KINEMATIC ANALYSIS OF BEVEL PLANETARY GEARS BY USING THE INSTANTANEOUS AXIS OF ROTATION

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În lucrare se prezintă o nouă metodă pentru analiza cinematică a mecanismelor planetare cu roţi dinţate conice. Metodele tradiţionale de analiză cinematică a mecanismelor planetare cu roţi dinţate cilindrice utilizează metoda lui Willis sau metoda contururilor, ţinînd cont de semnul raportului de transmitere. Totuşi, acest lucru nu este posibil pentru un mecanism planetar conic simplu (cu două roţi în angrenare) deoarece nu sunt paraleli toţi vectorii viteză unghiulară şi prin urmare metode adiţionale sunt necesare. În acest caz se utilizează o relaţie vectorială între vitezele unghiulare relative în raport cu braţul port-satelit şi vectorii ataşaţi roţilor care formează angrenajul. Pe baza schemei cinematice a mecanismului se realizează schema vectorilor unitari care pot fi utilizaţi pentru vizualizarea poziţiei iniţiale a mecanismului.

In this paper we present a new method for the kinematic analysis of planetary mechanisms with bevel gears. The traditional methods for the kinematic analysis of planetary mechanisms with cylindrical gears employ the Willis method or the contours method considering the sign of the gear ratio. This issue is not possible for single mesh bevel gear planetary mechanism (with two meshing gears) because not all angular velocity vectors are parallel to one another and consequently, additional methods are required. In this case it is used a vector relationship between the relative angular velocities related to the planet carrier and the vector attached to the gears composing the mesh. Based on the kinematic scheme of the mechanism a unit vector scheme is accomplished which can be used to visualize the initial position of the mechanism.

Keywords: planetary mechanisms, bevel gears, kinematic

1. Introduction

The kinematics and dynamics of the planetary mechanisms requires the knowledge of the analytical expressions of the kinematic parameters. From the kinematic point of view the epicyclic mechanisms with cylindrical gears can be studied by the Willis method [3], [5], [6], [7], [9], [14], [15], [17] complying with the sign convention for the transmission ratio depending on the gear type. The

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exterior mesh has the sign minus while the interior mesh has the sign plus. The issue can be solved using the contours method too [10].

In papers [1] and [2] the authors determine the transmission ratio sign using a visual inspection of the kinematic scheme.

In the case of conical planetary gears (figure 1.*a*) with a single gearing not all angular velocity vectors are parallel, so the Willis method cannot be used.

Generally, in the bevel planetary gears case the transmission ratio can be written in the following form:

$$i_{23}^{1} = \frac{\left|\overline{\omega}_2 - \overline{\omega}_1\right|}{\left|\overline{\omega}_3 - \overline{\omega}_1\right|} = \frac{z_3}{z_2}, \tag{1}$$

showing that the transmission ratio sign cannot be determined.

If the determination of the element 2 angular velocity is asked, the vector relations can be written under the form of:

$$\overline{\omega}_2 = \overline{\omega}_1 + \overline{\omega}_{21}; \tag{2}$$
$$\overline{\omega}_2 = \overline{\omega}_3 + \overline{\omega}_{23},$$

on the basis of which the vector polygon in fig. 1,*b*. can be achieved.
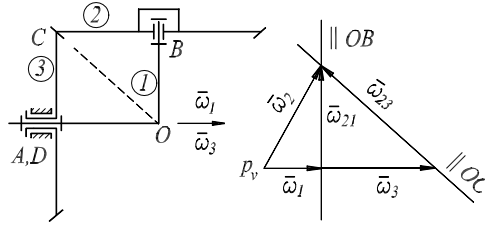

Fig 1. The bevel planetary gears: a) kinematic scheme;
b) the angular velocities polygon

It is to be observed that the method is hard and it is not convenient to analytical calculation fulfilment.

2. The instantaneous axis method

This paper present a relation between the kinematic and the geometric parameters of the spherical planetary gears with a single mesh, in order to allow the application of the analytic and numerical-analytic methods.

Figure 2 presents: *a*) the kinematic scheme, *b*) the structural scheme, *c*) the multipole scheme and *d*) the vector scheme [10] of bevel planetary gears having two degrees of mobility. This mechanism consists of the sun gear 3, the planet gear 2 and the planet carrier 1.

The structural relation of the mechanism [10] is $Z(0) + R(1) + R(3) + (R - RT)(2)$, which means that the mechanism consists of the basis $Z(0)$, the motor groups $R(1)$, respectively $R(3)$, as well as ρ-dyad $(R\text{-}RT)(2)$.

The final Denavit-Hartenberg trihedral [4], [13] $O(\bar{n}_0, \bar{e} \times \bar{n}_0, \bar{e}_1)$ with the $\bar{e}_1$ unitary vector oriented by OO' axis is attached to the basis of the mechanism. The unitary vector $\bar{e}_2$ of the $O'B$ axis and the unitary vector $\bar{e}_3$ under a given direction in the planet gear plane 2 are attached too.

The unitary vectors $\bar{e}_1$ and $\bar{e}_2$ have been attached to the principal axes of the mechanism.

The two cones, polhodiec 2 and herpolhodiec 3, tangent under the commune generating line, are rolling down one over the other without slipping. Let's consider a point P situated on the *instantaneous axis* of the relative movement of the gears 2 and 3, and Q and R its projections on OO' and $O'B$ axes. In the point P (and in any other one on the instantaneous axis of the relative motion), there is the equality:

$$\bar{\omega}_{31} \times \overline{QP} = \bar{\omega}_{21} \times \overline{RP} \tag{3}$$

or, by using the unitary vectors of the basis trihedral, the relation (3) becomes:

$$\omega_{31} \, \bar{e}_1 \times QP \, \bar{n}_0 = \omega_{21} \, \bar{n}_0 \times RP(-\bar{e}_1). \tag{4}$$

The angular velocities $\bar{\omega}_{31}$ and $\bar{\omega}_{21}$ are considered positive in the sense of the unitary vectors $\bar{e}_1$ and $\bar{e}_2$ respectively.

For the calculation facility, the $O(\bar{i}, \bar{j}, \bar{k})$ trihedral is superposed over the basis trihedral $O(\bar{n}_0, \bar{e} \times \bar{n}_0, \bar{e}_1)$. The relation (4) becomes:

$$\omega_{31} \, QP \, \bar{j} = \omega_{21} \, RP \, \bar{j}, \tag{5}$$

which, under the projection on $\bar{j}$ direction, leads to the relative angular velocity determination of the element 2 related to 1, namely:

$$\omega_{21} = \omega_{31} \frac{QP}{RP} \tag{6}$$

Considering that the angular velocities of the elements **1** and **3** are known, as well as the teeth numbers of the two gears, the following relation is obtained:

$$\omega_{21} = \omega_{31} \frac{z_3}{z_2}, \tag{7}$$

where $\omega_{31} = \omega_3 - \omega_1$.

In the same way the relative angular acceleration ε_{21} can be determined.
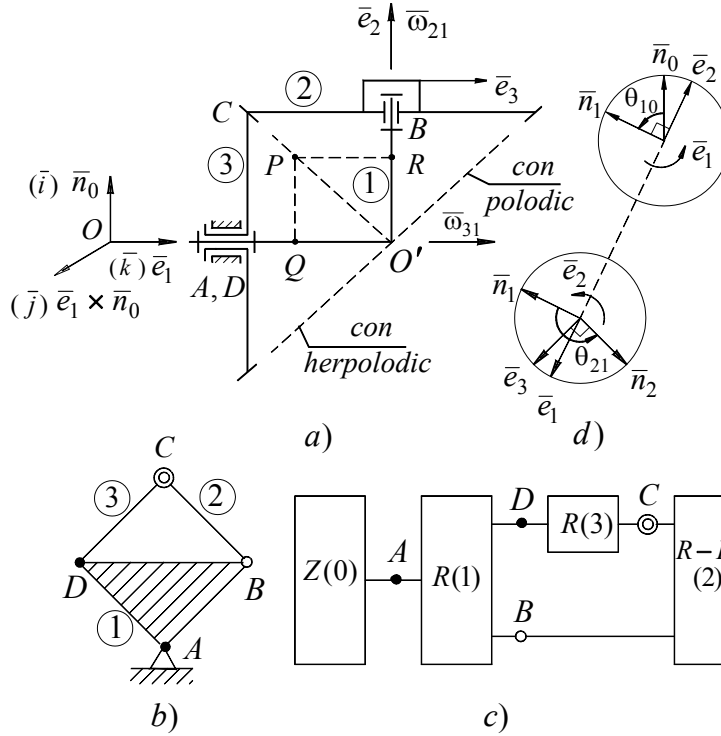

Fig 2. The bevel planetary gear:
a) the kinematic scheme gear with the emphasizing of the axis unitary vectors;
b) the structural scheme; c) the multipolar scheme, d) the unitary vector scheme.

To determine the axial rotation angle θ_{21}, the equation (7), of relative angular velocities, is integrated. There results:

$$\theta_{21} = \int \omega_{31} \frac{z_3}{z_2}\, dt + C \qquad (8)$$

or

$$\theta_{21} = (\theta_{30} - \theta_{10}) \frac{z_3}{z_2} + C, \qquad (8')$$

where θ_{30} is the angle between the unitary vector $\bar{n}_0$ and a unitary vector marked on the gear **3**.

The determination of integration constant is done by using the unitary vectors scheme from figure 2.*d* (for setting the initial position of the mechanism).

Thus, for $\theta_{10} = \pi / 2$, $\theta_{30} = 0$ and $\theta_{21} = \pi$, there results:

$$C = \pi(1 + \frac{z_3}{2z_2}).$$
(9)

Because for writing the relation (3) it was used the instantaneous axis of the relative motion between gears 2 and 3, we named the method after its name.

It is mentioned the fact that the method is also applicable to ordinary or planetary mechanism with cylindrical gears.

Another method to demonstrate the relation between the relative angular velocities ω_{21}, ω_{31} and the teeth numbers of **2** and **3** gears is obtained by using the chains of relative motions [13], [16], [18]. Thus, for the mechanism in figure 2 in point *D*, the vector relation can be written as:

$$\overline{V}_{D_{31}} = \overline{V}_{D_{32}} + \overline{V}_{D_{21}}.$$
(10)

Taking into account the fact that:

$$\overline{V}_{D_{32}} = \overline{V}_{C_{32}} + \overline{V}_{D_{32}C_{32}}, \ \overline{V}_{D_{21}} = \overline{V}_{B_{21}} + \overline{V}_{D_{21}B_{21}}, \ \overline{V}_{B_{21}} = \overline{V}_{C_{32}} = \overline{V}_{D_{31}} = \overline{0},$$

the relation (10) becomes:

$$\overline{\omega}_{32} \ x \ \overline{CD} + \overline{\omega}_{21} \ x \ \overline{BD} = \overline{0}$$
(11)

or

$$\overline{\omega}_{31} \ x \ \overline{CD} + \overline{\omega}_{21} \ x \ \overline{BC} = \overline{0},$$
(11')

where $\overline{\omega}_{31} = \overline{\omega}_{32} + \overline{\omega}_{21}$, $\overline{BD} = \overline{BC} + \overline{CD}$.

If the unitary vectors system $O(\overline{i}, \overline{j}, \overline{k})$ is considered, the relation (11') becomes:

$$\omega_{31} \overline{k} \times CD(-\overline{i}) + \omega_{21} \overline{i} \times BC(-\overline{k}) = \overline{0}.$$
(12)

After accomplishing the vector products, the projection on the directionof the unit vector $\overline{j}$ and expressing the dimensions of *BC* and *CD* as functions of the teeth numbers and gears module, the same relation (7) is obtained.

3. Computational example

Fig. 3.*a* schematizes an orientation mechanism with bars and gears having three degrees of mobility. In the figure 3.*b*, *c* and *d* the structural scheme, the multipolar scheme, the unit vectors scheme of the mechanism respectively, are presented.

The mechanism consists of the basis Z(0), the motor groups *R*(8), *R*(9), *R*(10) and the ρ-dyads (*R-RT*)(1), (*R-RT*)(2), (*R-RT*)(3), (*R-RT*)(4), (*R-RT*)(5), (*R-RT*)(6) and (*R-RT*)(7). The angles of the axial rotations are emphasized with the help of

the unit vectors scheme.

The unit vectors scheme is used to visualize the current position of the mechanism and to specify its initial position. The principal axes of the mechanism are OO', $O'O''$ and $O''P$.

To the principal axes of the mechanism the unit vectors $\bar{e}_1$, $\bar{e}_2$ and $\bar{e}_3$ respectively, have been attached. To the manipulated object axis, or to a segment of it, the unit vector $\bar{e}_4$ has been attached.

3.a. Direct kinematic analysis

There are known:
 - the generalized coordinates θ_{80}, θ_{90}, $\theta_{10,0}$, ω_{80}, ω_{90}, $\omega_{10,0}$, ε_{80}, ε_{90}, $\varepsilon_{10,0}$, from the motors joints A, B and C;
 - the initial position of the mechanism:

$$\theta_{10}^0 = \frac{\pi}{2},\ \theta_{80}^0 = 0,\ \theta_{90}^0 = 0,\ \theta_{10,0}^0 = 0,\ \theta_{21}^0 = \pi/2,\ \theta_{32}^0 = \pi/2\,;$$

 - the axial displacements: $s_1 = OO'$, $s_2 = O'O''$, $s_3 = O''P$, $s_4 = PT$;
 - the crossing lengths (the distances between the unit vectors $\bar{e}_1, \bar{e}_2, \bar{e}_3$ and $\bar{e}_4$): $a_1 = a_2 = a_3 = 0$;
 - the crossing angles (the angles between unit vectors $\bar{e}_1$, $\bar{e}_2$, $\bar{e}_3$ and $\bar{e}_4$ of the considered axes): $\alpha_1 = \alpha_2 = \alpha_3 = \dfrac{\pi}{2}$.

The following parameters have been determined:
 - the relative angular velocities: ω_{10}, ω_{21}, ω_{60}, ω_{70}, ω_{51}, ω_{61}, ω_{71}, ω_{41}, ω_{42}, ω_{32};
 - the relative angular accelerations ε_{10}, ε_{21}, ε_{60}, ε_{70}, ε_{51}, ε_{61}, ε_{71}, ε_{41}, ε_{42}, ε_{32};
 - the angles of the axial rotations θ_{10}, θ_{21} and θ_{32}, about the unit vectors $\bar{e}_1$, $\bar{e}_2$ and $\bar{e}_3$ respectively;

To determine the relative angular velocities and accelerations the instantaneous axis method is used, as follows:
- the joint D: $\bar{\omega}_{80} \times \overline{AD} = \bar{\omega}_{10} \times \overline{ED}$; after the vectors replacement by the correspondent dimensions and unit vectors, there results: $\omega_{80}\bar{i} \times AD\bar{k} = \omega_{10}\bar{k} \times ED(\text{-}\bar{i})$,
or $\omega_{80}z_8(-\bar{j}) = \omega_{10}z_1(-\bar{j})$. After the projection on the unit vector $\bar{j}$ direction, the

relative angular velocity ω_{10} is obtained, namely:

$$\omega_{10} = \omega_{80}\,\frac{z_8}{z_1};$$

- the joint G: $\overline{\omega}_{90} \times \overline{BG} = \overline{\omega}_{60} \times \overline{FG}$, whence it results $\omega_{60} = -\omega_{90}\,\dfrac{z_9}{z_6};$

- the joint Q: $\overline{\omega}_{10.0} \times \overline{CQ} = \overline{\omega}_{70} \times \overline{NQ}$, whence it results $\omega_{70} = \omega_{10.0}\,\dfrac{z_{10}}{z_7};$

- the joint H: $\overline{\omega}_{41} \times \overline{IH} = \overline{\omega}_{61} \times \overline{FH}$, whence it results $\omega_{41} = \omega_{61}\,\dfrac{z_6}{z_{4'}},$

where $\omega_{61} = \omega_{60} - \omega_{10} = -\omega_{90}\,\dfrac{z_9}{z_6} - \omega_{80}\,\dfrac{z_8}{z_1};$

- the joint M: $\overline{\omega}_{51} \times \overline{LM} = \overline{\omega}_{71} \times \overline{NM}$, whence it results $\omega_{51} = -\omega_{71}\,\dfrac{z_7}{z_{5'}},$

where $\omega_{71} = \omega_{70} - \omega_{10} = \omega_{10.0}\,\dfrac{z_{10}}{z_7} - \omega_{80}\,\dfrac{z_8}{z_1};$

- the joint K: $\overline{\omega}_{21} \times \overline{JK} = \overline{\omega}_{51} \times \overline{LK}$, whence it results $\omega_{21} = -\omega_{51}\,\dfrac{z_5}{z_2};$

- the joint R: $\overline{\omega}_{32} \times \overline{SR} = \overline{\omega}_{42} \times \overline{IR}$, whence it results $\omega_{32} = -\omega_{42}\,\dfrac{z_4}{z_3},$

where $\omega_{42} = \omega_{41} - \omega_{21} = \omega_{61}\,\dfrac{z_6}{z_{4'}} + \omega_{51}\,\dfrac{z_5}{z_2}.$

After the replacements, it results:

$$\omega_{10} = \omega_{80}\,\frac{z_8}{z_1};$$

$$\omega_{21} = -\left(\omega_{80}\,\frac{z_8}{z_1}\frac{z_7}{z_{5'}} - \omega_{10.0}\,\frac{z_{10}}{z_{5'}}\right)\frac{z_5}{z_2};$$

$$\omega_{32} = \omega_{80}\left(\frac{z_6}{z_{4'}} - \frac{z_7}{z_{5'}}\frac{z_5}{z_2}\right)\frac{z_8}{z_1}\frac{z_4}{z_3} + \omega_{90}\,\frac{z_9}{z_{4'}}\frac{z_4}{z_3} + \omega_{10.0}\,\frac{z_{10}}{z_{5'}}\frac{z_5}{z_2}\frac{z_4}{z_3}.$$

The relative angular accelerations are determined in the same way:

$$\varepsilon_{10} = \varepsilon_{80}\,\frac{z_8}{z_1};\ \varepsilon_{60} = -\varepsilon_{90}\,\frac{z_9}{z_6};\ \varepsilon_{70} = \varepsilon_{10.0}\,\frac{z_{10}}{z_7};$$

$$\varepsilon_{61} = \varepsilon_{60} - \varepsilon_{10} = -\varepsilon_{90}\,\frac{z_9}{z_6} - \varepsilon_{80}\,\frac{z_8}{z_1};$$

$$\varepsilon_{41} = \varepsilon_{61}\,\frac{z_6}{z_{4'}}\,;\ \varepsilon_{71} = \varepsilon_{70} - \varepsilon_{10} = \varepsilon_{10.0}\,\frac{z_{10}}{z_7} - \varepsilon_{80}\,\frac{z_8}{z_1}\,;\ \varepsilon_{51} = -\varepsilon_{71}\,\frac{z_7}{z_{5'}}\,;$$

$$\varepsilon_{21} = -\varepsilon_{51}\,\frac{z_5}{z_2}\,;\ \varepsilon_{42} = \varepsilon_{41} - \varepsilon_{21} = \varepsilon_{61}\,\frac{z_6}{z_{4'}} + \varepsilon_{51}\,\frac{z_5}{z_2}\,;\ \varepsilon_{32} = -\varepsilon_{42}\,\frac{z_3}{z_4}\,.$$

After the replacements the following relations are obtained:

$$\varepsilon_{10} = \varepsilon_{80}\,\frac{z_8}{z_1}\,;$$

$$\varepsilon_{21} = -(\varepsilon_{80}\,\frac{z_8}{z_1}\,\frac{z_7}{z_{5'}} - \varepsilon_{10.0}\,\frac{z_{10}}{z_{5'}})\,\frac{z_5}{z_2}\,;$$

$$\varepsilon_{32} = \varepsilon_{80}\left(\frac{z_6}{z_{4'}} - \frac{z_7}{z_{5'}}\,\frac{z_5}{z_2}\right)\frac{z_8}{z_1}\,\frac{z_4}{z_3} + \varepsilon_{90}\,\frac{z_9}{z_{4'}}\,\frac{z_4}{z_3} + \varepsilon_{10.0}\,\frac{z_{10}}{z_{5'}}\,\frac{z_5}{z_2}\,\frac{z_4}{z_3}\,.$$

For the axial rotation angles θ_{10}, θ_{21} and θ_{32} determination, the transmission functions of the corresponding relative angular velocities are integrated:

$$\theta_{10} = \int \omega_{80}\,\frac{z_8}{z_1}\mathrm{dt} + C_1\,;$$

$$\theta_{21} = -\int (\omega_{80}\,\frac{z_8}{z_1}\,\frac{z_7}{z_{5'}} - \omega_{10.0}\,\frac{z_{10}}{z_{5'}})\,\frac{z_5}{z_2}\mathrm{dt} + C_2\,;$$

$$\theta_{32} = \int [\omega_{80}\,(\frac{z_6}{z_{4'}} - \frac{z_5}{z_2}\,\frac{z_7}{z_{5'}})\,\frac{z_8}{z_1}\,\frac{z_4}{z_3} + \omega_{90}\,\frac{z_9}{z_{4©}}\,\frac{z_4}{z_3} + \omega_{10.0}\,\frac{z_5}{z_2}\,\frac{z_4}{z_3}\,\frac{z_{10}}{z_{5'}}]\mathrm{dt} + C_3$$

After the integration of the above equations the integration constants are determined, and it results:

$$\theta_{10} = \theta_{80}\,\frac{z_8}{z_1} + \frac{\pi}{2}\,;\ \theta_{21} = -(\theta_{80}\,\frac{z_8}{z_1}\,\frac{z_7}{z_{5'}} - \theta_{10.0}\,\frac{z_{10}}{z_{5'}})\,\frac{z_5}{z_2} + \frac{\pi}{2}\,;$$

$$\theta_{32} = \theta_{80}\,(\frac{z_6}{z_{4'}} - \frac{z_5}{z_2}\,\frac{z_7}{z_{5'}})\,\frac{z_8}{z_1}\,\frac{z_4}{z_3} + \theta_{90}\,\frac{z_9}{z_{4'}}\,\frac{z_4}{z_3} + \theta_{10.0}\,\frac{z_5}{z_2}\,\frac{z_4}{z_3}\,\frac{z_{10}}{z_{5'}} + \frac{\pi}{2}$$

3.b. Inverse kinematic analyses

In the case of the back kinematic analysis two cases can appear, namely:

1) the positions, velocities and accelerations of the tracing point T are known, namely XT, YT, ZT, $\dot{X}T, \dot{Y}T, \dot{Z}T$, $\ddot{X}T, \ddot{Y}T, \ddot{Z}T$, and the kinematic parameters are required in the motor joints *A*, *B* and *C,* namely:

$\theta_{80}, \theta_{90}, \theta_{10.0}, \omega_{80}, \omega_{90}, \omega_{10.0}, \varepsilon_{80}, \varepsilon_{90}, \varepsilon_{10.0}\,;$

2) the relative angular velocities ω_{10}, ω_{21} and ω_{32} are known and the mechanism initial position and the kinematic parameters in the motor joints *A, B* and *C* are required. For the first case the knowledge of the mechanism constructive characteristics is needed, as presented at 2.a. paragraph.

Taking into account the mechanism constructive characteristics the position vector of the tracing point T has the expression: $\bar{r}_T = s_1\bar{e}_1 + s_2\bar{e}_2 + s_3\bar{e}_3 + s_4\bar{e}_4$, so that the T point coordinates are:

$$X_T = s_2 A_2 + s_3 A_3 + s_4 A_4;$$
$$Y_T = s_2 B_2 + s_3 B_3 + s_4 B_4;$$
$$Z_T = s_1 C_1 + s_2 C_2 + s_3 C_3 + s_4 C_4,$$

where:

$$C_1 = 1, \ A_2 = \sin\theta_{10}, \ B_2 = -\cos\theta_{10}, \ C_2 = 0,$$
$$A_3 = \cos\theta_{10}\sin\theta_{21}, \ B_3 = \sin\theta_{10}\sin\theta_{21}, \ C_3 = -\cos\theta_{21},$$
$$A_4 = \sin\theta_{10}\cos\theta_{32} - \cos\theta_{10}\cos\theta_{21}\sin\theta_{32},$$
$$B_4 = \cos\theta_{10}\cos\theta_{32} + \sin\theta_{10}\cos\theta_{21}\sin\theta_{32},$$
$$C_4 = \sin\theta_{21}\sin\theta_{32}$$
$$s_1 = OO', \ s_2 = O'O'' \quad s_3 = O''P \quad s_4 = PT \ ([11], [13]).$$

After the replacements, it results:

$$X_T = s_2\sin\theta_{10} + s_3\cos\theta_{10}\sin\theta_{21} - s_4(\sin\theta_{10}\cos\theta_{32} - \cos\theta_{10}\cos\theta_{21}\sin\theta_{32});$$

$$Y_T = -s_2\cos\theta_{10} + s_3\sin\theta_{10}\sin\theta_{21} + s_4(\cos\theta_{10}\cos\theta_{32} + \sin\theta_{10}\cos\theta_{21}\sin\theta_{32});$$

$$Z_T = s_1 - s_3\cos\theta_{21} + s_4\sin\theta_{21}\sin\theta_{32}.$$

The above equations constitute a non-linear system with the unknown θ_{10}, θ_{21}, θ_{32}, a system which can be solved by an adequate numerical method.

By deriving with respect to the time the position relations of the tracing point T, a linear equation system in the unknowns ω_{10}, ω_{21} and ω_{32} is obtained. By deriving the velocities relations of the tracing point T, a linear system in the unknowns ε_{10}, ε_{21} and ε_{32} is obtained.

Further on, it follows the determination of the motor joints variables *A, B* and *C*, that is: $\theta_{80}, \theta_{90}, \theta_{10.0}, \omega_{80}., \ \omega_{80}, \omega_{90}, \omega_{10.0}, \varepsilon_{80}, \varepsilon_{90}$ and $\varepsilon_{10.0}$. By using the relations of the forward kinematic analysis the following relations are obtained:

$$\theta_{80} = (\theta_{10} - C_1)\frac{z_1}{z_8};$$

$$\theta_{90} = (\theta_{10} - C_1)(2\frac{z_5}{z_2}\frac{z_7}{z_{5'}}\frac{z_{4'}}{z_9} - \frac{z_6}{z_9}) - (\theta_{21} - C_2)\frac{z_{4'}}{z_9} + (\theta_{32} - C_3)\frac{z_3}{z_4}\frac{z_{4'}}{z_9} \; ;$$

$$\theta_{10.0} = (\theta_{10} - C_1)\frac{z_7}{z_{10}} + (\theta_{21} - C_2)\frac{z_{5'}}{z_5}\frac{z_2}{z_{10}} \; .$$

The integration constants are settled for the initial position of the mechanism. For the considered initial position there results: $C_1 = \dfrac{\pi}{2}$, $C_2 = \dfrac{3\pi}{2}$, $C_3 = \dfrac{\pi}{2}$.

The expressions of the angular velocities and accelerations in the mechanism motor joints are:

$$\omega_{80} = \omega_{10}\frac{z_1}{z_8} \; ;$$

$$\omega_{90} = \omega_{10}(2\frac{z_5}{z_2}\frac{z_7}{z_{5'}}\frac{z_{4'}}{z_9} - \frac{z_6}{z_9}) - \omega_{21}\frac{z_{4'}}{z_9} + \omega_{32}\frac{z_3}{z_4}\frac{z_{4'}}{z_9} \; ;$$

$$\omega_{10.0} = \omega_{10}\frac{z_7}{z_{10}} + \omega_{21}\frac{z_{5'}}{z_5}\frac{z_2}{z_{10}} \; ,$$

respectively,

$$\varepsilon_{80} = \varepsilon_{10}\frac{z_1}{z_8} \; ;$$

$$\varepsilon_{90} = \varepsilon_{10}(2\frac{z_5}{z_2}\frac{z_7}{z_{5'}}\frac{z_{4'}}{z_9} - \frac{z_6}{z_9}) - \varepsilon_{21}\frac{z_{4'}}{z_9} + \varepsilon_{32}\frac{z_3}{z_4}\frac{z_{4'}}{z_9}$$

$$\varepsilon_{10.0} = \varepsilon_{10}\frac{z_7}{z_{10}} + \varepsilon_{21}\frac{z_{5'}}{z_5}\frac{z_2}{z_{10}} \; .$$

In the case of the second variant, namely when the relative angular velocities ω_{10}, ω_{21} şi ω_{32} are known, the angular velocities ω_{80}, ω_{80} and $\omega_{10.0}$ of the motor joints A, B and C are known. By the integration of the angular velocities equations the angles θ_{80}, θ_{80} and $\theta_{10.0}$ are achieved . The integration constants are determined in the same way.

3. Conclusions

The instantaneous axis method facilitates the determination of gears mechanisms kinematic parameters and it simultaneously allows the use of the analytic methods in the study of these mechanisms. The unit vectors scheme allows the visualization of the relative positions of the mechanisms elements which leads to the accomplishment of a complete image over the orientation of the mechanism.

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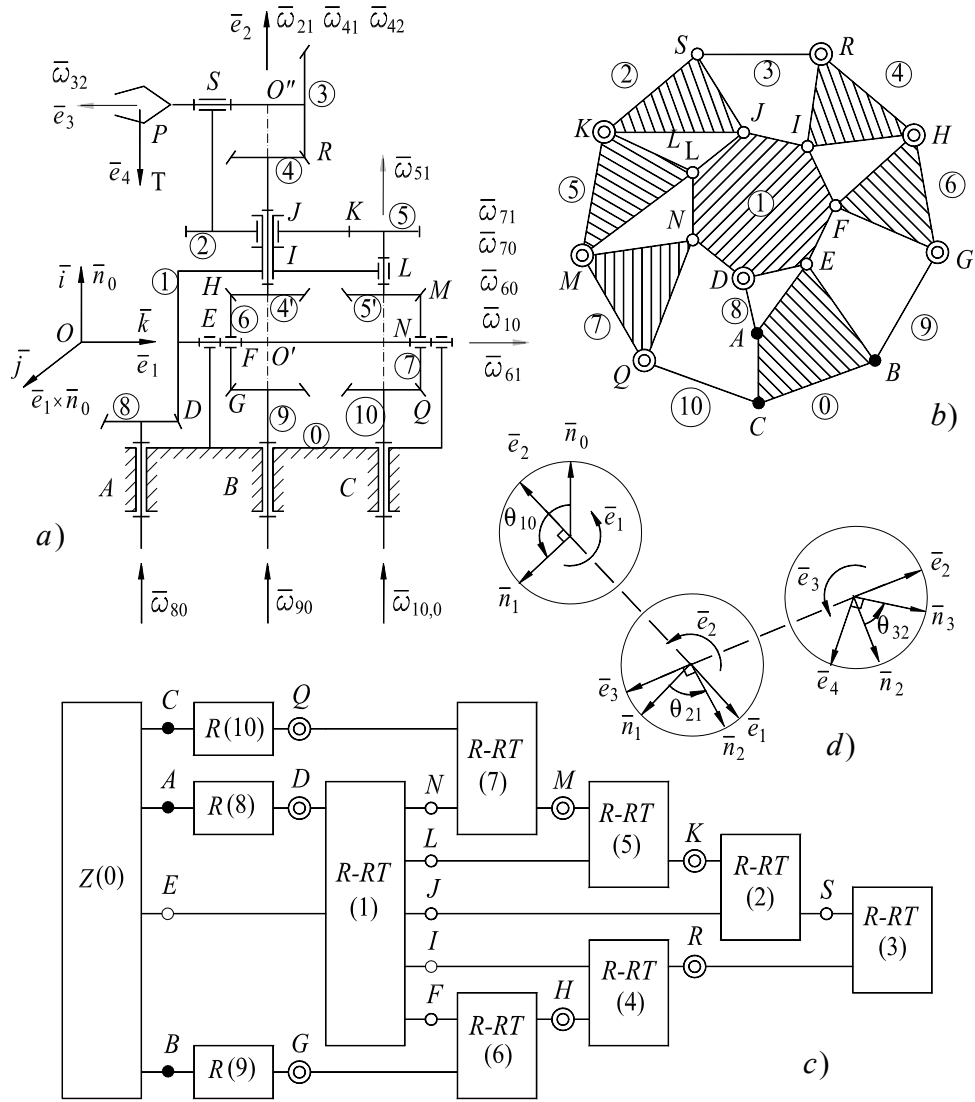
Fig. 3. The kinematic, structural, multipole and unitary vector scheme
of an orientation mechanism