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STATIC AND TRANSIENT STRESS ANALYSIS OF NECURON-NECURON ELLIPTICAL GEAR TRANSMISSION

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Elliptical gear transmissions although not as widely used as cylindrical ones, offer the advantage of varying the rotation speed within the same turn without the need of complicated mechanisms or driving motors. In the meantime, the elliptical gears present some particularities regarding the teeth geometry that will influence the stress/strain distribution. Since the teeth for an elliptical gear are not identical, a comprehensive analysis of the stress/strain distribution is more complex because it implies analysis for different positions of the gears. Keeping this in mind, the authors present in the paper analytical (for the contact pressure – Hertz formula), 2D and 3D static and transient finite element analysis (FEA) for an elliptical gear transmission with fixed distance between shafts and straight teeth. To fully describe the materials involved in the analysis, a device for measuring the static friction coefficient has been developed by the authors. Analytical results were compared with FEA ones. The results obtained by the authors were compared with the one obtained by other researchers. Conclusions and discussions regarding the stress distribution are presented.

Keywords: Elliptical gears, Non-circular gears, Elliptical gears stress analysis, Finite element, Necuron friction coefficient, Transient FEA

1. Introduction

The elliptical gears are widely used in robotics, printers or other transmissions where a variable speed is required. The geometry and generation of elliptical gears is well documented [1-8]. The literature is not so rich in presenting stress/strain/displacement distributions using finite element analysis for elliptical gears. The particularity of this type of transmissions is that the teeth profiles are not identical for the whole circumference of this type of gear. The differences are more important when the ratio of the ellipses axes are higher. This is why the authors tried to perform a number of finite element analysis (2D and 3D, static and transient structural) for an elliptical gear transmission, with gears made of Necuron.

The **workflow** the authors followed was:

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- Experimental determination of the static friction coefficient for different material pairs (Necuron-Necuron, Necuron-Aluminum, Necuron-Bronze);
- Analytical calculation of the contact pressure (Hertz formula adapted for elliptical gears);
- Validation of the settings for a 2D (static structural, plane stress analysis) by comparison with previous Hertz formula result;
- 3D static structural analysis and comparison with a 2D (static structural, plane stress) analysis;
- 3D transient structural analysis.

2. Initial data

The elliptical gear transmission considered for the analysis is presented in Figure 1. The main geometric elements of the transmission are detailed in Table 1.

The gears material considered for the case study was Necuron, with main mechanical properties presented in Table 2

Table 1

Main geometric properties of the gear

Number of teeth/lobes	38/2
Addendum Coefficient	1
Clearance Coefficient	0.335
Module	2 mm
Pressure Angle	20 degrees
Thickness	8.5 mm
Major/Minor Pitch Radius	45.5/29.5 mm

Table 2

Main mechanical properties for Necuron (determined experimentally by authors)

Density	1109 kg/m^3
Poisson coefficient	.23
Young Modulus	1424.37 MPa
Yield stress	53.0046 MPa

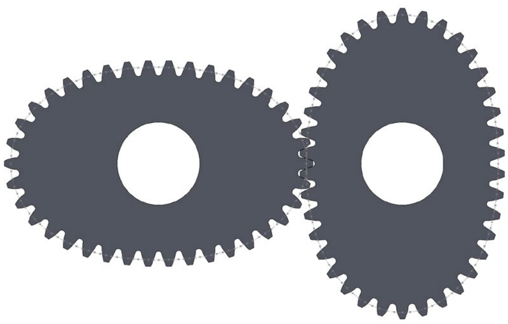


Fig. 1 The elliptical gear transmission

Necuron is a polymeric material with high strength to bending, compression and abrasion.

It also has a very fine structure a smooth and paintable surface and very good processing properties. Necuron contains no halogens, plasticizer or solvent, and it is manufactured fluorocarbohydrate-free. Necuron does not contain any fillers that release harmful dust during machining.

It is used mainly for tools for sheet metal deformation (in the automotive industry for example), molds for plastic deformation of metals, models for copying etc. It can be glued with K8 and K13 adhesives.

All the above mentioned properties make Necuron a material to be considered for many applications, including gear transmissions.

One mention that the gears were generated using a reference rack.

3. Experimental work

In order to determine the static friction coefficient between the pair of gears the authors designed and realized a specialized device, presented in Figure 2.

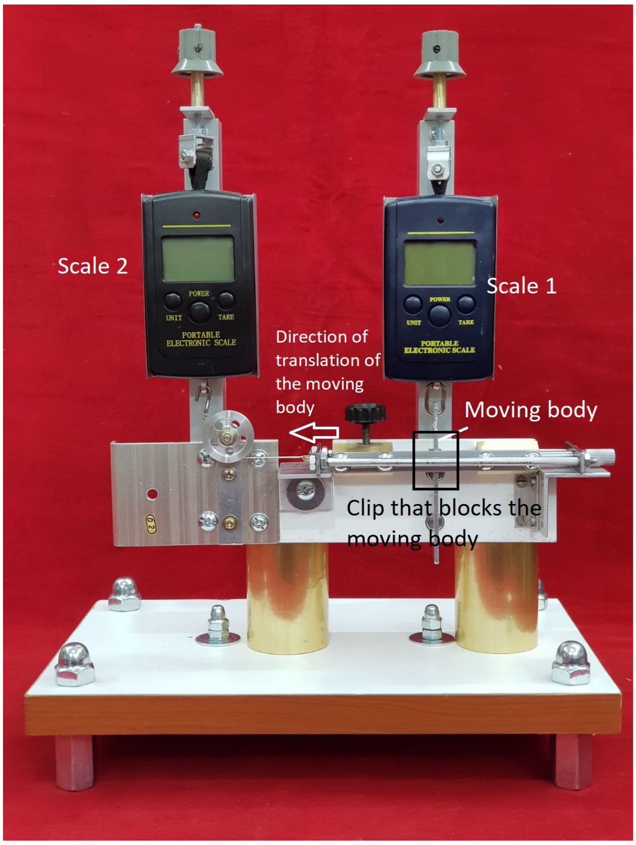


Fig. 2 Apparatus for determining the friction coefficient

Basically, a so called moving body is clipped with a system of two jaws actioned through Scale 1, capable to measure with precision the normal force actioning on the moving body. The same moving body is pulled with a controlled force through Scale 2. When the pulling force is greater than the friction force, the moving body will start to slip. Every test is video recorded. The video file is after that analyzed with the VLC software that automatically detects movement. At that very moment, the value displayed by Scale 1 and Scale 2 are read.

These two values will allow a very straightforward calculation of the static friction coefficient. In order to assure reliable results, 201 tests have been performed for every pair of materials and the values provided were statistically analyzed (Kolmogorov-Smirnof and Shapiro-Wilk tests) for evaluating the normal distribution of data [9].

Fig. 3 presents the result of the Shapiro-Wilk test. The friction coefficient resulted from the experiment, for a Necuron-Necuron material pair was 0.107.

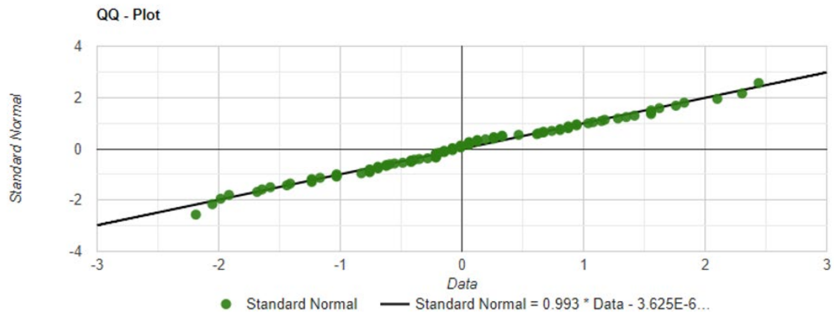


Fig. 3 QQ plot for the Shapiro-Wilk test

4. Calculation of the contact pressure with Hertz formula

As already mentioned, for elliptical gear, the teeth profiles varies along the pitch curve, so contact pressure is also different for different positions along the pitch path. In our case the gears are positioned such as the major axis are perpendicular, with the major axis of the driving gear horizontal. The Hertz formula [1] was used by the authors as a benchmark for the results obtained using the Finite Element Method (FEM). The Hertz formula [1] was used by the authors as a benchmark for the results obtained using the Finite Element Method (FEM).

The following equations were used :

$$b = 2 * \sqrt{2 * F_n * \frac{R}{\pi * B * E^{'}}} \tag{1}$$

Where:

$$R = \frac{R1 * R2}{R1 + R2} \tag{2}$$

R1 and R2 being the radii of the equivalent contact circles (see Figure 4). R1=8.96 mm, R2=32.505 mm.

$$E' = \frac{E}{2*(1-\nu^2)} \quad (3)$$

Where E is the Young modulus of Necuron (both gears are made of the same material), ϑ is the Poisson coefficient for Necuron (0.23, see Table 2).

$$\sigma = \frac{4}{\pi} * \frac{F_n}{2*B*b} \quad (4)$$

Where F_n is the normal force, B the gear width (8.5 mm, see Table 1) and b, calculated with equation (1) is the semi width of the contact region.

$$F_n = F * \cos(\alpha) \quad (5)$$

$$F = \frac{M_t}{R'} \quad (6)$$

Where:

- α is the pressure angle (20 degrees - see Table 1);
- M_t the torque the gears transmit. Mt=5000 N*mm.;
- R' the distance from the gear center to the point where two teeth are in contact. R'=29.02 mm.

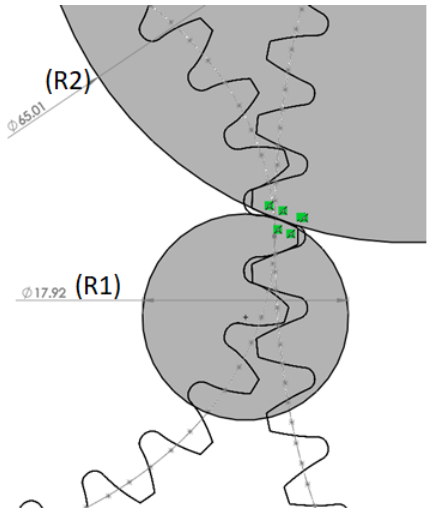


Fig. 4 Geometric details for the Hertz formula

After calculation, the contact pressure was 18.015 MPa. At this stage, the following statements should be made:

- For the elliptical gears, the teeth are not identical, so for one quarter of the gear, a contact pressure for each pair of teeth should be calculated;

- The case one considered was for the gears positioned such as the major axis are perpendicular, with the major axis of the driving gear horizontal;
- The R1 and R2 were determined as radii of two circles defined by three points of the tooth profile, in the very vicinity of the contact point between the profiles of the teeth.

5. Finite element model validation using a 2D model (plane stress)

In order to validate a finite element model for the future analysis, the authors considered a static structural plane stress analysis for only one pair of teeth being in contact (ANSYS 2021R1 was used). In the case of a real gear, the contact pressure would be spread over more than a pair of teeth, so it would have not been suitable for a comparison with the one calculated with the Hertz formula - equations (1) to (6).

The gears material considered for the case study was Necuron, with main mechanical properties presented already in Table 2

Comparison between analytical results for the contact pressure and FEA have also been done in [10, 11, 12], some authors even still using photoelasticity [11]. Although this approach do not capture the stress or contact pressure variation on the teeth width, it has the great advantage that significantly reduces the calculation effort. Different element sizes were considered, from 0.4 to 1mm (see Table 3), and the results were compared with the analytical one (Hertz formula).

Fig. 5 presents the geometric model devised for this analysis.

To further reduce the number of elements, one used quadratic elements, with curvature capturing feature ON. The contact between the teeth was modeled as Frictional (with the frictional coefficient 0.107) and Augmented Lagrange formulation has been used [13].

The results (contact pressure) obtained for different element sizes are presented in Table 3.

Fig. 5 Geometric model for the plane stress analysis in the case of only one pair of teeth being in contact

Table 3

Contact pressure for different element sizes

Element size [mm]	Contact pressure (MPa]	Absolute difference to Hertz formula [MPa]	Relative difference to Hertz formula [%]
0.4	18.89	0.88	4.87%
0.5	18.40	0.38	2.13%
0.6	18.33	0.32	1.77%
0.7	18.65	0.64	3.55%
0.8	18.09	0.07	0.39%
1.0	20.96	2.94	16.33%

The analysis of the values obtained for the contact pressure varying the element size, allows us to appreciate that element sizes between 0.4 to 0.8 mm will produce reliable results. The domain 0.4-0.8 mm for the average element size will be used for the future analysis, static or transient, 2D or 3D.

6. Comparison between stress distributions for 2D (plane stress) and 3D static structural analysis

Since the final analysis will be a 3D Transient structural, the authors considered of interest to compare results obtained after a 2D and a 3D Static structural analysis. This comparison was intended to offer confidence in the results. Again, quadratic hexa/prism elements were used, with Augmented Lagrange formulation. Table 4 presents a comparison between contact pressure and Von Mises equivalent stress for a 2D (plane stress) and a 3D Static structural analysis. The table also offers information about the absolute and relative difference between results. This time we are dealing with complete gears, not only with one pair of teeth, this is why the values are smaller than the ones in Table 3, since the contact pressure, or equivalent stress is spread over more than one pair of teeth. One also mentions that for the contact pressure the values for the driving gear were considered in Table 4 (and Figure 6).

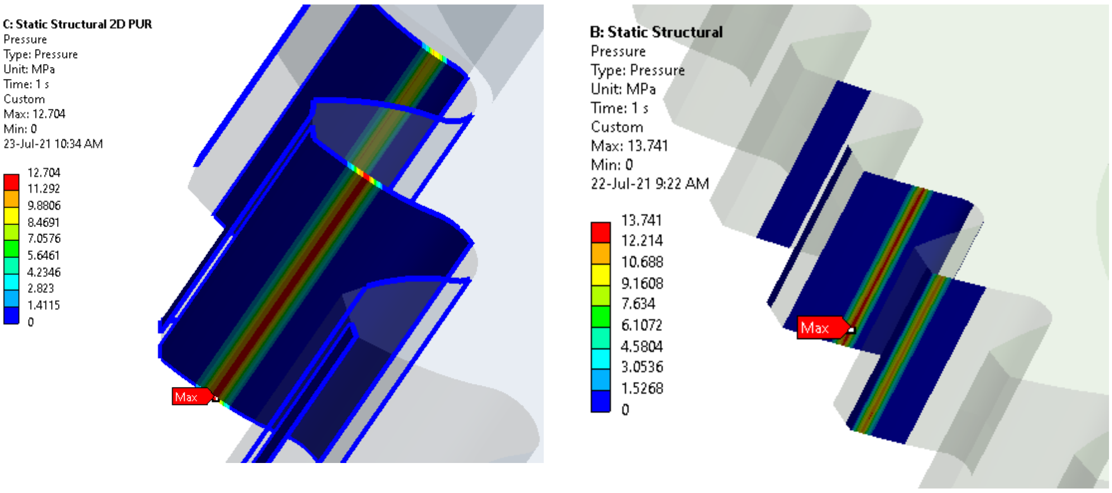
Table 4

2D versus 3D results for contact pressure and equivalent stress (maximal values)

	Contact pressure [MPa]	Equivalent stress Von Mises [MPa]
2D	12.704	13.767
3D	13.741	13.857
Absolute difference [MPa]	-1.037	-0.090
Relative difference [%]	8.10	0.60

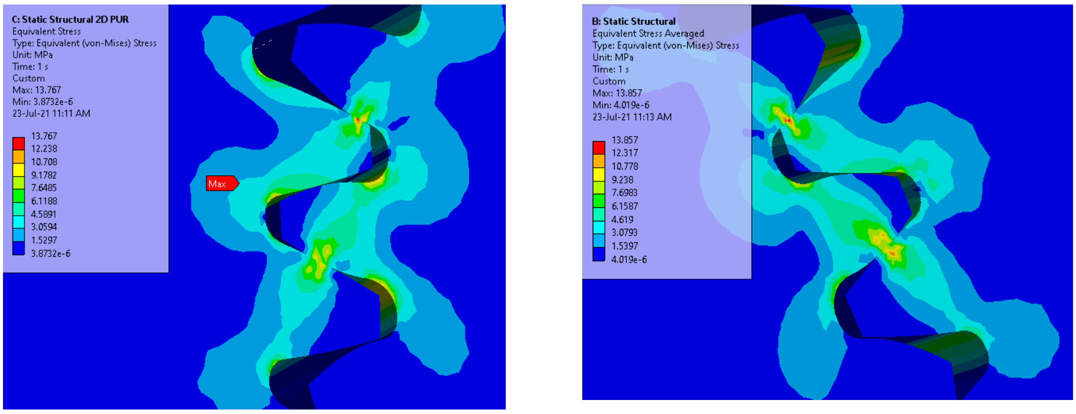
Fig. 6 displays the contact pressure distributions for a 2D and a 3D static structural analysis.

Fig. 7 presents the equivalent stress (Von Mises) distribution for the plane stress (2D) analysis, respectively the 3D analysis.



a. Contact pressure 2D analysis b. Contact pressure 3D analysis

Fig. 6 Contact pressure, 2D and 3D



a. Equivalent stress 2D analysis b. Equivalent stress 3D analysis

Fig. 7 Equivalent stress, 2D and 3D

7. Transient structural analysis

As already mentioned, in the case of elliptical gears, the teeth are not identical, so a comprehensive analysis implies in the case of a static structural analysis a number of static structural analysis, for different positions appreciated as relevant, of the gears [14, 15]. One solution would be a transient structural analysis. In the case of the transient structural analysis, if it is performed for a complete rotation, it will offer access to any desired position results. More than that, in the case of the transient structural analysis, the effect of the angular speed of the gears will be measured.

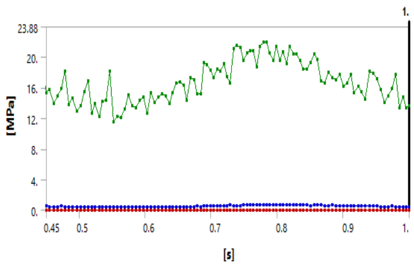
The transient structural analysis has been performed with the following settings:

- Element size in the contact area of the teeth: 0.5 mm;

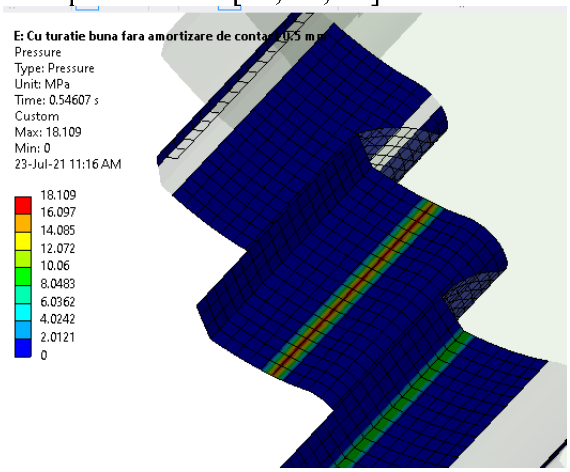
- Quadrilateral elements, with curvature capture;
- Pair of materials: Necuron-Necuron;
- Angular speed of the gears: 6.28 rad/sec (1 rot/sec), starting from 0 and growing linearly to the maximal value in 0.5 seconds;
- Total duration of analysis: 1 sec;
- Statical frictional coefficient for the contact: 0.107;
- Resistant torque 5 Nm.

Contact pressure, equivalent (VonMises) stress, frictional stress and sliding distance have been obtained and assessed.

Fig. 8.a presents the variation of the contact pressure, with a focus on the final 0.5 seconds, when the rotational speed of the gears is constant, while in Fig. 8.b, the contact pressure distribution is depicted (for the moment 0.54607 sec). As one will observe for other quantities (equivalent stress especially), the variation in time of the contact pressure presents peaks. This analysis of the results shows that these peaks occur when a tooth engage (touches) another tooth on the other gear. Since we are dealing with a transient structural analysis, it is normal that this moment will generate a peak value. Fig. 9.a and 9.b present the distribution of the contact pressure and equivalent stress over the width of the teeth for a static structural and respectively a transient structural analysis. The curves are similar with the ones presented in [16, 19, 20].



a. Contact pressure variation for the final 0.5 seconds



b. Contact pressure (for time 0.54607)

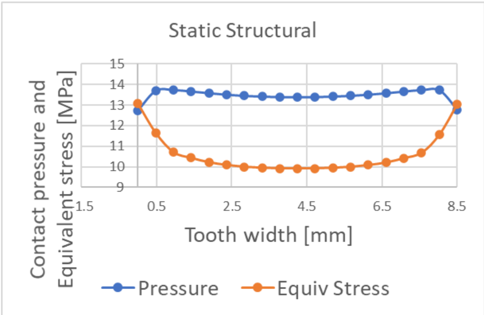
Fig. 8 Contact pressure variation and distribution

In Table 5 values of contact pressure and equivalent stress for static structural respectively transient structural analysis are compared. One mention should be made: for the transient structural analysis one has considered the moment when the gears have the same reciprocal position as in the case of the static structural analysis.

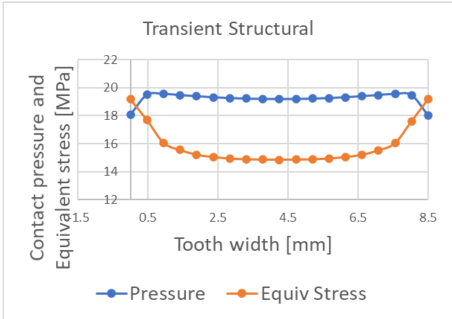
Table 5

Static and Transient structural analysis results comparison

	Contact pressure [MPa]	Equiv. stress Von Mises [MPa]
Static structural	13.741	13.193
Transient structural	19.564	19.186
Relative difference [%]	42.3	45.42



a. Contact pressure and equivalent stress distribution over the teeth width - Static structural



b. Contact pressure and equivalent stress distribution over the teeth width - Transient structural

Fig. 9 Contact pressure and equivalent stress distribution

8. Discussions

In the case of the 2D (plane stress) analysis, for the model with only one pair of teeth in contact (see Fig. 3), for average elements dimensions between 0.4 and 0.8 mm, the results are in good accordance with the analytical ones - equations (1) to (6). As can be seen in Fig. 6.b, the maximal value for the contact pressure is registered at a short distance from the tooth margin, in accordance with [14, 16]. The results obtained with 2D and 3D analysis are very similar (see Table 4) for the equivalent stress. For the contact pressure the difference between 2D and 3D analysis are slightly more important (see also Table 4). Our interpretation is that in the case of the 2D approach, the contact stress is constant along the tooth axial direction, while in the case of the 3D approach an elliptical like shape for the contact pressure, considering the margin effect appears. This behavior is also confirmed in [14].

As can be seen in Figs. 8.a the contact pressure (the equivalent stress also) exhibits abrupt peak values. This kind of time evolution is also present in [17]. These values correspond to the moments when the teeth of the driving gear engage the teeth of the driven gear. The abrupt peaks, the authors reckon, are due only partially to the shock of the teeth engaging other teeth. The other reason is that the tooth edge where the equivalent stress is maximal is prone to produce stress singularities.

Another issue of interest was to determine the moment (or position of the two gears) for the period with constant value for the angular velocity, when the equivalent stress reaches the maximal value. The result is presented in Figure 10 and is in accordance with other sources [18].

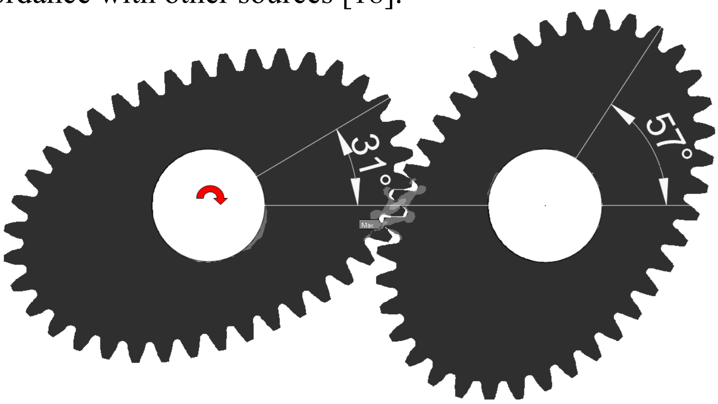


Fig. 10 Gears position for maximal stress (transient analysis)

9. Conclusions

The transient structural analysis is a time consuming approach. For the considered model, with the average element size of 0.5 mm, the running lasted 19 hours (on an above the average PC). This effort is justified, on one hand by the fact that the transient structural analysis allows the assessment of the structure response for the entire duration of the movement, and on the other one, by the fact that it takes into account the dynamic effect, so we can evaluate the influence of the angular speed for example.

The relative difference between the two cases (as can be seen in Table 5) is significant, surpassing 42%.

The fact that elliptical gears have teeth with different geometries along the pitch curve complicates the stress analysis in the sense that a simple static approach is not sufficient since it will capture the stress distribution only in one of the positions of the two gears. The solution is either a succession of static analysis, or a transient one.

The transient analysis also offers the advantage that it will capture the dynamic effect of the gears in movement. A transient analysis has also drawbacks, the most important being the fact that it is time consuming. This can be, at least partially, surpassed by simplifying the gears geometry, or by performing the analysis for a reduced time interval, where the stress is assumed to be maximal.

In the case of elliptical gears with straight teeth, for calibration or static analysis, a plane stress approach is also useful. For the transient structural case, the plane stress analysis has convergence problems.

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