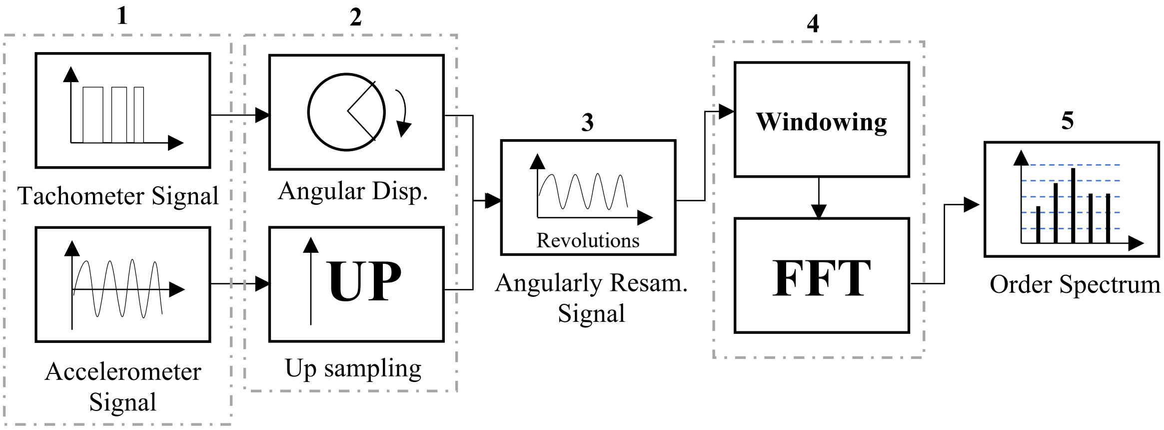


Fig. 2 Order tracking algorithm

Where θ is the angular position, t is time, b_0, b_1 and b_2 are coefficients obtained by fitting three successive tachometer pulses arrival times (t_1, t_2, t_3):

$$\theta(t_1) = 0$$
$$\theta(t_2) = \Delta\Phi$$
$$\theta(t_3) = 2\Delta\Phi$$

$$\begin{pmatrix} 0 \\ \Delta\theta \\ 2\Delta\theta \end{pmatrix} = \begin{bmatrix} 1 & t_1 & t_1^2 \\ 1 & t_2 & t_2^2 \\ 1 & t_3 & t_3^2 \end{bmatrix} \begin{Bmatrix} b_0 \\ b_1 \\ b_2 \end{Bmatrix} \tag{14}$$

After the vector b is obtained, Equation 15 can be used to find the resampling times:

$$t = \frac{1}{2b_2}\left[\sqrt{4b_2(\theta - b_0) + b_1^2} - b_1\right] \tag{15}$$

After the resampling is done, FT is applied on the new signal to obtain an order spectrum that visualize the periodicities as a function of shaft speed harmonics or orders instead of frequencies.
In our case, envelope order spectrum is favored over the classical order spectrum, due to the efficiency of the envelope analysis as a cyclostationarity tool that allows the demodulation of the signal and the extraction of the general energy fluctuation of the whole impulses train rather than the energy of the impulses themselves.

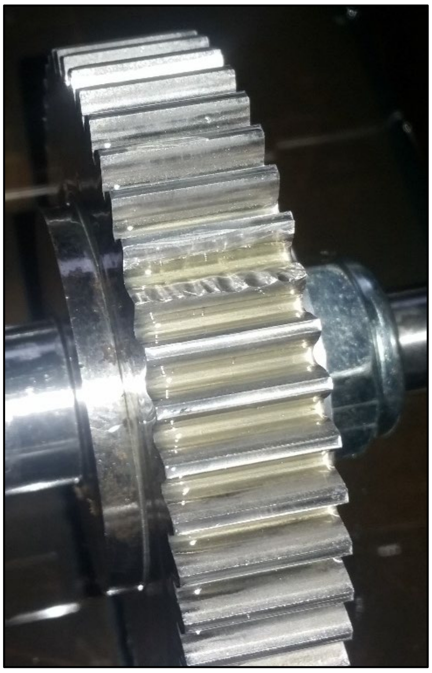
3. Application and results discussion

Test rig description

The approach has been tested with signals captured from the three-stage gearbox presented in (Fig.3), it has three shafts guided by six healthy bearings. Four spur gears are mounted on these shafts, one of them has a fault on one of its teeth. The ensemble is partly immersed into oil to provide lubrication. The gearbox is powered by a three-phase 2.2 KW electric motor having a speed range that goes up to 2860 Rev/min, controlled by an electrical frequency variator that allows tests to be carried out under both constant and variable speed conditions. The gearbox is attached to the motor with a flexible coupling. An electrically controlled brake is used to simulate loads.

Measurements

Using a Brüel & Kjær accelerometer installed on the bearing, the vibration signal shown in Fig.4 has been measured with a sampling frequency equal to 16.384 Hz and a total of 265.554 samples, while the driving and the driven shafts with their gears, have been experiencing an acceleration between 400 and 870 RPM (for the driving gear) and between 328 and 714 RPM (for the driven gear) as represented in Fig.5. The speed variation which has been achieved using a programmable electrical frequency variator attached to the motor, lasted for about sixteen seconds. The shafts angular movement have been followed by a Brüel & Kjær laser probe that gives a pulse each time a reflective tape put on the shaft comes in front of the laser beam. The vibration and speed signals have been measured concurrently using a multi-channel analyzer.



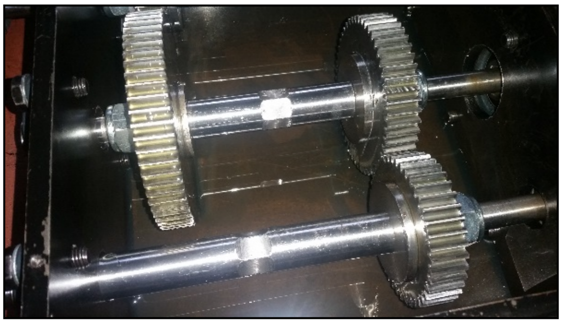


Fig. 3 The test rig

Signal processing

As mentioned before, the first operation consists of decomposing the measured signal into IMFs using CEEMDAN. This procedure has given the modes represented in Fig.6. The second step would be the selection of the optimal modes that best reflect the gears dynamic response. The choice has been made upon the following observations: The meshing order (42) is well covered by the first two modes. It is also noticed that Kurtosis values are close to each other especially for the three last modes. However, comparing the order spectrums of the obtained IMFs shows that the first IMF's order spectrum contains one extra meshing order harmonic (84) than the others, and the first time domain signal has the highest Kurtosis value, expressing that the first mode includes both meshing components and susceptibly the defect signs (Fig.7), and must be the best candidate for the next stage.

The extracted signal must now be decomposed depending on its structure, in order

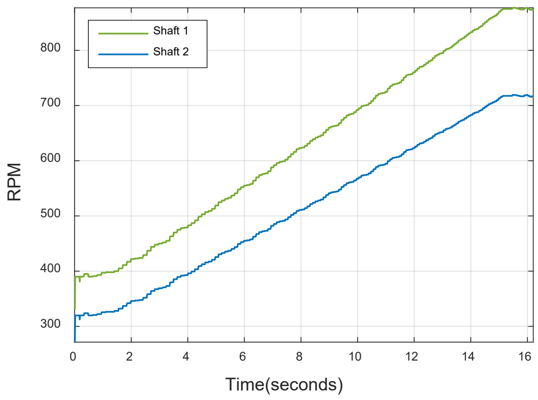


Fig. 5 The speed variation curve

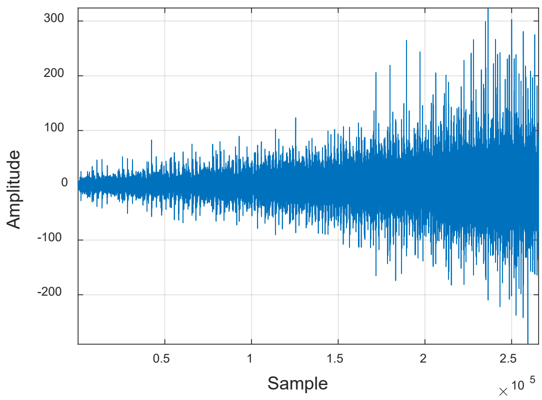


Fig. 4 The captured vibration signal

to take away the meshing components and leave the impulse train caused by the defect as a remaining. This is done over the passage of the selected mode envelope through the constructed WMRA filters, which allows the separation of the components that have the same structure as that of the chosen wavelet mother by the cross-correlation premise. The described process has given the vectors represented in Fig.8. The analysis of the given results leads to the selection of the third vector as an optimal vector that clearly shows what appears to be the shocks induced by the fault (Fig.9).

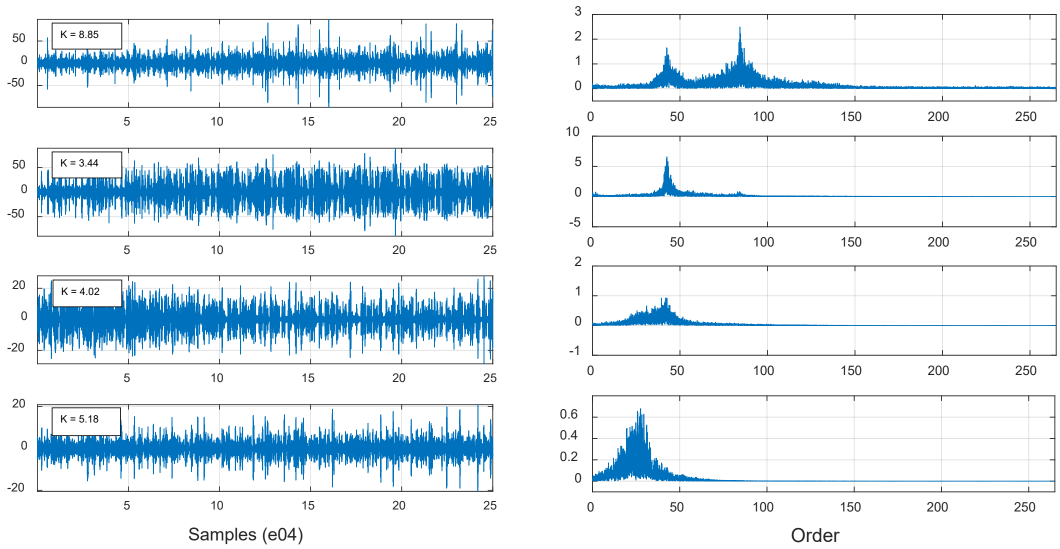


Fig. 6 The obtained IMFs with their corresponding order spectrum

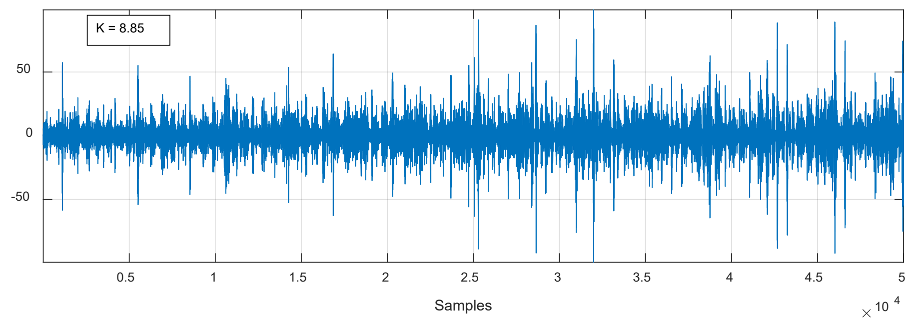


Fig. 7 The optimal IMF

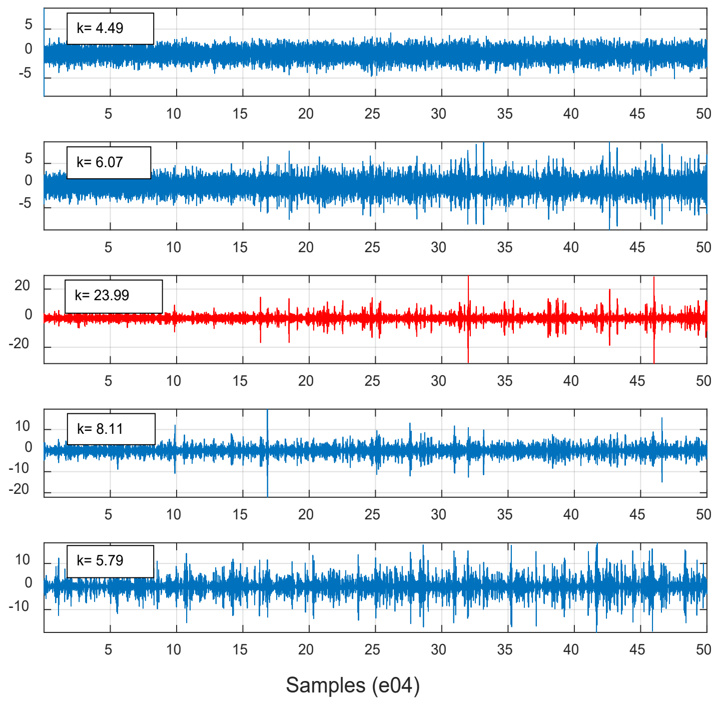


Fig. 8 The details obtained after WMRA application

For confirmation, a final operation is executed, including the selected vector envelope order spectrum calculation and representation. The signal envelope is calculated and then angularly resampled as explained before. After that, a windowed FFT is lastly applied on the obtained envelope to achieve the final result. A spur gear faulty tooth would make contact with another once each revolution, this would be represented in the order spectrum with a peak of order one and its harmonics, which is the same result obtained here (Fig.10).

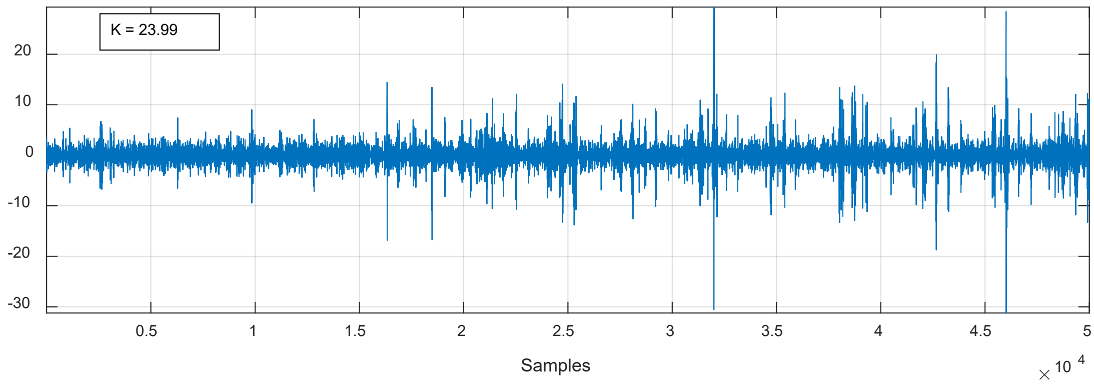


Fig. 9 The optimal detail

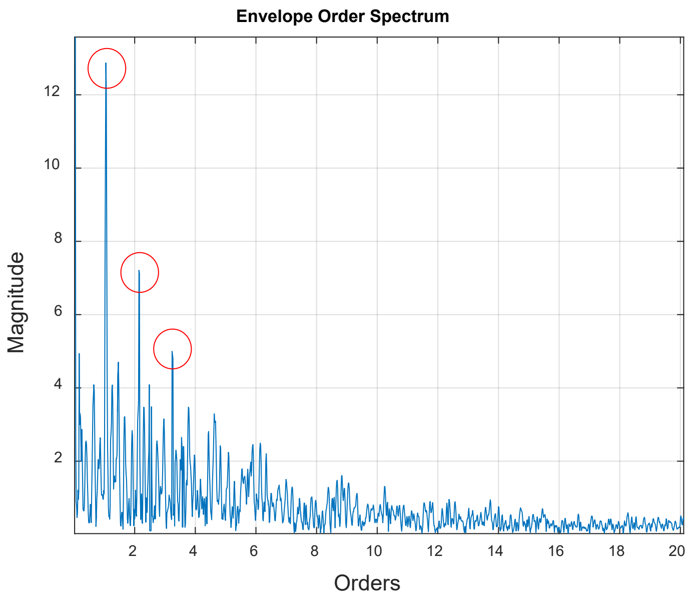


Fig. 10 The Envelope Order Spectrum

A comparison between the obtained results, shows that the combination of WMRA with the empirical decomposition, has not only increased the Kurtosis value which is a key parameter in fault detection, but also contributed in enhancing the envelope order spectrum; Fig.11 represents the provided result from the calculation of the envelope order spectrum directly over the IMF obtained by CEEMDAN, which shows that, contrary to the order spectrum obtained by the proposed approach, the first two peaks are faded out, which is ambiguous and may lead to a false or uncertain decision.

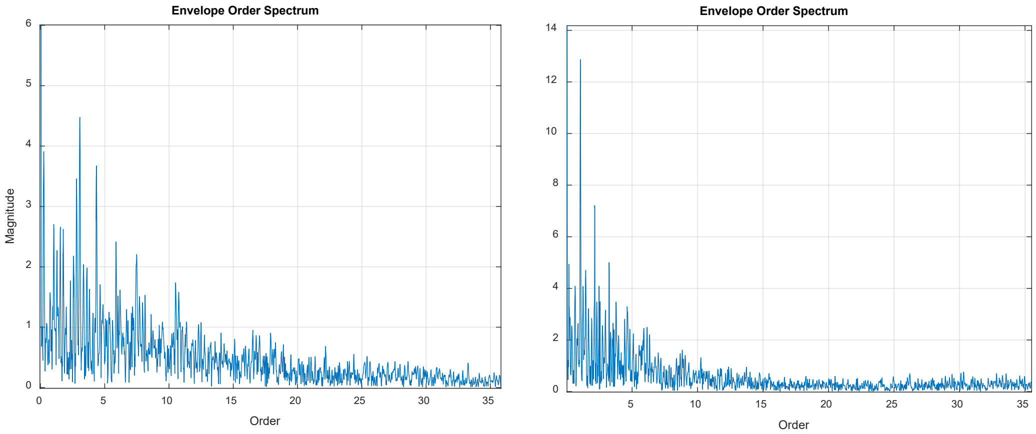


Fig. 11 Envelope Order spectrum obtained after applying: CEEMDAN only (Left), Proposed technique (Right)

4. Conclusion

In this paper, a hybrid method has been tested for the detection of spur gear faults under variable speed. It is based on Complete Empirical Mode Decomposition with Adaptive Noise and Wavelet Multi-Resolution Analysis, with the assistance of Order Tracking Analysis. The CEEMDAN has been used in order to separate the gears vibration response from other signal components. The WMRA has been then used to isolate the impulse train caused by faults, from the optimal mode envelope. The angular resampling has been used to remove the speed variation effects and help in performing an envelope order spectrum that successfully highlights the characteristic order of the fault. The proposed technique has been proven effective with data from a test rig.

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