NONLINEAR ANALYSIS OF THE ATTRACTOR ASSOCIATED TO CT IMAGES OF TRAUMATIC BRAIN INJURIES

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Scopul acestui articol este de a dezvolta algoritmi având ca punct de plecare metode din teoria sistemelor dinamice haotice (serii de timp) şi de a-l aplica la studiul imaginilor CT reprezentând traumatisme cerebrale. Studiul statistic a arătat că dimensiunea de corelaţie a atractorului asociat unei serii de timp discriminează între ţesuturile traumatizate şi cele normale.

The goal of the paper is to develop methods and algorithms based on the theory of chaotic dynamical systems theory (time series) and to apply them to study CT images of traumatic brain injuries. The statistical analysis shows that the use of the correlation dimension of the attractor of a time series improves the diagnosis for the traumatic brain injuries.

Key words: Time series, attractor, correlation dimension, traumatic brain injuries.

1. Introduction

Nonlinear analysis of time series as well as fractal dimension analysis have been successfully used in the last decades to investigate single and multivariable signals ([1],[4],[10],[11],[12],[15],[16],[21]). Nonlinear methods were developed in the last decades as part of deterministic chaos theory to study the behavior of chaotic dynamical systems from physics, biology, medicine and chemistry. In medical imaging (as part of noninvasive medicine), these techniques can be applied in the analysis of CT and MRI images. In this article, we investigate the possibility of applying nonlinear methods to analyze CT images of traumatic brain injuries.

The fidelity limits of CT investigations are related to several external factors as the technical limits of the CT equipment, collaboration with the patient, the attention and experience of the imagistic doctor or the CT protocol. Therefore,

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more than 50% of traumatic brain lesions detected during medical-legal autopsies are constantly under-diagnosed or have an incorrect and incomplete CT diagnosis ([18], [19]). This is one of the main arguments for an improvement of the noninvasive diagnosis methods.

2. Mathematical background

If (X,d) is a metric space (state space) and $T \neq \varnothing$ is a time set, a dynamical system is a map $\Psi : T \times X \to X$ such that $\Psi(0,x) = x$ and $\Psi(t,\Psi(s,x)) = \Psi(t+s,x)$, $\forall t,s \in T, \forall x \in X$.

A dynamical system Ψ is called chaotic if:

- Ψ is sensitive dependent to the initial conditions
- Ψ is topological transitive
- Ψ generates dense periodic orbits

A set $K \neq \varnothing$ is called the attractor of the system Ψ if it is closed, Ψ-invariant and there exists a neighborhood V of K such that $\lim_{t\to\infty} d(\Psi(t,x),K) = 0, \forall x \in V$. If the attractor has a non-integer Hausdorff dimension then it is called a strange attractor.

The basin of attraction of the attractor K is the largest open neighborhood satisfying the above condition.

A physical measure on the space X is a real valued map $\mu : X \to \mathbb{R}$. Let $\tau \in T$ be a delay, let $t \in T$ be a fixed moment and $x \in X$ be a fixed state. A sequence of measurements:

$$\mu(\Psi(t,x)), \mu(\Psi(t+\tau,x)), \mu(\Psi(t+2\tau,x)), \ldots, \mu(\Psi(t+(j-1)\tau,x)), \ldots$$

is called a time series associated to the system Ψ starting from $(t,x) \in T \times X$.

The main mathematical result about strange attractors is Taken's embedding theorem ([20],[7],[8],[4]):

Let $\Psi : \mathbb{R} \times X \to X$ be a dynamical system of class C^2 and let $\mu : X \to \mathbb{R}$ be a measurement of class C^2. Let $t \in \mathbb{R}$ and let $\tau > 0$ be a time delay. If K is the compact attractor of Ψ with box-counting dimension b, then the map $h : K \to \mathbb{R}^{2b+1}$ is generically injective, hence it is an embedding of the attractor in the space $\mathbb{R}^{2b+1}$. This allows one to reconstruct the attractor starting from the time series in a higher dimensional space (after computing the box-counting dimension) ([10],[11],[12]).

After the reconstruction of the attractor, one can compute its correlation dimension by using the formula:

$$D_C = \lim_{\varepsilon\to 0} \frac{\ln C(\varepsilon)}{\ln \varepsilon},$$

where, for every $\varepsilon > 0$, $C(\varepsilon)$ is the correlation integral:

$$C(\varepsilon) = \lim_{n\to\infty} \frac{1}{n^2} \sum_{i,j=1}^{n} H(\varepsilon - \|y_i - y_j\|)$$

where y_i is a point in the embedding of the attractor and H is the Heaviside function.

3. Algorithms for CT images analysis

For a rectangular area of interest in a given CT image we follow the procedure described below:

First, we compute the box-counting dimension by using the box-counting algorithm for gray-level images [4].

Then we divide the area into horizontal strips of pixels of a fixed height L and by concatenating them we obtain a long strip of columns (of height L) of pixels. We generate a time series of length W, denoted by $(\mu(t))_t$ by letting $\mu(t)$ be equal to the mean value of the gray level of the pixels in the t^{th} column ([2],[4],[7],[8],[9],[10],[11],[12],[13],[14],[16],[17],[21]).

Next, we proceed to the reconstruction of the attractor associated to the obtained time series. Thus, we select a certain reconstruction time delay, τ. For a fixed embedding dimension, d, and for an integer t, we consider the vector $(\mu(t), \mu(t+\tau), \mu(t+2\tau), ..., \mu(t+(d-1)\tau))$ which is a point in the d-dimensional embedding space. The attractor K is the set of all the points obtained for $t = 1, 2, ..., W-(d-1)$.

For a particular class of CT images (traumatic brain injuries in our case), the appropriate values of L, τ, d are determined by applying various experimental techniques such as the ones implemented in MatLab (Nonlinear analysis toolbox).

To compute the correlation dimension, D_C, of the attractor we do the following:

Let r be in an appropriate subset R of $\{0, 1, 2, ..., \lfloor 255\sqrt{d} \rfloor\}$ and

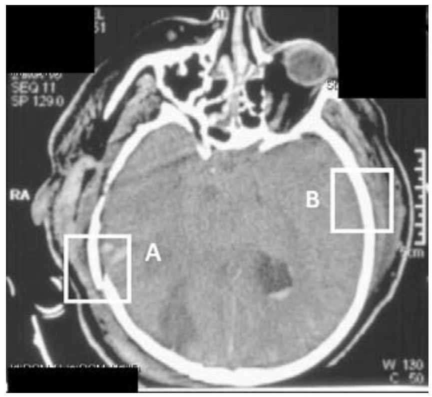
$$C(r) = \text{card}\{(y_i, y_j) \in K^2 \mid \|y_i, y_j\| \le r, i \ne j\}.$$

We approximate the correlation dimension of the attractor K in the embedding dimension d with the slope of the regression line for the set of points, $(\ln r, \ln C(r))$, $r \in R$.

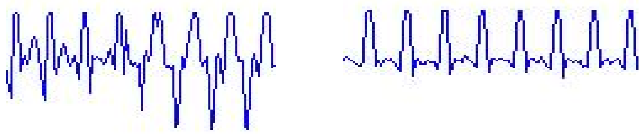
In applications, we compute the correlation dimension $D_C(d)$, for different embedding dimensions d. The discrimination factor between different types of tissues is the slope of the regression line of the points $\{(d, D_c(d)) \mid d = 2, 3, ..., 12\}$.

By using the above procedure, we analyzed CT images of traumatic brain injuries. For every image we selected different areas of normal or injured tissue and computed the box-counting dimension and the correlation dimension of the associated time series.

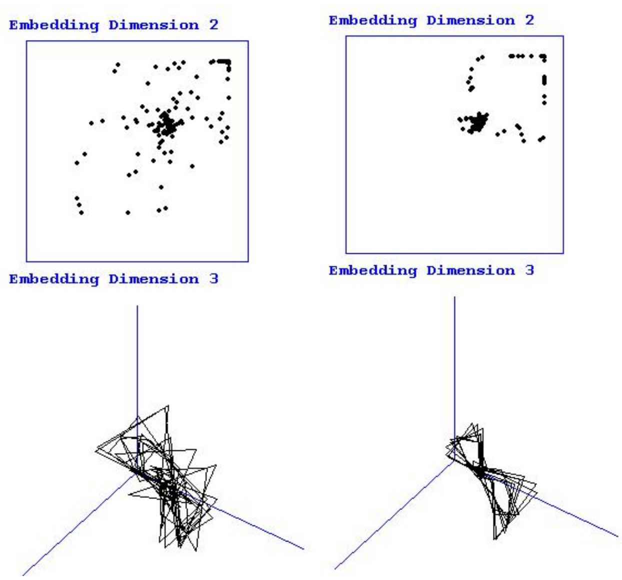
Below, we apply the algorithms to a sample CT image with traumatic brain injuries.



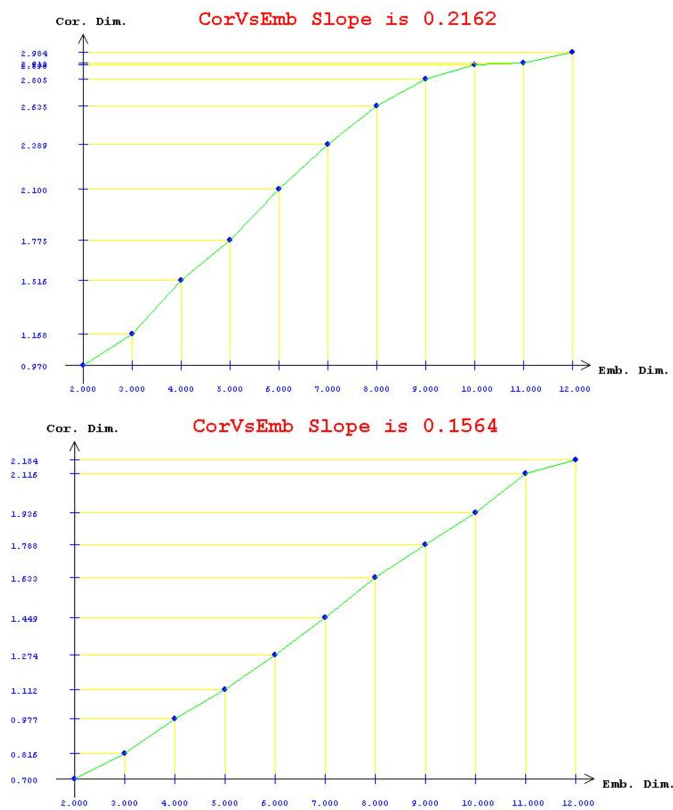
CT image and two area of interest (A – injured, B – normal)



Time series for A (left) and for B (right)



Reconstructed attractor in 2 and 3 dimension for A (left) and for B (right)



The correlation dimension versus the embedding dimension and the slope of the regression line (CorVsEmb) for A (left) and B(right)

4. Statistical analysis and results

By using the results of the previous algorithms, we analyzed a sample of 30 CT images with traumatic brain injuries due to car accidents.

A bootstrap analysis with a jackknife correction for errors ([5]) is run to assess the accuracy of our primary statistical results of ACD. We applied this technique to obtain more reliable statistical estimators (mean, standard deviation and confidence intervals for the mean) in order to make statistical inference, that is, to decide the ACD power of discrimination between the injured and normal tissue.

Our approach is based on a nonparametric bootstrap method which relies on the empirical distribution function of data (refer to Efron and Tibshirani for detailed discussions ([6])).

We considered the case where the sample is drawn from an unspecified probability distribution, with the observations $x_1, x_2, ..., x_n$ viewed as realizations of independent random variables with common distribution function. We denote the interest estimate parameter as $\hat{\theta}$ (mean and standard deviation). A Monte Carlo resampling with replacement was conducted for 1000 generated bootstrap samples, with the jackknife procedure applied at each step (i.e. randomly exclude one of the values x_j and compute the statistic of interest, $\hat{\theta}$ from this resample, denoted $\hat{\theta}^*$). The empirical distribution of the resulting values $\hat{\theta}_1^*, \hat{\theta}_2^*, ..., \hat{\theta}_{1000}^*$ is an approximation to the distribution function of $\hat{\theta}^*$. The bootstrap statistic $\hat{\theta}$ is approximated by taking the average of $\hat{\theta}_1^*, \hat{\theta}_2^*, ..., \hat{\theta}_{1000}^*$ values. The confidence intervals for the mean were computed with the percentile method applied to the bootstrap distribution.

The above algorithm and the random generator of the bootstrap samples was implemented as a MatLab routine. The bootstrap analysis was conducted for both types of tissues (ACD for injured tissue and for normal tissue). The statistics of interest were the mean, standard deviation and the confidence interval for the mean.

The results of the nonparametric bootstrap algorithm are presented below:

	ACD		
	Sample mean (SD)	Bootstrap mean (SD)	Confidence interval 95% (bootstrap mean)
normal tissue	0.156 (0.028)	0.157 (0.030)	(0.154 , 0.159)
injured tissue	0.221 (0.045)	0.220 (0.042)	(0.216 , 0.223)

The values obtained by the bootstrap algorithm proved that the estimators are reliable and we tested the difference of ACD mean between the two groups (Kolmogorov-Smirnov test). The result proved that the approximated correlation dimension discriminate well between the groups (p=0.003).

5. Conclusions

As a consequence of the previous section, we conclude that the approximated correlation dimension discriminate well between injured and normal tissue. Therefore the use of ACD can be considered an important improvement of noninvasive methods for diagnosis in the traumatic brain injuries. By improving the CT noninvasive diagnosis methods, the evolution of the traumatic cerebral lesions and subdural and epidural hematoma can be more accurate predicted during the multiple CT examinations of the first 1-2 days after the car accident. Thus, by having more information with regard to the evolution of lesions with cerebral hemorrhagic, the cerebral trauma can be classified in one of the pre-established patterns determined through a statistic study of autopsies and better forecasts concerning the evolution of patients and the surgical technique to be used in the following period can be made. These projections will allow prediction of cerebral lesions with surgical indication or invasive treatment and the optimum surgery moment, resulting in a higher survival rate for the patients.

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