FRACTAL APPROACH FOR ERODATED WEAR OF SURFACES BY SOLIDE PARTICLES

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The goal of this study was to examine the micro-geometry of eroded surfaces of a helicopter blade segment, on the second and third compressor stage, and on a carbon fiber woven composite sample, as resulted from the impact with sand particles. By recording of the roughness height using a spherical head stylus, the Abbott-Firestone bearing curve has been plotted. The random nature of the roughness height is described through statistical analysis. It was found that the distribution frequency follows Weibull's law. By using the structured function, the fractal nature of the micro-geometry of the eroded surface, the Weierstrass – Mandelbrot (WM) function was determined.

Keyword: abrasive, erosion wear**,** roughness, fractal, helicopter blade.

1. Introduction

Roughness is of great importance in surface response in relation to mechanical bading. An important concept is to construct an analytical description that represents well the topography of the surface for some functional stadies. The original topography is modified in friction phenomena. The wear of machine parts is the progressive damage and material loss which occurs on component surfaces as a result of the relative motion.

The solid particle erosion behavior of metals and alloys has been characterized over the past years [1-5]. The "cutting" or "micromachining" models of Finnie and Bitter [6] computes the volume of the crater generated in the eroding material when it is impacted by a hard angular particle at a given velocity and angle of incidence. This model can explain the experimentally observed maximum of the erosion rate at intermediate impact angles and the effect of velocity to the erosive rate, only for annealed pure metals and not for alloys.

The cutting model predicts an inverse relationship between the erosive rate and of material and its hardness.

The other erosive models include the fatigue models of Hutchings [7], Follansbec et.al. and Ratner and Styller [8]. These models predict the erosion rate of a material to be inversely related to the product of its strength and fracture strain.

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Sundararajan and Shewmon propose a model for localization of deformation in the eroding material underneath the impacting particle and the lip formation [9]. The critical condition for the onset of localization can be transformed into a critical strain criterion. The erosion rate is inversely related to the critical strain and proportional to the volume of material underneath the impacting particle undergoing plastic deformation. The state of art the of erosion wear and an application for a helicopter blade were published by Tudor et al. [10].
The effect of erosion wear can be evaluated by analysis of topography of surface. The irregularity of the surface profile can be measured with conventional surface roughness parameters such as R_q, R_{max}, R_z, etc.
Fractal geometry as a tool for the characterization of surface topography has gained much attention in recent years [11-17]. In previous papers there are variable definitions of the fractal dimension. This is due in part to the observations that fractal geometry can reflect the natural and intrinsic properties of random phenomena and that it can overcome several disadvantages of conventional statistics and random process methods of surface analysis.
The objective of this paper is to establish classical and fractal characteristics for eroded surface of helicopter blade by the solid particle.

2. Characterization of surface topography by conventional methods

The experimental measurements have been carried on a blade segment from the main rotor of the Allouette helicopter and also on the second and third compressor stage from the R11F300 turbine engine. Another test has been made on a carbon fiber woven composite sample. The R11F300 turbine engine has a compressor 6-stage low pressure and 3-stage high pressure combustor; Can-annular, 10 flame tubes Turbine: Single-stage high pressure, single-stage low pressure axial. The second and third compressor stage from the R11F300 turbine engine are made of overall – inconel, steel, aluminum, stainless Steel, titanium, paint, brass, preservative coating, asbestos.

Table 1

The technical parameters of the R11-F300 engine

Maximum thrust load		Overall pressure ratio	Turbine inlet temperature	Specific fuel consumption		Thrust-to-weight ratio
Military power	38.7 kN (8.708 lbf)	8.9:1	955 °C	At idle	97 kg/(h·kN) (0.95 lb/(h·lbf))	53.9 N/kg (5.5:1)
With afterburner	60.6 kN (13.635 lbf)		(1.750 °F)	With afterburner	242 kg/(h·kN) (2.37 lb/(h·lbf))	

The erosion tests were carried on with at a mean velocity of 96 m/sec for different time intervals (t_1=5min, t_2=10min).

Weight measurements of the samples were made after each time interval in order to determine the weight loss due to the erosion process.

Table 2

The parameters of the sand blaster installation

Blowing velocity	Mass flow per minute	Nozzle diameter
96 [m/s]	2 [kg/min]	20[mm]

In order to analyses the quality of the eroded surfaces for the three samples, the profile of each sample were recorded with the specialized machines provided by the UPG Ploiesti – Kema Technik and by the UPB FIMM – Mitutoio.

The resulted parameters for the y_i profile are listed below:

The amplitude parameters:

$N = length(y)$ length;

$R_a = \frac{1}{N}\sum_{i=1}^{N}|y_i|$ arithmetic average;

$R_q = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(y_i)^2}$ the mean square root;

$R_p = (\max(y))$ prominent maximum height of the profile;

$R_v = mean(y)$ maximum depth of the profile;

$R_t = \max(y) - \min(y)$ maximum height of the profile;

$R_{sk} = \frac{1}{NR_q^{\ 4}}\sum_{i=1}^{N}(y_i)^4$

Skewness the Gauss R_{sk}=0 Skewness;

$R_{ku} = \frac{1}{NR_q^4}\sum_{i=1}^{N}(y_i)^4$ Gauss R_{ku}=3 Kurtosis.

Functional parameters:

The Abbott-Firestone bearing curve:

$z = sort(y) + |\min(y)|$; $za = \frac{z}{\max(z)}$

The following table provides the statistical data determined from the measured samples.

Table 3

Characteristical parameter for the micro-geometry of the eroded surfaces

Characteristical parameter	Carbon fiber composite	Helicopter blade	Stage 02	Stage 03
R_a	3.079	0.418	2.524	0.494
R_q	3.667	0.57	3.099	0.639
R_p	7.92	0.96	7.76	1.32
R_v	0.639	0.078	0.028	-0.307
R_t	17.45	3.31	15.12	3.35
R_{sk}	0.446	-1.396	-0.059	-1.473
R_{ku}	2.34	6.52	2.559	3.678

The bearing curves are centralized in Fig.1:

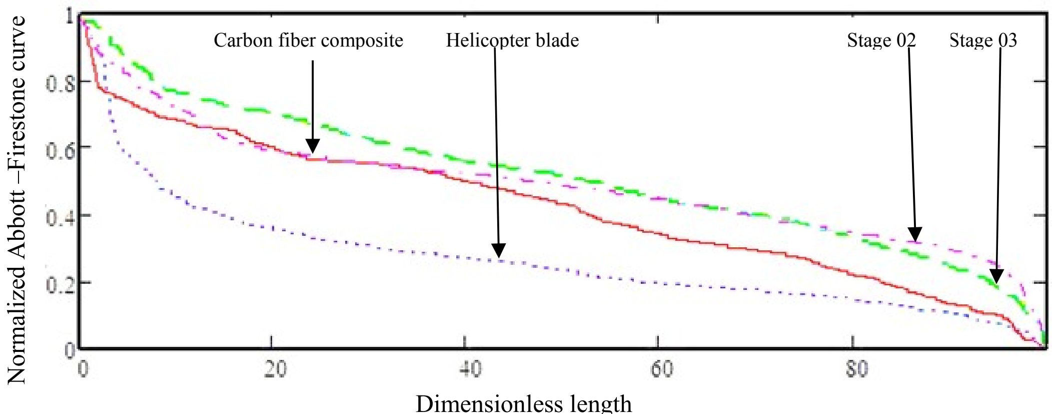

Fig. 1. Normalized Abbott –Firestone bearing area curve.

3. Roughness distribution of eroded surfaces

The erosive wear of surfaces subjected to the impact with the hard abrasive particles with a random geometry distribution has a significant effect over the initial roughness of the surface.

In this case, the experimental results for the height of the roughness must be subjected to statistical tests. For corectly establishing if the nature is random or non-random the von Neumann (1941), statistical method was applied, which hypothesizes the random risk ε if [18]:

$$M_1 \le r_{N;\varepsilon}, \tag{1}$$

$$\text{where: } M_1 = \frac{\delta^2}{2s^2} \text{ and } r_{N;\varepsilon} = 1 - |u_\varepsilon| \sqrt{\frac{N-2}{N^2-1}}, \tag{2}$$

$$\text{with } \delta^2 = \frac{1}{N-1}\sum_{i=1}^{N-1}\left(y_{i+1}-y_i\right)^2, \qquad s^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(y_i - y_m\right)^2 \tag{3}$$

and u_ε –qvantile of normal standard distribution;

$$F_0\left(u_\varepsilon\right) = \varepsilon \text{ cu } F_0(u) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{u} e^{-z^2} dz \tag{4}$$

The next table presents the results regarding the random nature of the micro-geometry of the eroded surface for the 4 tested surfaces.

Table 4

The eroded surface roughness height characterization

Roughness height characterisation	Carbon fiber composite	Helicopter blade	Stage 02	Stage 03
N	1990	1990	1990	1990
M_1	$4.50 \cdot 10^{-3}$	$2.86 \cdot 10^{-3}$	$1.74 \cdot 10^{-3}$	$4.32 \cdot 10^{-3}$

u_ε ($u_{0.001}$)	3.09	3.09	3.09	3.09
u_ε ($u_{0.05}$)	1.64	1.64	1.64	1.64
$R_{N;\varepsilon}$ ($R_{N;0.001}$)	0.93	0.93	0.93	0.93
$R_{N;\varepsilon}$ ($R_{N;0.001}$)	0.96	0.96	0.96	0.96
Caracter	random	random	random	random

From the above table we observe that the roughness height is random. In this case, the theoretical distribution law – which best fits the experimental data is chosen. For this, the Cramer-von Mises statistical test is applied. The test consists of [18]:

- sorting of the results: $ys = sort(y)$;

$$C = \frac{1}{12N} + \sum_{i=1}^{N}\left[F_Y(ys_i) - \frac{2i-1}{2N}\right]^2, \tag{5}$$

where $F_Y(ys_i)$ is the calculated value for the proposed distribution law assumed to be true with the risk ε;

- comparing the statistical function C with the calculated value from Stephens and Maag (1968) $C_{1-\varepsilon}$ and taking the decision: accept the distribution function if $C \le C_{1-\varepsilon}$. Typical values for the used constants are:
- $C_{1-\varepsilon}$: $C_{0.99} = 0.743; C_{0.95} = 0.461; C_{0.90} = 0.347.$

Through the application of the main distribution laws (negative exponential, Gauss, Weibull, bi-parametrical or tri-parametrical) for the current measurements it was found that the Weibull bi-parametrical law best fits the dataset:

$$F(t,\beta,\delta) = 1 - \exp\left[-\left(\frac{t}{\delta}\right)^{\beta}\right]. \tag{6}$$

In this case, the distribution parameters (β, δ) are determined through the maximal plausibility method, by solving the system of equations (7). Thus, the calculated values for the parameters β, δ and the statistic function C are presented in the table below.

It must be stated that the profile of micro-geometry were displaced with 10 μm in order to calculate the β, δ logarithmic functions, but without modifying the distribution law for the roughness height:

$$Ec(\beta) = \frac{\sum_i\left[(t_i)^{\beta}\ln(t_i)\right] + (n-n)(t_n)^{\beta}\ln(t_n)}{\sum_i (t_i)^{\beta} + (n-n)(t_n)^{\beta}} - \frac{1}{n}\left(\sum_i \ln(t_i)\right) - \frac{1}{\beta} \tag{7}$$

$$\delta = \left[\frac{1}{n}\left[\sum_i (t_i)^{\beta_e} + (n-n)(t_n)^{\beta_e}\right]\right]^{\beta_e} \tag{7'}.$$

Table 5

The Weibull repartition parameters for the roughness height

Weibull repartition parameters	Carbon fiber composite	Helicopter blade	Stage 02	Stage 03
β	3.26	26.21	3.63	18.87
δ	11.85	10.30	11.13	9.95
C	128.03	124.75	111.95	127.096

Figure 2 shows the probability density function of the roughness height for the carbon fiber sample, the Alouette main rotor blade sample and also the 2nd and 3rd compressor blade from the R11F300 turbine engine.

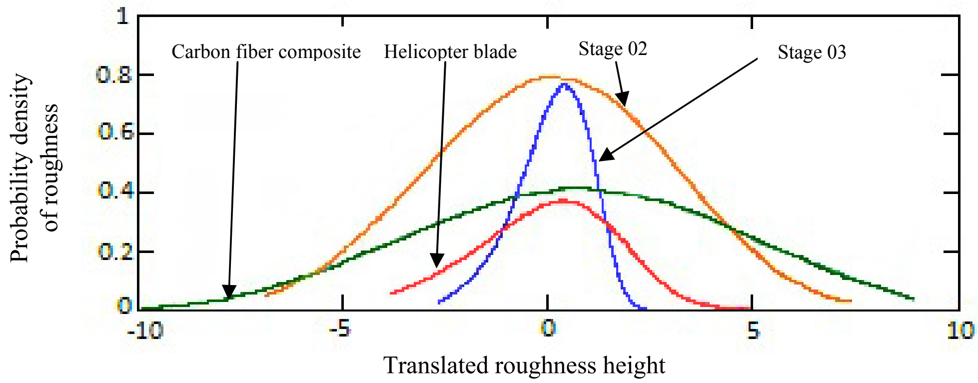

Fig. 2. Probability density of roughness the Alouette main rotor blade sample and also the 2nd and 3rd compressor blade from the R11F300 turbine engine.

From the obtained profile of micro- geometry it becomes apparent that the valleys have wear edges, specific to ductile materials. The shape and probability density function is consistent with the fatigue deterioration criteria. Therefore, it is found that the erosion process with solid particles is a complex phenomenon of cumulative effects of fatigue and micro-cutting.

Fig. 3 exemplifies a part of the profile of micro-geometry (y_i) of an eroded surface, as a function to of digitalized point i, and a detail of one of the cavities resulted through the erosion process.

4. Fractal geometry of eroded helicopter blade surfaces

Fractal geometry reveals natural properties of random and unpredictable phenomena [11]. Several methods have been developed to characterize the dimension of a fractal set, such the compass dimension, box dimension, mass dimensions and area – perimeter dimension. All of these methods can be easily computed for self-affine fractals [17].

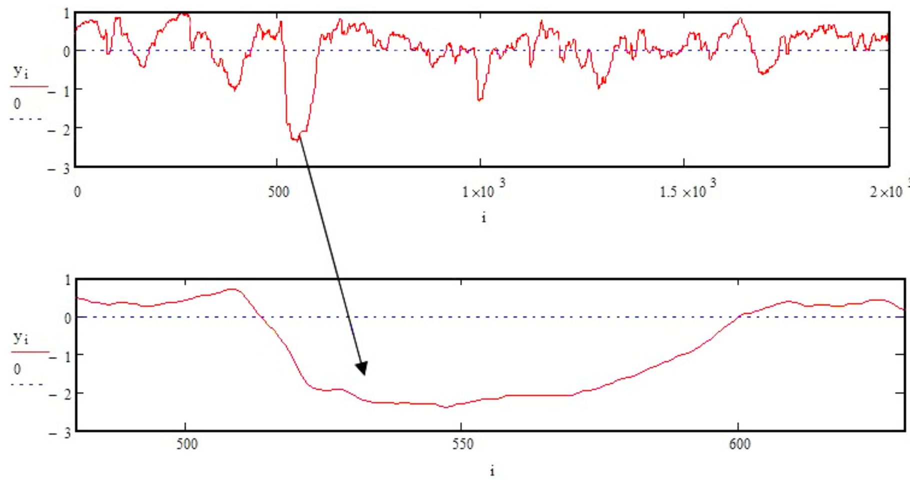

Fig. 3. Quality of surface made with erosion solid particles.

In our study we use the structure function method to calculate fractal parameters *D* (fractal dimension) and *L* (topothesy). The structure function is defined as the variance of profiles increment $y(x+\lambda)-y(x)$. This increment is assumed to have a Gaussian distribution with zero mean and the following variance [11]:

$$\left\langle \left[y(x+\lambda)-y(x)\right]^2 \right\rangle = \frac{2C_1}{4-2D}\sin\left[\frac{\pi}{2}(2D-3)\right]\Gamma(2D-3)\left|\lambda\right|^{4-2D}, \tag{8}$$

where λ is any displacement along the *x* direction, C_1 is a constant, *D* is the fractal dimension of the y(x) function and Γ( ) is the gamma function. It can be seen that the chord joining *y* values separated by a distance λ has a finite mean – square slope. If there is a displacement λ = L such that the chord has an r.m.s. slope of unity statistically, then a concise formula can be written, namely:

$$\left\langle \left[y(x+L)-y(x)\right]^2 \right\rangle / L^2 = 1. \tag{9}$$

By comparing eqns. (8) and (9), an equation relating the structure function with fractal geometry parameters is derived:

$$\left\langle \left[y(x+\lambda)-y(x)\right]^2 \right\rangle = L^{2D-2}\left|\lambda\right|^{4-2D}. \tag{10}$$

Such a function is both stationary and isotropic. From the above relations *D* and *L* can be expressed as:

$$D = \frac{4-D_s}{2} \quad 0 \prec D_s \prec 2; \tag{11}$$

$$L = 10^{\frac{C_2}{2D-2}}, \tag{12}$$

Where D_s is the slope of structure function (10) plotted on double logarithmic coordinates and C_2 is the intersection of the structure function curve with the y (ordinate) axis.
The structure function (S) of roughness discrete values from eroded surface (y) can be written as:

$$S(N,k) = \frac{1}{N-k} \sum_{i=1}^{N-K} \left(y_{i+k} - y_i \right)^2 \tag{13}$$

Where k is the increment of x ordinate and N is the length of the y vector.
For example, Figure 4 shows the structure function (S(N,k)) in the logarithmic axes and the straight line ($y_f(x_f)$) which approximates this function for eroded surface presented in figure 3.

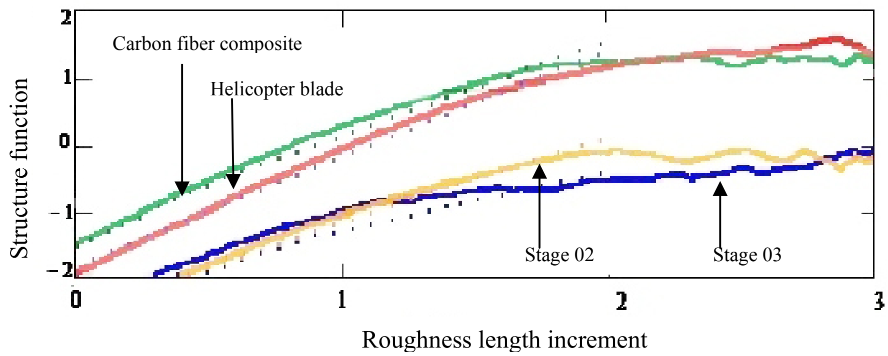

Fig. 4. Structure function of eroded surface from figure 3.

The slope of this function is calculated by use three points of structure function. Thus, for the example of helicopter blade, the slope is D_s= 1.37 and the fractal parameter D = 1.31.
The second fractal parameters L (topothesy) can be determined using eq. (12), after evaluation of the ordinate C_2 of the structure function (13). Thus,

$$C_2 = \log(S(N,k)) - D_s \cdot \log(k) . \tag{14}$$

We adopt the increment k = 5 and N = 1990. In this case, $C_2 = -2.59$ and $L = 7.8 \cdot 10^{-5}$.
The fractal parameters D (fractal dimension), L (topothesy) and characteristic length (G) are determined for all eroded surfaces and are presented in the table 6.

Table 6

Fractal parameters for eroded surfaces.

Fractal parameters	Carbon fiber composite	Helicopter blade	Stage 02	Stage 03
D	1.17	1.31	1.26	1.51
L	$4.8 \cdot 10^{-6}$	$7.8 \cdot 10^{-5}$	$3.41 \cdot 10^{-3}$	$5.69 \cdot 10^{-3}$
G	$1.71 \cdot 10^{-6}$	$3.93 \cdot 10^{-5}$	$1.58 \cdot 10^{-3}$	$3.22 \cdot 10^{-3}$

The scale-independent of fractal parameters D and L has been doubted in [16, 17]. The absolute scale-independent of parameters D and L is impossible sometimes, and also unnecessary [19, 20].

It was found that the Weierstrass – Mandelbrot function (WM model) could characterize the self-affine profile lines of rough surfaces. In Fig. 5, the deviation of the profile Z(x) from its mean line Z(x) = 0 is described as:

$$Z(x) = G^{(D-1)} \sum_{n=n_1}^{\infty} \frac{\cos(2\pi\gamma^n x)}{\gamma^{(2-D)n}} , \tag{15}$$

where parameter D is fractal dimension of surface profile (for a physically continuous surface 1 < D < 2); G is the characteristic length scale of surface that determines the position of spectrum along power axis, and is invariant with respect to all frequencies of roughness; $\gamma = 1.5$ can provide both the phase randomization and high spectral density; the number of wavelength in a certain level is 1.5 times the number of wavelengths in the previous level [16]; n is an integer, n_1 is the minimum of n which is determined by measuring length integer $\gamma^{n_1} = 1/L_s$, L_s is the sampling length.

The characteristic length scale can be evaluated as a function of topothesy (L) and of fractal dimension D, $G = L/C_4$, with:

$$C_4 = \left\{ \frac{\Gamma(2D-3)\sin[\pi(2D-3)/2]}{2-D} \right\}^{1/(2D-1)} \tag{16}$$

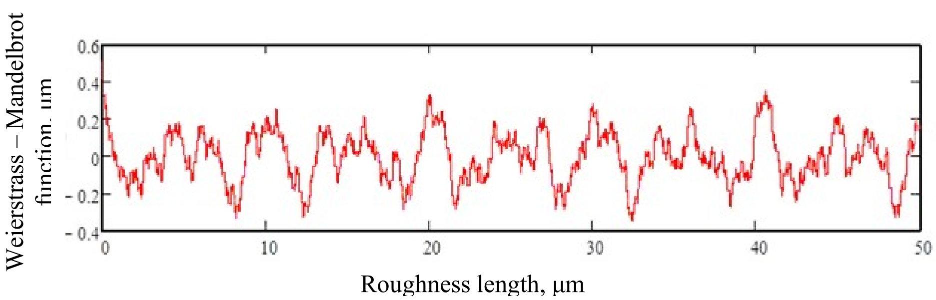

Fig. 5. The Weierstrass – Mandelbrot function (WM) of eroded surface from figure 3.

The WM function can be used to define the curvature at the asperity tip or cavity at level *n* and the critical deformation at beginning of yielding becomes at the randomized abrasive rigid particles. These aspects will be analyzed in a future paper.

5. Conclusions

1. The profiles of eroded surface of helicopter blade by the solid particles are random property.

2. The roughness of surface has a Weibull distribution.
3. The cavities wear traces show ductile behaviour of helicopter blade material.
4. The fractal character of eroded surface has been demonstrated by the structure function of engineering rough surfaces from the helicopter blade.

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