A WEAR MODEL OF CUTTER TOOTH ON BRITTLE MATERIAL

Andrei TUDOR[1], Monica VLASE[2]

Uzura unui dinte tăietor folosit la maşinile de construcţii este cel mai important parametru care defineşte durabilitatea echipamentelor de săpare.
Proprietăţile mecanice ale materialelor din beton armat sau piatră ce urmează a fi săpate şi geometria dintelui tăietor definesc distribuţia presiunii pe suprafaţa de contact şi forţele de săpare.
Proprietăţile reologice ale materialului sunt definite astfel încât deformaţiile locale sunt proporţionale cu presiunea în acel punct. Aceste deformaţii sunt independente de presiunea în punctele din vecinătate.
Scopul acestei lucrari este de a defini un model matematic pentru distribuţia presiunii de contact şi a forţelor de lucru, precum şi pentru a determina profilul uzurii dintelui tăietor.

The wear of a cutting tooth from building machine is the most important parameter, which defines the excavator or pickier tool durability.
The mechanical properties of the ferro-concrete and the stone material and the geometry of the cutter tooth define the pressure distribution on the contact surface and the working forces.
The rheological properties of material are defined that the local deformations are proportional with the pressure in one point. These deformations are independent on the neighbor pressure.
The aim of this paper is to define the mathematical model of the contact pressure distribution, the working forces and the profile of the worn cutter tooth.

Keywords: contact pressure; wear model; durability; cutter contact.

1. Introduction

Its wearing influences the endurance of the cutter tooth of excavator or pickier tool from building machine. This parameter can be determined taking into account the pressure action on the cutting sides of the cutter tooth during working process but also its geometry.

The rheological properties of the ferro-concrete and the stone material are considered to be the Winkler mechanical properties (the elasticity is proportional only to the deformation for every contact point and do not depend on the neighbour points) [1, 2, 3].

[1] Professor, University POLITEHNICA of Bucharest, Romania, e-mail: tudor@meca.omtr.pub.ro
[2] Lecturer, Technical University of Civil Engeneering of Bucharest, Romania

To analyse the tribological contact between cutter tooth and stone or concrete material, we propose a theoretical model, similar with [4,5].

It is considered a two-dimensional cutter tooth (1), which can be characterized by the γ and α angles (fig.1). The cutter tooth (1) has a contact with the concrete or stone material (2).

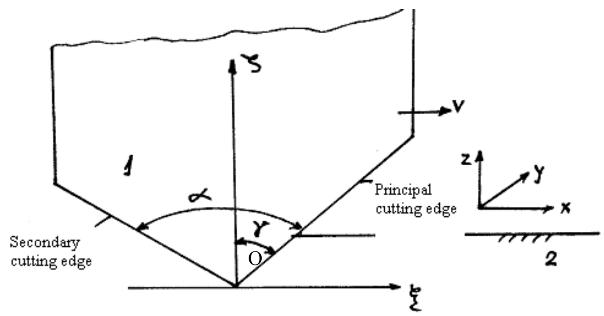

Fig.1. The geometric model for the cutter tooth and material contact.

In the working process, the cutter tooth deforms the material elastically and plastically up to a certain contact pressure, which it is called the critical pressure. The crack appears into the material at the critical pressure and then it propagates up to the free surfaces of the material.

2. Contact pressure model

It is considered a cutter tooth under the form of an excavator machine. The analysis starts from the simplified hypothesis that considers the cutting tool as being bi-dimensional, as the third dimension does not influence in a significant way the process that will be analyzed further on.

Attaching to the tool the axis system (ξ,ζ) having the origin O and to the land material (concrete or stone) the axis system (x, y, z), it is obtained:

$$\begin{aligned} x &= \xi + vt \\ z &= \zeta - c(t) \end{aligned} \tag{1}$$

where:

v – excavating speed; *c*(*t*) – excavating depth; *t* – time.

In the axis system (ξ,ζ), the geometry of the tooth nose is characterized by the function $f(\xi,t)$.

In the excavating process, the tooth deforms elastically the land material under the critical cracking pressure p^*.

The elastically and plastically deformation ($w(x, t)$) is proportional with the material pressure [$\vec{p}_2(x,t)$]:

$$w(x,t) = k\vec{p}_2(x,t) \tag{2}$$

where: *k* – the material elasticity characteristic [6]:

$$k = \frac{(1+\nu)(1-2\nu)h}{(1-\nu)E} \tag{3}$$

where: ν – Poisson's coefficient;

E – elastic modulus;

h – material layer thickness.

The contact condition of the tooth with the working aria, f(ξ,t) can be written under the differential form [4,5]:

$$k_v \vec{p}_2(x,t) + k\frac{d\vec{p}_2(x,t)}{dt} = \frac{dc(t)}{dt} - \frac{df(x-vt,t)}{dt} \tag{4}$$

where k_v is the specific plastic deformation velocity parameter of material.

In the coordinate system (ξ,ζ), the equation (4) becomes:

$$\begin{aligned} & k_v p(\xi,t) + k\left(\frac{\partial p(\xi,t)}{\partial t} - \frac{\partial p(\xi,t)}{\partial \xi} v\right) = \frac{dc(t)}{dt} \\ & + \frac{\partial f(\xi,t)}{\partial \xi} v - \frac{\partial f(\xi,t)}{\partial t} \end{aligned} \tag{5}$$

where: $p(\xi,t) = \vec{p}_2(\xi + vt, t)$.

The wear rate of the tooth is considered to be depending on the result obtained by multiplying the sliding speed (v) with the normal pressure on the contact surface (p_n):

$$\frac{\partial f(\xi,t)}{\partial t} = k_w p_n(\xi,t) \cdot \frac{v}{\cos(\beta_n)} \tag{6}$$

where: k_w – wear parameter of the tooth material. This parameter is a modified Archard parameter [5, 6].

Based on the differential geometry elements, the angle β_n can be defined using the derivative function f(ξ,t), $tg\beta_n = \frac{\partial f(\xi,t)}{\partial \xi}$.

Therefore, the active profile of the cutter tooth varies in the wear process, thus eq. (6) can be write:

$$\frac{\partial f(\xi,t)}{\partial t} = k_w p(\xi,t) v \left[1 + \left(\frac{\partial f(\xi,t)}{\partial \xi}\right)^2\right] \tag{7}$$

For the excavation material type, the contact pressure on the tooth can be determined following the differential equation (5). Then, the form of the worn-out profile of the tool is evaluated (equation 7), considering the excavation process stationary, with the concrete or stone cracking and wearing of the cutter tooth.

3. Determination of the pressure distribution on the edges of the cutter tooth

Stationary process with concrete or stone-cracking and wearing of the cutter tooth (sharp nose)

The case of the angular cutter tooth is considered as having the wear rate under the form of:

$$\frac{\partial f(\xi,t)}{\partial t} = \begin{cases} k_w p(\xi,t) v \left(1 + m_{u1}^2\right) & \text{for } \xi \le 0 \\ k_w p(\xi,t) v \left(1 + m_{u2}^2\right) & \text{for } \xi > 0 \end{cases} \tag{8}$$

where: $m_{u1} = \tan(\pi/2+\alpha-\gamma)$ - the angular coefficient of the secondary edge;

$m_{u2} = ctg(\gamma)$ - the angular coefficient of the principal edge.

Taking this into account the solution of the differential equation (5), having the unknown data $p(\xi, t)$, becomes:

$$p_a = \begin{cases} A_{12} + B_{12} \exp\left(\frac{1}{B_{12}} \xi_a + ct_{12}\right) \\ \text{for } \xi_a \le 0 \\ A_{22} + B_{22} \exp\left(\frac{1}{B_{22}} \xi_a + ct_{22}\right) \\ \text{for } \xi_a > 0 \end{cases} \tag{9}$$

where:

$$A_{12} = \frac{c_0 + \frac{m_{u1}}{kp^*}}{\frac{1}{kv}\left[k_v + k_w v\left(1 + m_{u1}^2\right)\right]}$$

$$B_{12} = \frac{1}{\frac{a}{kv}\left[k_v + k_w v\left(1 + m_{u1}^2\right)\right]} \tag{10}$$

$$A_{22} = \frac{c_0 + \frac{m_{u2}}{kp^*}}{\frac{1}{kv}\left[k_v + k_w v\left(1 + m_{u2}^2\right)\right]}$$

$$B_{22} = \frac{1}{\frac{a}{kv}\left[k_v + k_w v\left(1 + m_{u2}^2\right)\right]}$$

with: c_0 – initial cutter depth in working material;

a – constant value on the ξ axe (fig. 3).

Integration constants are determined to the limit: $p_a(1) = p_a(b_{a2})$, where $b_{a2} = b_2/a$, (b_2 is ξ-coordinate of the contact point on the secondary edge).
The pressure distribution is obtained using the expression:

$$p_a(\xi_a) = \begin{cases} A_{12}\left[1-\exp\left(\dfrac{\xi_a - b_{a2}}{B_{12}}\right)\right] = p_{a1} \\ \text{for } \xi_a \le 0 \\ A_{22}\left[1-\exp\left(\dfrac{\xi_a - 1}{B_{22}}\right)\right] = p_{a2} \\ \text{for } \xi_a > 0 \end{cases} \tag{11}$$

The dimensionless abscissa on the secondary cutting side of the cutting tool is deduced out of the condition of pressure continuity in point $\xi_a = 0$:

$$b_{a2} = -B_{12} \ln\left\{1 - \frac{A_{22}}{A_{12}}\left[1-\exp\left(-\frac{1}{B_{22}}\right)\right]\right\} \tag{12}$$

In Fig. 2 is presented the distribution of non-dimensional pressures $p_a(\xi_{a1}, \xi_{a2})$ for different values of the wearing coefficient k_w and of the speed parameter $v_a = v/p^*k_v$; $p_a = p/p^*$.
(v_a = 0.1, 0.5, 10; k_w = 0.0000001, 0.000001, 0; $\beta = 70^o$, $\gamma = 15^o$; $p^* = 10^4$ MPa)

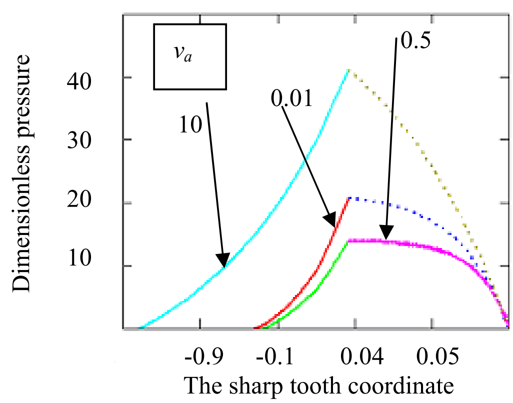

Fig. 2. Pressure distribution on active faces of the tooth.

Stationary process with concrete or stone-cracking and wearing of the cutter tooth (rounded nose)

It is considered a tool characterized by angles α and γ and the connection radius *r* between the two edges (fig. 3).

In this case, the differential pressure equation will have different expression for the three areas of the cutting edges: EB-secondary cutter edge, BC-rounded end angle of the tooth, CI-main cutter tooth edge.
The solution of the differential equation (5), in this case, for the intervals $\xi_a \in [\xi_{aE}, \xi_{aB}]$ and $\xi_a \in [\xi_{aC}, \xi_{aI}]$ is:

$$\begin{aligned} p_a(\xi_a) &= A_{12}\left[1-\exp\left(\frac{\xi_a-\xi_{aE}}{B_{12}}\right)\right] \\ p_a(\xi_a) &= A_{22}\left[1-\exp\left(\frac{\xi_a-1}{B_{22}}\right)\right] \end{aligned} \tag{13}$$

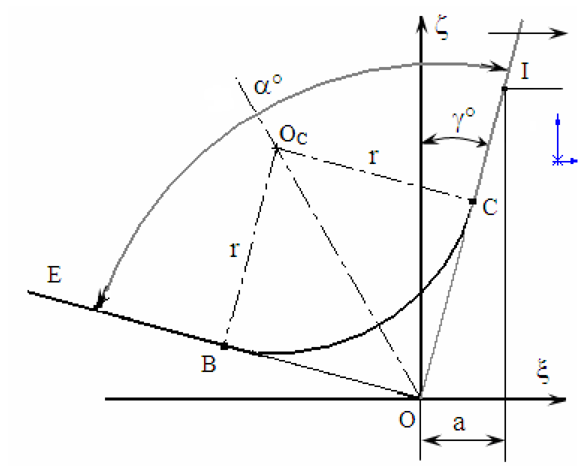

Fig. 3. The shape of the rounded tool.

For the interval $\xi_a \in [\xi_{aB}, \xi_{aC}]$, the solution can be written under the form:

$$\begin{aligned} p_a(\xi_a) &= exp\left[\int_{\xi_{aE}}^{\xi_a} g_1(\xi_a)d\xi_a\right]\cdot \\ &\cdot\left\{\int_{\xi_{aE}}^{\xi_a} g_2(\xi_a)exp\left[-\int_{\xi_{aE}}^{\xi_a} g_1(\xi_a)d\xi_a\right]d\xi_a + ct_3\right\} \end{aligned} \tag{14}$$

where: $g_1(\xi_a) = \frac{a}{kv}\left[k_v + k_w v\left(1+g^2(\xi_a)\right)\right]$; $g_2(\xi_a) = -\frac{c_0}{kv}\frac{a}{p^*} - \frac{av}{kvp^*}g(\xi_a)$

The integration constant ct3 and the integration limit ξ_{aE} are determined from the continuity condition of pressure in points C and B:
Therefore we obtain:

$$ct_3 = A_{12}\left[1-\exp\left(\frac{\xi_{aB}-\xi_{aE}}{B_{12}}\right)\right]$$

$$\xi_{aE} = \xi_{aB} - B_{12}\ln\left\{\begin{array}{l} 1-\dfrac{A_{22}\left[1-\exp\left(\dfrac{\xi_{aC}-1}{B_{22}}\right)\right]}{A_{12}\exp\left[\displaystyle\int_{\xi_{aB}}^{\xi_{aC}} g_1(\xi_a)d\xi_a\right]}+ \\ +\dfrac{1}{A_{12}}\displaystyle\int_{\xi_{aB}}^{\xi_{aC}} g_2(\xi_a)\exp\left[-\int_{\xi_{aB}}^{\xi_{aC}} g_1(\xi_a)d\xi_a\right]d\xi_a \end{array}\right\} \qquad (15)$$

The equation (14), determined using constants ct_3 and ξ_{aE} (obtained from the relation (15) using MathCAD program) is used for representing the pressure distribution on BC area of the tool nose (Fig. 4).

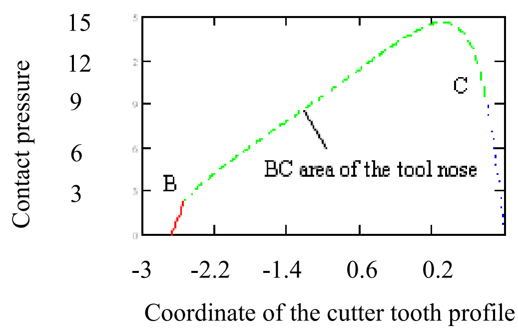

Fig. 4. Pressure distribution to the nose of the rounded tooth

4. Determination of worn out profile of the cutter tooth

The case of completely sharpen tooth

From the equation (9), taking in to account the wear rate under the form (8), was deduced the differential equation, having the unknown data p(ξ, t)

$$\frac{dp_a}{d\xi_a}(\xi_a, t_a) = \frac{a}{kv}\left[k_v + k_w v\left(1+m_{u1,2}^2\right)\right]p_a - \\ - \frac{c_0}{kv}\frac{a}{p^*} - \frac{m_{u1,2}}{k}\frac{a}{p^*} \qquad (16)$$

with $\xi_a=\xi/a$; $t_a=t/t^*$ and $p_a=p/p^*$

Integrating correspondently with the time, it is obtained the equation of the relative profile ($f_a = f/a$):

$$f_a(\xi_a, t_a) = \begin{cases} f_{oa}(\xi_a) + k_w p^* t_a (1 + m_{u1}^2) p_{a1}(\xi_a) \\ \quad for\ b_{a2} < \xi \le \xi_{as1}^* \\ f_{oa}(\xi_a) + k_w t_a (1 + m_{u1}^2) \\ \quad for\ \xi_{as1}^* \le \xi \le 0 \\ f_{oa}(\xi_a) + k_w t_a (1 + m_{u2}^2) \\ \quad for\ 0 \le \xi \le \xi_{as2}^* \\ f_{oa}(\xi_a) + k_w p^* t_a (1 + m_{u2}^2) p_{a2}(\xi_a) \\ \quad for\ \xi_{as2}^* < \xi \le 1 \end{cases} \tag{17}$$

where: $t_a = t/t^*$ is the non-dimensional contact time of the tool with the material characterized by critical pressure p^*, elasticity parameter k and specific plastic deformation velocity parameter k_v.
In Figure 5 is presented the form of the worn tooth.

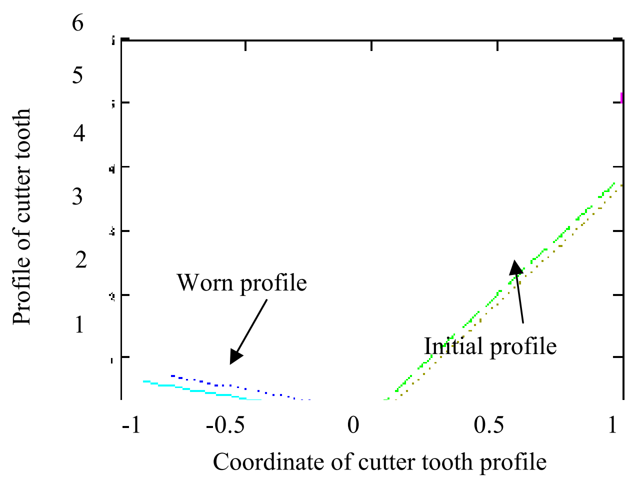

Fig. 5. The profile of the worn-out cutter tooth.

The case of the rounded tooth nose

The form of the worn out profile of the tooth $f_a(\xi_a, t_a)$ is determined integrating the equation:

$$\frac{\partial f_a(\xi_a, t_a)}{\partial t_a} =$$

$$k_w p^* p_a(\xi_a, t_a) v \left[1 + \left(\frac{\partial f_a(\xi_a, t_a)}{\xi_a} \right)^2 \right] =$$

$$= \begin{cases} k_w p^* p_a(\xi_a, t_a) v \left[1 + m_{u1}^2 \right] \\ for \ \xi_{aE} \le \xi_a \le \xi_{aB} \\ k_w p^* p_a(\xi_a, t_a) v \left[1 + m_{u2}^2 \right] \\ for \ \xi_{aC} \le \xi_a \le 1 \\ k_w p^* p_a(\xi_a, t_a) v \cdot \\ \cdot \left\{ 1 + \left[\frac{\xi_a - \xi_{aO_c}}{\sqrt{r_a^2 - (\xi_a - \xi_{aO_c})^2}} \right]^2 \right\} \\ for \ \xi_{aB} < \xi_a \le \xi_{aC} \end{cases} \quad (18)$$

Considering that at the moment t =0, the profile is the initial one:

$$f_a(\xi_a, t_a) = f_{oa}(\xi_a) + \\ + k_w p * p_a(\xi_a) t_a \left[1 + \left(\frac{\partial f_a(\xi_a)}{\partial \xi_a} \right)^2 \right] \quad (19)$$

The effect of cutter geometry (α, γ angles), relative time and wear coefficient about the form of cutter teeth corner is shown in the Figures 6, 7 respectively.

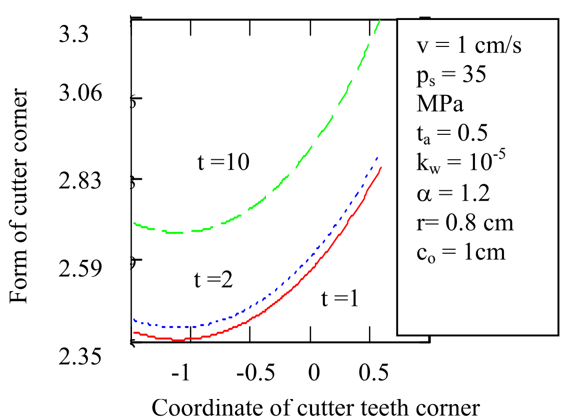

Fig.6: Effect of wear and time on form of cutter

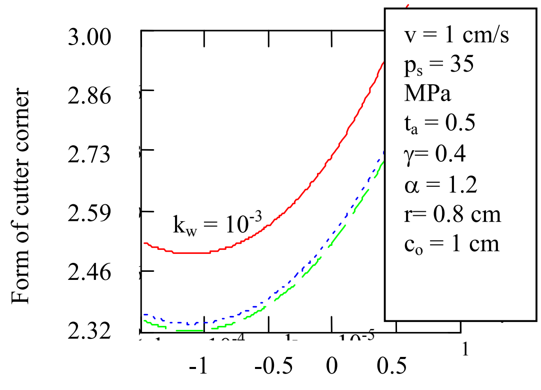

Fig.7. Effect of wear coefficient on cutter form.

5. Conclusions

Analyzing the pressure distribution and the profile of the worn cutting tool in both cases of the tool nose (sharp and rounded) it is obtained some important conclusions:

- The maximum pressure point is situated around the C point (fig. 4), where the wear is maximum.
- The maximum pressure contact point corresponds with the maximum wear point, even the contact pressure is bigger than the critical Winkler material pressure;
- The wear of the cutting tool can be evaluated by the modifying of the cutting angle, which involves the decrease of the cutting productivity;
- The optimization of the tool life can be realized if are correctly chosen the cutting angle, the value of the rake angle and the tool shape.

R E F E R E N C E S

[1] *O. G. Checkina, I.G. Goryacheva and V.G. Krasnik,* "The model for tool wear in rock cutting", Wear*,* 198, pp.33-38, 1996.

[2] *I.G. Goryacheva*, "Changes in contact characteristics of junction during wear", in Int.Congress EUROTRIB-1989*,* **Vol.3,** pp.360-366, 1989.

[3] *P. Munoz-Escalona, and Z. Cassier*, "Influence of the critical cutting speed on the surface finish of turned steel", Wear, 218, pp.103-109, 1998.

[4] *A. Tudor,* ***C****. Tache, A. Tache,* "A Cutter Model for Manufacturing Winkler Brittle Material", AIMETA International Tribology Conference, September 20-22, L'Aquila, Italy, pp. 320-327, 2000.

[5] *A. Tudor, C. Tache, A. Tache,*"A wear model of cut for manufacturing the Winkler brittle material" 7 th International Conference, on Tribology, September 4-5, Budapest, Hungary, pp. 66-69, 2000.

[6] *A. Tudor, M. Dumitru, M. Negriu, I. Radu,* "A wear erosion – corrosion mape of carbide and ceramic composites" U.P.B. Sci. Bull. Series D, Vol. 63, No 2, 2001, pp.53-60.