THE SUSPENSIONS DYNAMICS IN BIOGAS REACTORS

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Lucrarea prezintă soluția analitică pentru curgerea laminară şi permanentă în reactoare cilindrice şi soluția numerică a curgerii în reactoare cu forme mai complicate. Peste această curgere se suprapune mişcarea naturală a unor suspensii solide şi se obține dinamica acestora în curentul de lichid. În cazul unui reactor pentru producerea de biogaz soluțiile obținute pot fi utilizate în vederea optimizării procesului de metanogeneză, respectiv obținerea unei cantității mai mari de biogaz cu o reducere a consumului energetic.

The paper presents analytical solution for the laminar permanent flow in cylindrical reactors and the numerical solution for flow in reactors with more complicated geometry. Over this flow the natural movement of solids suspensions overlaps and their dynamics in liquid is obtained. In the case of a reactor used for biogas production the obtained solutions can be used to optimize the anaerobic digestion process and achieve a larger quantity of biogas with a lower energy consumption.

Keywords: biogas, homogenization reactors, suspensions dynamic

1. Introduction

The issue of homogeneous concentration of suspension in mixing reactors could be applied in chemical industry, metallurgy, food industry [1] and recently to anaerobic digesters (AD) in order to produce biogas. The hydraulic and thermal homogenization of two-phase or three-phase fluid (liquid + suspensions + bubbles gas) is carried out with mixing/homogenization devices as intubated or free propellers [1].

In the simple geometry AD case, the liquid fluid flow solutions obtained by the authors differ from those published in special publications [2-4].

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In Romania, the first research on the numerical solving of this problem occur in 2000 [5], then continue, but refers only to liquid flow in the AD conic bottom [5, 6].

2. The flow analytical solution in cylindrical reactors

A cylindrical reactor with height much longer than diameter ($H>>2r_1$) is considered. The permanent flow of the incompressible and heavy liquid is provided by an intubated propeller (fig. 1) which assures an ascendant flow between the two coaxial cylinders; the rotating component of the liquid velocity in the inner tube is neglected, at least at the Z_t level.

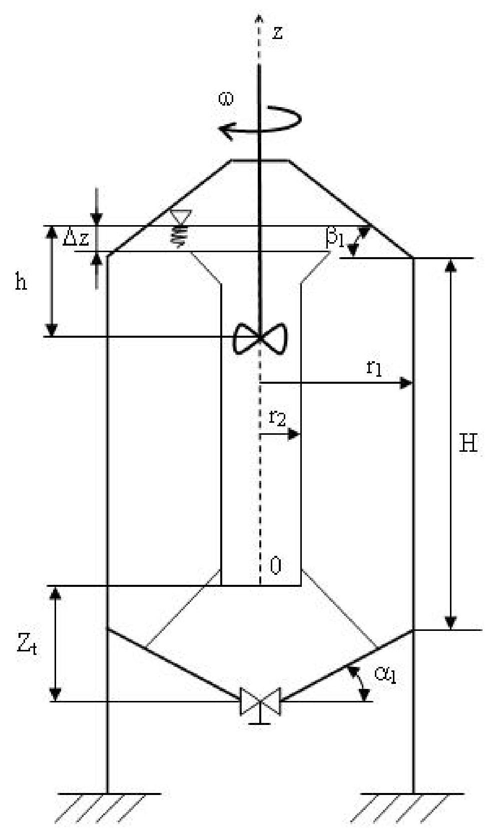

Fig. 1 – Intubated propeller reactor

Navier-Stokes equations and equation of mass conservation written in cylindrical coordinates are:

$$
\begin{aligned}
&v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}+v_z\frac{\partial v_r}{\partial z}-\frac{v_\theta^2}{r}=X_m-\frac{1}{\rho}\frac{\partial p}{\partial r}+\upsilon\Delta v_r;\\
&v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}+v_z\frac{\partial v_\theta}{\partial z}+v_r\frac{v_\theta}{r}=Y_m-\frac{1}{\rho}\frac{\partial p}{\partial \theta}+\upsilon\Delta v_\theta;\\
&v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}=Z_m-\frac{1}{\rho}\frac{\partial p}{\partial \theta}+\upsilon\Delta v_z;\\
&\frac{\partial v_r}{\partial r}+\frac{1}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{\partial v_z}{\partial z}+\frac{v_r}{r}=0
\end{aligned}
, \qquad (1)
$$

with masic unit force components $X_m=0,\quad Y_m=0,\quad Z_m=-g.$

Considering the simplified hypothesis of the laminar flow, the stream lines are parallel with Oz, so v_r=0 and v_θ=0.

In addition, if the motion has axial symmetry ($v_r=0, v_\theta=0$), Navier-Stokes equations become

$$
\begin{aligned}
&0=\frac{1}{\rho}\frac{\partial p}{\partial r};\\
&0=\frac{1}{\rho}\frac{\partial p}{\partial \theta};\\
&0=-g-\frac{1}{\rho}\frac{\partial p}{\partial z}+\upsilon\left(\frac{d^2v_z}{dr^2}+\frac{1}{r}\frac{dv}{dr}\right);
\end{aligned}
, \qquad (2)
$$

From the continuity equation, it results that v_z component depends only on the radius r.

The last equation, written as

$$
\frac{d^2v_z}{dr^2}+\frac{1}{r}\frac{dv}{dr}=\frac{g}{\nu}+\frac{1}{\eta}\frac{dp}{dz}\,, \qquad (3)
$$

shows that the equality is possible only if each member of the equation is a constant,

$$
\frac{d^2v_z}{dr^2}+\frac{1}{r}\frac{dv}{dr}=k;\quad \frac{g}{\nu}+\frac{1}{\eta}\frac{dp}{dz}=k \qquad (3')
$$

To integrate, the differential equation of velocity is written as

$$
\frac{1}{r}\frac{d}{dr}\left(r\frac{dv_z}{dr}\right)=k
$$

It is obtained:

$$
v_z=\frac{k}{4}r^2+c_1\ln r+c_2 \qquad (4)
$$

The boundary conditions for velocities

$$r = r_1; \quad v_z = 0; \quad r = r_2; \quad v_z = 0; \tag{5}$$

provide the constants c_1 and c_2:

$$c_1 = -\frac{k}{4}\frac{\left(r_2^2 - r_1^2\right)}{\ln\frac{r_2}{r_1}}; \qquad c_2 = \frac{k}{4}\frac{r_2^2\ln r_1 - r_1^2\ln r_2}{\ln\frac{r_2}{r_1}} \tag{6}$$

Thus, the distribution for velocities is obtained as:

$$v_z = \frac{k}{4}\left[r^2 - \frac{r_2^2 - r_1^2}{\ln\frac{r_2}{r_1}} - \ln r + \frac{r_2^2\ln r_1 - r_1^2\ln r_2}{\ln\frac{r_2}{r_1}}\right], \tag{7}$$

solution that is different from those published in [2, p151], [3, p. 276], [4, p. 59].

Denoting:

$$\alpha = \frac{r_2^2 - r_1^2}{\ln\frac{r_2}{r_1}}; \quad \beta = \frac{r_2^2\ln r_1 - r_1^2\ln r_2}{\ln\frac{r_2}{r_1}}, \tag{8}$$

the solution has the expression

$$v_z = \frac{k}{4}\left(r^2 - \alpha\ln r + \beta\right) \tag{9},$$

with constants α and β positive by defined and k constant negative by defined.

Liquid flow rate is calculated considering the elementary flow rate that crosses the infinitesimal distance *dr:*

$$Q = \int_{r_1}^{r_2} v_z 2\pi r dr = \frac{\pi k}{8}\left[r_2^4 - r_1^4 - \frac{\left(r_2^2 - r_1^2\right)}{\ln\frac{r_2}{r_1}}\right] \tag{10},$$

which is same expression as the one published in [4].

The average velocity has the expression:

$$v_{zm} = \frac{k}{8}\left[r_2^2 - r_1^2 - \frac{\left(r_2^2 - r_1^2\right)}{\ln\frac{r_2}{r_1}}\right] \tag{11},$$

Reynolds number of flow is expressed with the relationship:

$$\mathrm{Re} = \frac{2}{\upsilon\pi}\frac{Q}{r_2 + r_1} \tag{12}$$

It is noted that velocity has a maximum value situated at the distance r_m:

$$r_m = \sqrt{\frac{\alpha}{2}} = \sqrt{\frac{r_2^2 - r_1^2}{\ln r_2 / r_1}} \tag{13}$$

which does not depend on the Reynolds number but only on the reactor geometry.

For large values of the ratio H / (r_2-r_1), the conic bottom influence over the flow between cylinders is negligible.

The maximum velocity value is:

$$v_{z\max} = \frac{k}{8}\left[\alpha - \alpha \ln\frac{\alpha}{\beta} + 2\beta\right] \tag{14}$$

and depends on the flow rate only through constant *k* according to (10).

The relations (13) and (14) correspond to the case $H/(r_1 - r_2) \to \infty$.

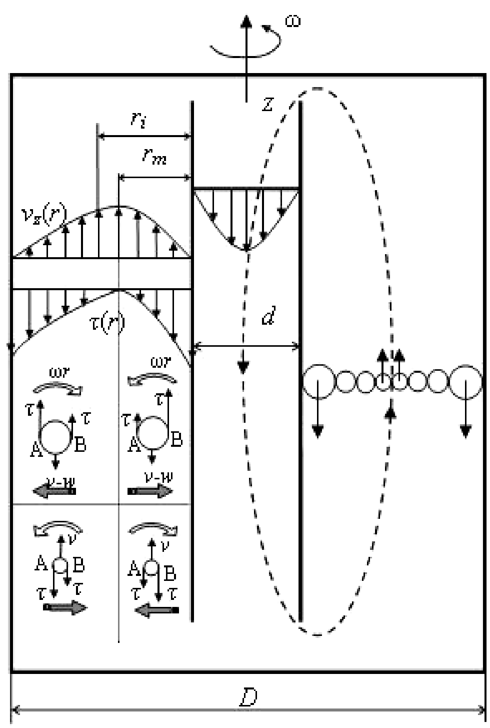

Fig. 2 – The particles motion in the cylindrical part of intubated propeller reactor

In Figure 2 were represented variations of velocity $v_z(r)$ and friction viscous shear stress $\tau(r)$ with

$$\tau(r) = \eta\frac{\mathrm{d}v_z}{\mathrm{d}r} = \frac{k\eta}{4}\left(2r - \frac{\alpha}{r}\right) \tag{15}$$

wich are variable with the radius ($\tau_A \neq \tau_B$), in the case of a solid spherical particle which moves into liquid. τ is a function of viscosity and also a function of flow rate through the *k* constant according with (10).

Over this solved flow solid spherical particles of different diameters and/or density, with terminal velocity *w* overlap.

Friction viscous shear stress τ imparts an instant rotational motion that, composed with vertical motion, produces the radial particles movement (Magnus effect), depending on their size/density.

Thus a complicated particles motion (translation and rotation) results, which leads to radius and high variable concentrations.

If the solid suspensions are the subsistence for bacteria from the reactor, then the complicated particle motion is favorable to their development and anaerobic digestion process.

3. The flow numerical solution

In the case of reactors with complicated geometry (fig. 1), flow in the conic bottom area is no more one-dimensional and the Navier-Stokes equations solution for the whole flow field is numerically obtained applying the method of Taylor finite series expansion [8].

It is considered the reactor from Figure 1, where the liquid phase flow is laminar, permanent, with axial symmetry.

The following dimensionless terms are introduced:

$$\begin{aligned} & r = \frac{R}{R_t},\ z = \frac{Z}{R_t}, \\ & v_r = \frac{V_r}{V_m},\ v_z = \frac{V_z}{V_m}, \\ & p = \frac{P}{\rho V_m^2},\ \psi = \frac{\Psi}{V_m R_t^2} = \frac{\Psi \pi}{Q},\ \mathrm{Re} = \frac{V_m R_t}{\upsilon} \end{aligned} \qquad (16)$$

where

- R_t – the inner tube radius,
- v_r and v_z - the dimensionless components of velocity,
- V_m - the average velocity,
- Ψ - the stream function,

Navier-Stokes equations are dimensionlessed. By eliminating the pressure and introducing the dimensionless streamfunction, one obtains the partial derivatives equation:

$$\psi_{r^4}^{IV}+2\psi_{r^2z^2}^{IV}+\psi_{z^4}^{IV}-2\frac{\psi_{r^3}^{III}+\psi_{rz^2}^{III}}{r}+\frac{3}{r^2}\left(\psi_{r^2}^{II}-\frac{\psi_r^{I}}{r}\right)=$$
$$\mathrm{Re}\left(\frac{3\psi_z^{I}\psi_r^{I}}{r^3}+\frac{\psi_{rz}^{II}\psi_r^{I}-2\psi_{z^2}^{II}\psi_z^{I}-3\psi_{r^2}^{II}\psi_z^{I}}{r^2}+\right. \tag{17}$$
$$\left.+\frac{\psi_{r^3}^{III}\psi_z^{I}-\psi_{z^3}^{III}\psi_r^{I}-\psi_{r^2z}^{III}\psi_r^{I}+\psi_{rz^2}^{III}\psi_z^{I}}{r}\right)$$

The streamline function verifies identically the conservation mass equation.

The boundary conditions of velocity are:

- velocity is zero on the solid walls,
- out of the inner tube $v_r = 0$ and v_z has a parabolic distribution given by the Hagen-Poiseuille flow.

The following conditions for the stream function were imposed in order to solve the problem numerically:

- $\Psi=0$ on the solid wall of reactor
- $\Psi=\pi$ on the lateral of the inner tube

Flow field is covered by a rectangular network with $\Delta r=h$ and $\Delta z=k$ steps.

Finite Taylor expansions for Ψ streamfunction are used and therefore the derivatives including fourth order are:

$$\psi_r^{I}=\frac{\psi_1-\psi_3}{2h};\quad \psi_z^{I}=\frac{\psi_2-\psi_4}{2k};\quad \psi_{r^2}^{II}=\frac{\psi_1-2\psi_0+\psi_3}{h^2};$$
$$\psi_{z^2}^{II}=\frac{\psi_2-2\psi_0+\psi_4}{k^2};\quad \psi_{r^3}^{III}=\frac{\psi_9-2\psi_1+2\psi_3-\psi_{11}}{2h^3};$$
$$\psi_{z^3}^{III}=\frac{\psi_{10}-2\psi_2+2\psi_4-\psi_{12}}{2k^3};$$
$$\psi_{r^2z}^{III}=\frac{\psi_5+\psi_6-\psi_7-\psi_8-2\psi_2+2\psi_4}{2h^2k}; \tag{18}$$
$$\psi_{r^4}^{IV}=\frac{\psi_9-4\psi_1+6\psi_0-4\psi_3+\psi_{11}}{h^4};$$
$$\psi_{r^2z^2}^{IV}=\frac{\psi_5+\psi_6+\psi_7+\psi_8-2(\psi_1+\psi_2+\psi_3+\psi_4)+4\psi_0}{h^2k^2}$$

where ψ_1, ..., ψ_{12} represent the streamfunction values for 1, 2 ,..., n points (nodes).

The relations (18) are replaced in the partial derivatives equation (17).

By distributing the corresponding terms of ψ_0 and Re, an algebraic formula associated with the partial derivatives equation is obtained:

$$\alpha\psi_0=\beta+\mathrm{Re}\,\delta \tag{19}$$

where the coefficients α, β and δ have the expressions:

$$\alpha = 6\left(\frac{1}{h^4}+\frac{1}{k^4}-\frac{1}{r^2h^2}\right)+\frac{8}{h^2k^2}-\mathrm{Re}\left(\frac{3}{h^2}+\frac{2}{k^2}\right)\frac{\psi_2-\psi_4}{r^2k};$$

$$\beta = \left[4\left(\frac{1}{h^2}+\frac{1}{k^2}\right)-\frac{3}{r^2}\right]\frac{\psi_1+\psi_3}{h^2}-\left[2\left(\frac{1}{h^2}+\frac{1}{k^2}\right)-\frac{3}{2r^2}\right]\frac{\psi_1-\psi_3}{rh}+$$

$$+4\left(\frac{1}{h^2}+\frac{1}{k^2}\right)\frac{\psi_2+\psi_4}{k^2}-\frac{\psi_9+\psi_{11}}{h^4}-\frac{\psi_{10}+\psi_{12}}{k^4}-2\frac{\psi_5+\psi_6+\psi_7+\psi_8}{h^2k^2}+$$

$$+\frac{\psi_9-\psi_{11}}{rh^3}+\frac{\psi_5-\psi_6-\psi_7+\psi_8}{rhk^2}; \qquad (20)$$

$$\delta_1 = \frac{\psi_1-\psi_3}{4hk}\left(3\frac{\psi_2-\psi_4}{r}-\frac{\psi_{10}-\psi_{12}}{k^2}-\frac{\psi_5+\psi_6-\psi_7-\psi_8}{h^2}+\frac{\psi_5-\psi_6+\psi_7-\psi_8}{2rh}\right);$$

$$\delta_2 = \frac{\psi_2-\psi_4}{k}\left(\frac{\psi_9-\psi_{11}}{9h^3}-3\frac{\psi_1+\psi_3}{2rh^2}-\frac{\psi_2+\psi_4}{rk^2}+\frac{\psi_5-\psi_6-\psi_7+\psi_8}{4hk^2}\right)$$

$$\delta = \frac{\delta_1+\delta_2}{r}$$

The network points (nodes) are situated in vicinity of the reactor walls and treated separately according to the reactor geometry in that area, by applying the reflex principle and continuity streamfunction [8], [9].

Figure 3 represents the flow spectrum for a Re = 15.

The differences between the streamfunction values obtained for the last two iterations are less than 10^{-3}.

Stability and convergence of numerical solution of equation (19) is ensured by solving similar problems [9].

In the reactor cylindrical area the velocity and friction viscous shear stress distributions are similar to those obtained analytically and streamlines are parallel to the axis Oz, so the Magnus effect is conserving and the obtained spectre is physically correct.

Consequently solid particle dynamics is similar (fig. 2). In the conic bottom of the reactor, the particles rotation is negligible, with the tendency of sedimentation.

The distance between the inner tube end and reactor bottom determines the flow spectrum in this area. It is also to be noted that near the free surface, only light particles are drawn into the inner tube.

Thus, in order to accelerate the flow, the outer wall must be inclined in this area.

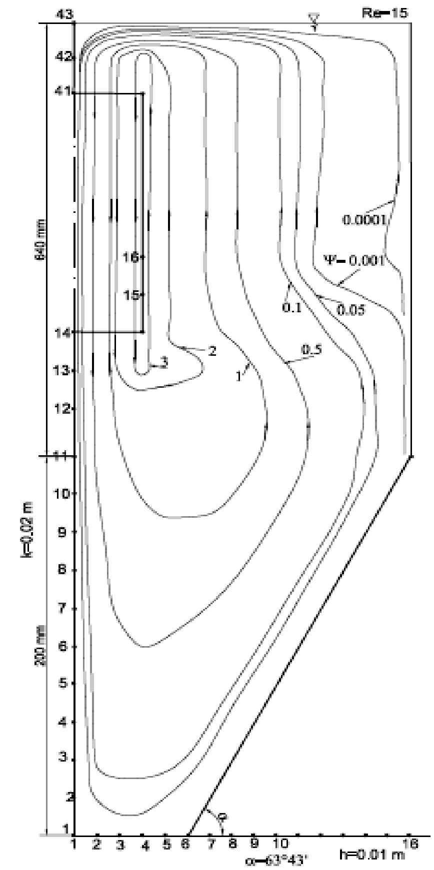

Fig. 3 – The flow spectre for Re= 15

4. Conclusions

Over laminar constant flow solved analytically in cylindrical reactors, natural suspension movement in small concentrations overlaps. A generation of particles "agitation" which results by composing of translation, rotation and radial motion is observed. The numerical solution highlights the reactor geometry influence on the suspension dynamics and advances constructive solutions (angles α, β and Z_t dimension). The spectrum flow analysis enables optimum selection regime for biological processes by choosing the appropriate reactor geometry and operating flow.

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