U.P.B. Sci. Bull., Series C, Vol. 74, Iss. 3, 2012 ISSN 1454-234x

ANALYSIS OF COUPLED OSCILLATORS BY SEMI-STATE VARIABLE METHOD

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Oscilatoarele controlate în tensiune şi rețelele de oscilatoare cuplate sunt intens studiate în decursul ultimilor ani datorită recentei abordări legate de utilizarea acestora în aplicațiile de tip radar şi în sistemele de comunicații. Defazajul dintre fiecare pereche de oscilatoare adiacente poate fi folosit pentru orientarea diagramei de radiație a unei rețele de antene. Pentru analiza acestui tip de circuit, în cadrul prezentei lucrării este propusă metoda variabilelor de semi-stare. Integrarea ecuațiilor neliniare de semi-stare este realizată funcția ode45 din Matlab. Validarea rezultatelor a fost efectuată prin compararea acestora cu cele obținute prin simulări PSpice. Două exemple demonstrative sunt analizate în acest sens.

Voltage controlled oscillators and coupled oscillators arrays are intensely studied in the last few years because of their use in beam-steering application. The phase-shift between each pair of coupled oscillators can be used to control the radiation pattern of a phased antenna array. The analysis of the behavior of this circuit is based on the semi-state variable approach. The nonlinear semi-state equations are integrated by using the ode45 routine from Matlab. The results were compared with the ones resulting from Pspice simulations. Two illustrative examples are given.

Keywords: voltage controlled oscillator, semi-state equations, circuit analysis, phase-shift

1. Introduction

In the last years the voltage controlled oscillators (VCOs) are used to control the phase in microwave antenna arrays as an alternative to electronic beam steering methods. These devices produce oscillatory output signals of high frequency [1] – [7]. If a constant phase progression throughout the oscillator's chain is established, the radiation pattern of a phased antenna array is steered in a particular direction. In the case of the linear array (Fig. 1), directing the beam to

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an angle α off broadside requires a phase shift Δφ between the adjacent oscillators. The angle α and the phase shift Δφ are related by the equation:

$$\alpha = \arcsin\left(\frac{\lambda_0}{2\pi d}\Delta\varphi\right), \qquad (1)$$

where d – represents the distance separating two antenna and λ_0 is the free-space wavelength [7], [8] – [10].

Simple Van der Pol oscillators, coupled through a resonant network that produces a constant magnitude and phase delay between the oscillators, provided a satisfactory model for a lot of applications [3], [7]. The analysis of this circuit can be performed using methods based on the semi-state variables.

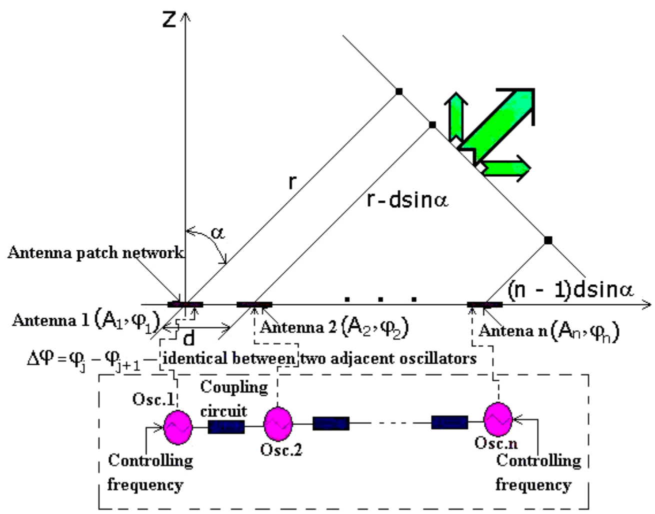

Fig. 1. An array of *n* – coupled oscillators.

The transient behavior of coupled oscillators can be analyzed by using Spice simulator, ADS program or Matlab. In this paper we propose an alternative consisting in a very flexible and general analysis procedure, the semi-state variable method. This procedure is a first step towards the identification of the parameters that describes the Van der Pol model. The computation of these parameters is not so simple because the nature of the non-linear behavior is very complex. The algorithm implies the generation of the semi-state equations in full-symbolic form by using - SESEGP program [11].

Because the semi-state equations are nonlinear, we have integrated them using the ode45 routine from Matlab. Once we obtained the output voltages waveforms for the coupled oscillators, we can compute the phase-shift.

2. Oscillator analysis using the semi-state variable equations

The analysis is performed on a pair of VCOs that have different free-running frequencies and are able to lock at a common frequency (synchronizing frequency) thanks to a resonant resistive circuit (Fig. 2).

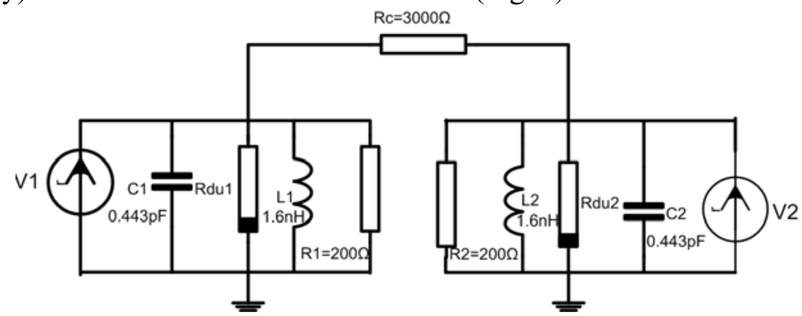

Fig. 2. Two oscillators coupled through a resistive network.

The synchronization is highly dependent on the coupling circuit parameters. The nonlinear characteristic of the two identical voltage-controlled nonlinear resistors has the following expression:

$$i = a_1 v - a_3 v,^3 \quad (2)$$

where: a_1 – is the negative conductance necessary to start the oscillation

$a_3 v^2$ – is the nonlinear conductance which simulates the saturation phenomenon.

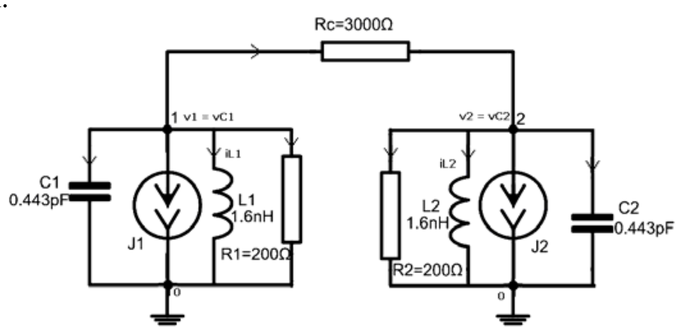

Fig. 3. Substitution of the voltage-controlled nonlinear resistors by ideal independent current sources.

In order to generate the semi-state equations in full-symbolic normal form it is adequate to substitute the voltage-controlled nonlinear circuit elements by ideal independent current sources (according to the substituting theorem [11]), as in Fig. 3.

Running the SESEGP program [11], the following nonlinear semi-state equations of the two oscillators, in the full-symbolic normal form, are obtained

$$\begin{aligned}
\frac{di_{L_1}}{dt} &= \frac{v_{C_1}}{L_1};\\
\frac{di_{L_2}}{dt} &= \frac{v_{C_2}}{L_2};\\
\frac{dv_{C_1}}{dt} &= -\frac{(R_c + R_1)v_{C_1}}{R_c R_1 C_1} + \frac{v_{C_2}}{R_c C_1} - \frac{i_{L_1}}{C_1} - J_1\\
\frac{dv_{C_2}}{dt} &= \frac{v_{C_1}}{R_c C_2} - \frac{(R_c + R_2)v_{C_2}}{R_c R_2 C_2} - \frac{i_{L_2}}{C_2} - J_2;
\end{aligned} \tag{2}$$

Considering $a_1 = -\ 0.0085$ and $a_3 = -\ 0.00071$ and using the ode45 routine from Matlab we integrate the ordinary differential equations (2) with the initial conditions: $v_{C1}(0)$ = 1.75 V, $v_{C2}(0)$ = 2.25, $i_{L1}(0)$ = 0, $i_{L2}(0)$ = 0. The two independent current sources behaviour is described by the following equations:

$$\begin{aligned}
J_1 &= a \cdot u_{C_1} - b \cdot u_{C_1}{}^3\\
J_2 &= a \cdot u_{C_2} - b \cdot u_{C_2}{}^3
\end{aligned} \tag{3}$$

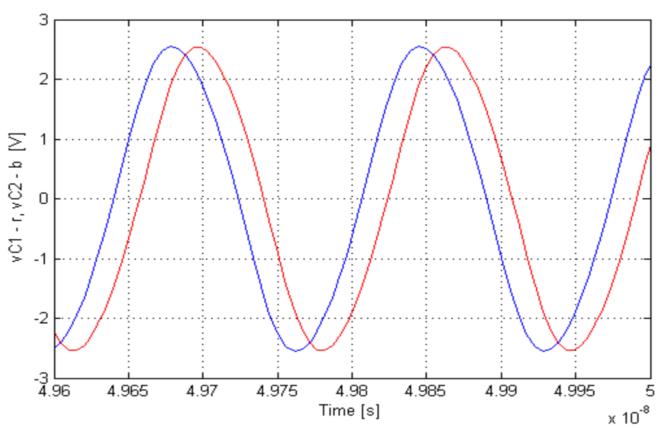

Fig. 4. Waveforms at each oscillator output, with Matlab.

In this way two sinusoidal waves are obtained at the output of each oscillator, at 6.0 GHz (Figs. 4 and 5) and $-34.56°$ out of phase, with amplitudes V_{os1m} = 2.5424 V, and V_{os2m} = 2.5424 V, respectively.

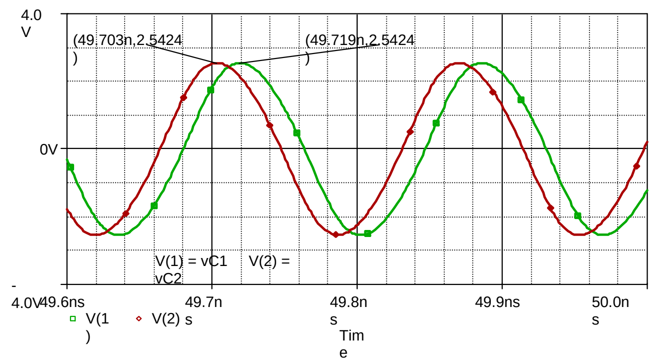

Fig. 5. Waveforms at each oscillator output, with Spice.

In Fig. 5 are shown the sinusoidal waveforms at the output of each oscillator, obtained with PSpice. We can remark that the numeric value of the phase shift obtained with semi-state equation procedure is close to that resulting from Pspice simulation.

We consider another circuit, represented in Fig. 6.

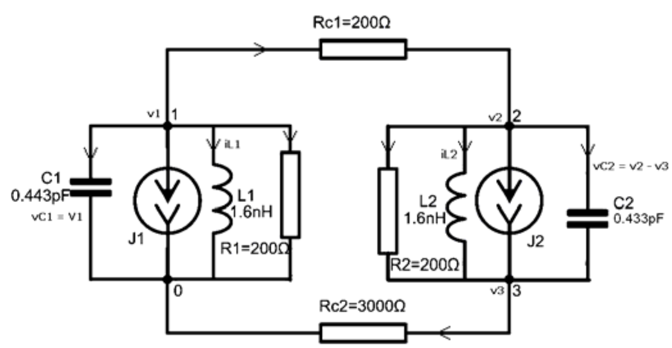

Fig. 6. Two oscillators coupled through two resistors.

Because, there isn't a path from any capacitors to the datum node, made-up only of capacitors, we can not determine the derivates of the independent variables from the equations (4). This is the reason for that, in order to analyze the circuit in Fig. 6 we will use the state equations in the full-symbolic form. To integrate these state equations we consider the following initial conditions: $v_{C1}(0) = v_1(0) = 1.75$ V, $i_{L1}(0) = 0$ A, $v_{C2}(0) = v_2(0) - v_3(0) = 2.25$ V, and $i_{L2}(0) = 0.0$ A and we use the ode45 routine from Matlab. These initial conditions were chosen so that the two oscillators can work. In this way at the output of each oscillator we get two sinusoidal waves at 6.0 GHz and $-28.04°$ out of phase, with amplitudes of: $V_{os1m} = 2.5484$ V and $V_{os2m} = 2.5483$ V, respectively.

$$\begin{aligned}
&\frac{\mathrm{d}v_1}{\mathrm{d}t} = -\frac{R_{C_1}+R_1}{C_1R_1R_{C_1}}v_1 - \frac{1}{C_1}i_{L_1} + \frac{1}{R_{C_1}\cdot C_1}v_2 - J_1;\\
&\frac{\mathrm{d}i_{L_1}}{\mathrm{d}t} = \frac{1}{L_1}v_1;\\
&\frac{\mathrm{d}i_{L_2}}{\mathrm{d}t} = \frac{1}{L_2}(v_2 - v_3);\\
&\frac{\mathrm{d}v_2}{\mathrm{d}t} - \frac{\mathrm{d}v_3}{\mathrm{d}t} = -\frac{R_2+R_{C_1}}{C_2R_2R_{C_1}}v_2 - \frac{1}{C_2}i_{L_2} + \frac{1}{R_2C_2}v_3 + \frac{v_1}{R_{C_2}C_2} - J_2.\\
&\frac{\mathrm{d}v_3}{\mathrm{d}t} - \frac{\mathrm{d}v_2}{\mathrm{d}t} = -\frac{R_2+R_{C_2}}{C_2R_2R_{C_2}}v_3 + \frac{1}{C_2}i_{L_2} + \frac{1}{R_2C_2}v_2 + \frac{v_1}{R_{C_2}C_2} + J_2.
\end{aligned} \tag{4}$$

The nonlinear characteristic of the two voltage-controlled nonlinear resistors is given in Fig. 7. In Figs. 8 and 9 are shown the sinusoidal waveforms at the output of each oscillator, obtained by Matlab and Spice, respectively.

We can remark that in the steady-state behaviour the voltage-controlled nonlinear resistors work on the linear portion of their nonlinear characteristics (Fig. 7). Consequently we can consider that the differential conductance of each oscillator is invariable and negative, and we can apply the complex representation. From the nonlinear characteristics of the two nonlinear resistors, in steady-state, it results that these conductances are equal and close to the value of – 0.005165 S.

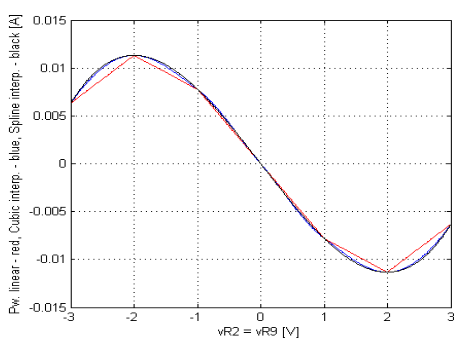

Fig. 7. The nonlinear characteristic of the two c.v. nonlinear resistors: a) Piecewise linear approximation – red; b) Cubic interpolation – blue; c) Spline interpolation - black.

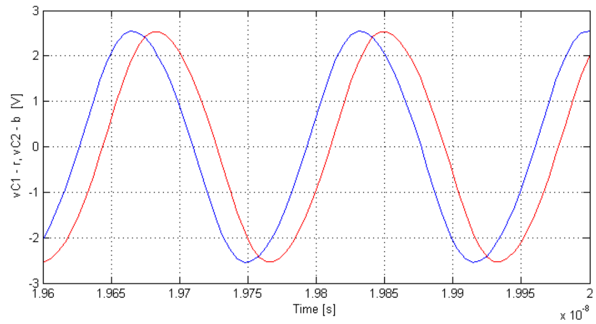

Fig. 8. Waveforms at each oscillator output, Matlab simulation.

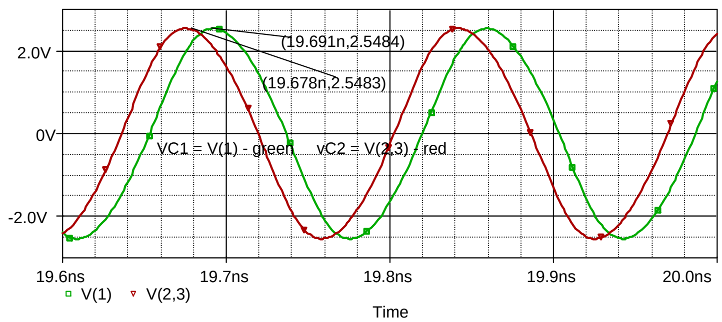

Fig. 9. Waveforms at each oscillator output, Spice simulation.

The phase-shift and the synchronization frequency obtained by simulating with Spice are identically with the ones obtained with Matlab simulations.

3. Conclusions

In this paper an efficient procedure was applied in order to compute the phase shift of the output voltages of two successive oscillators in a 1D antenna array. Our technique is based on a very flexible and general analysis procedure, the semi-state variable method and also on the state variable approach. For the two coupled oscillators the mathematical Van der Pol model is used. VCO designers need to test and implement control laws on simple systems, like the Van der Pol model, that reproduces well the dynamic of the real system. Since the semi-state equations and the state equations are nonlinear, in order to integrate them we call the most performing integrated procedure – the ode45 routine from Matlab. The results obtained by the proposed approach are in a good agreement with those computed by different simulators like Spice and ADS, showing the usefulness of such analysis.

We can remark that the numeric values of the phase shift fit near the synchronization frequency. Because the analysis is made around this particular frequency, at that moment we can say anything about the behavior outside this region

Acknowledgment

The work has been funded by the Sectoral Operational Programme Human Resources Development 2007-2013 of the Romanian Ministry of Labour, Family and Social Protection through the Financial Agreement POSDRU/6/1.5/S/19

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