STRENGTH ANALYSIS OF SHELLS ON THE BASIS OF PRINCIPLE OF CRITICAL ENERGY

Valeriu V. JINESCU[1], Angela CHELU[2], Vali-Ifigenia NICOLOF[3], Georgeta ROMAN (Urse)[4]

Nowadays the strength analysis of shells consists in: - analysis of crack free shell on the basis of analytical method or/and finite elements method far away from the crack; - fracture analysis at the tip of the crack with fracture mechanics concepts. The official rules for shells analysis as part of pressure equipment are insufficient, limited and sometimes difficult to apply to fracture analysis. In the paper a quite different method of shell strength analysis is proposed, based on the principle of critical energy, taking into account the deterioration due to crack. For cracked shells, on the basis of principle of critical energy, generalized failure and allowable envelope were obtained. The critical stress was correlated with the deterioration due to crack geometry (depth, length and width). Some experimental results of cracked tubular and cracked rectangular specimens have been used to show the dependence of the deterioration on the crack geometry and on the type of loading. Static single loaded and static double loading cracked specimens were analyzed as to obtain the values of deterioration, in a particular case. Some of the data depend only on crack length and depth. The crack width influence on the deterioration has been analyzed in the case of penetrate and unpenetrate crack in a specimen with rectangular section.

Keywords: strength of shells; deterioration due to crack; width of crack; critical stresses; principle of critical energy.

1. Introduction

Two problems arise in the calculation of shells as part of pressure equipment (PE), namely: - the calculation of crack free equipment; - the calculation of cracked equipment.

At present the two issues are dealt with distinctly [1]: - crackless equipment is treated on the basis of the classical relationships of loading below

[1] Prof., Industrial Process Equipment Department, University POLITEHNICA of Bucharest, Romania, e-mail: vvjinescu@yahoo.com
[2] Lecturer, Industrial Process Equipment Department, University POLITEHNICA of Bucharest, Romania, e-mail: angela.chelu@upb.ro
[3] Technical College „Carol I", 52nd Porumbacu Str., Bucharest, 060366, Romania, e-mail: yphygenia@ yahoo.com
[4] Asist., Industrial Process Equipment Department, University POLITEHNICA of Bucharest, Romania, e-mail: ursegeanina@yahoo.com

the yield stress [2]. These relationships are covered by international codes such as the ASME Code [3], the European Standard [4] or the British Standard [5]; - the cracked pressure equipment is currently analyzed based on the concepts of fracture mechanics. The calculation norms for the evaluation based on fracture mechanics are limited and insufficient [1].

On the basis of principle of critical energy [6; 7] in the paper one proposes: - a calculation method based on the deterioration caused by the existing crack; - a method for calculating the deterioration caused by the action of a single load, or by the superposition of different loads.

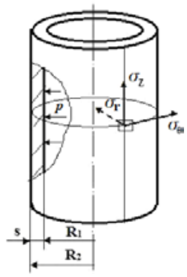
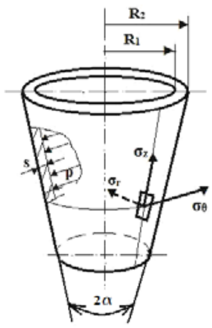
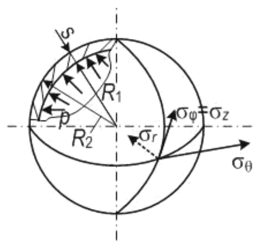
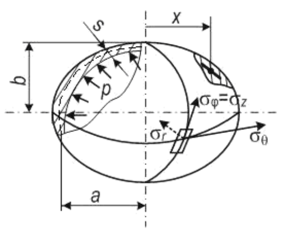
Low and medium pressure equipment consist of axially-symmetrical revolution shells, characterized by the dimensionless ratio (Table 1),

$$\beta = R_2/R_1 = 1 + s/R_1\ , \qquad (1)$$

where R_1 is the inner radius; R_2 is the outer radius, and $s=R_2-R_1$ is the wall thickness. The state of the stress in the shells is considered plane (Table 1); radial stress, σ_r, is neglected.

Table 1.

Main stresses in shells under inner pressure, p (processed after[2;8])

Shell / Stress	Cylindrical	Conical	Spherical	Ellipsoidal
				
σ_z	$\frac{p \cdot R_m}{2 \cdot s}$	$\frac{p \cdot R_m}{2s \cdot \cos\alpha}$	$\frac{p \cdot R_m}{2 \cdot s}$	$\frac{p \cdot a}{s} \cdot f_1$
σ_θ	$\frac{p \cdot R_m}{s}$	$\frac{p \cdot R_m}{s \cdot \cos\alpha}$	$\frac{p \cdot R_m}{2 \cdot s}$	$\frac{p \cdot a}{s} \cdot f_2$
R_m = 0.5 (R_1 + R_2) - the radius of the median surface in the calculated section; *a* and *b* - the large semi-axis and the small semi-axis of the median surface of the ellipsoid; *x* - coordinate in the direction of the large semi-axis.				
$f_1 = \sqrt{a^2 - (x/a)^2 \cdot (a^2 - b^2)} / (2b)$; $\quad f_2 = f_1 \cdot \left[2 - a^4 / \left(a^4 - x^2 \cdot (a^2 - b^2)\right)\right]$				

The main stresses, σ_z and σ_θ, in usual shells under internal pressure, *p*, have the expressions in Table 1. Nowadays one calculates the strength far from the crack and – separately – one analyzes the load at the tip of the crack in terms of fracture mechanics.

- The maximum stress in a shell with a rectilinear generator (Table 1) produced by the inner pressure is,

$$\sigma_\theta(p) = 2\sigma_z(p). \tag{2}$$

In current norms, the relations for assessing load values under internal pressure only are based on the third theory of strength (Tresca), according to which, since $\sigma_r(p) \approx 0$, the equivalent stress,

$$\sigma_{ech} = \sigma_{\max} - \sigma_{\min} = \sigma_\theta(p) - \sigma_r(p) = \sigma_\theta(p), \tag{3}$$

The design on the classical version, is based on condition,

$$\sigma_{ech} \le \sigma_{al}, \tag{4}$$

where σ_{al} is the allowable stress, calculated for the crack-free material,

$$\sigma_{al} = \max\left(\sigma_y / c_y ; \sigma_u / c_u\right), \tag{5}$$

where σ_y and σ_u are the yield stress and the ultimate stress, respectively; $c_y > 1$; $c_u > 1$ are safety coefficients.

Currently, cracks are not considered in the computation of PE strengths.

However, it is imperative that under the given operating conditions, if crack propagation is possible, it should not lead to brittle fracture.

Since stresses are generally lower than the yield stress when the vessel is under load, the risk assessment of crack is done by using, for example, the *stress intensity factor* concept [9],

$$K_I = Y \cdot \sigma \cdot \sqrt{\pi \cdot a}, \tag{6}$$

where σ is the normal stress acting perpendicular to the crack, a - the crack depth; Y - geometric factor depending on the crack shape and length.

Parameter K_I refers to the force acting at the tip of the crack and is a good criterion for the material fracture. In fracture mechanics, it is considered that *unstable breaking* occurs when it becomes equal to its critical value called *fracture toughness*, $K_{I,c}$, which is a measure of the material ability to resist crack growth. K_I is parameter of the structure (analogous to stress σ), while $K_{I,c}$ is a material parameter (analogous to yield stress, σ_y). The strength condition is,

$$K_I \le K_{I,al}, \tag{7}$$

where allowable stress intensity factor $K_{I,al} = K_{I,c} / c_K$, $c_K > 1$ is the safety coefficient. From relations (6) and (7) there results that for safety reasons, for a structure with cracks, stress is limited, namely,

$$\sigma \le K_{I,al} / \left(Y \cdot \sqrt{\pi \cdot a}\right). \tag{8}$$

At the same time, it is recommended that the crack depth meet the condition [3] $a \le a_{al}$, where the allowable depth $a_{al} = a_{cr} / c_a$ and $a_{cr} = a\,(K_{I,c})$, while $c_a > 1$ is the safety coefficient.

For pressure vessels in ASME Code, Section VIII recommend [3],

$$a_{al} \leq 0.25 \cdot s\,, \tag{9}$$

where *s* is the thickness of the wall.

Consequently, *nowadays we determine the safe load carrying capacity of a structure* subjected to stress σ, using two approaches:

– based on the classical strength of materials theory (4). Generally, the stress should not exceed the yield stress, σ_y or $\sigma_{al} \leq \sigma_y$;

– based on fracture mechanics (7); in the linear elastic fracture mechanics, the stress intensity factor should not exceed the fracture toughness ($K_I < K_{I,c}$).

2. Structure damage / deterioration

The concept of damage / deterioration was proposed by Kachanov [10] and the concept of damage degree was proposed by Gilormini et. al. [11].

Until our papers the damage concept was used for: to signal the fracture [12,13], the softening of material by the accumulation of damage parameter [14], or weaken as a result of microstructural damage [15], or of intergranular damage propagation and transgranular damage propagation [16]; to quantify the micro damage evolution [17]. The damage problem was solved on the other hand in connection with: damage accumulation and crack initiation [18-20], crack damage under cyclic loading [21,22], estimation the fatigue life [23, 24], damage models [25-28], damage mechanism [29,30], fatigue damage parameters evaluation [31;32] etc.

By deterioration the strength capacity of the mechanical structure decreases; it may be used decreasing the stress state. By damage the mechanical structure is destroyed. This is why we refer the deterioration instead of damage.

The critical stresses or the critical loads of the structures were not expressed in literature as dependent on the deterioration concept before the papers [33-40], as a result of the principle of critical energy [7;41;42] application.

Some proposals correlate the critical load with the reported crack depth.

For example, the burst pressure of a pipe calculated using ASME B31G code [43-45], as well as the critical bending moment [46] correlated with the reported crack depth. Not deterioration was defined in connection with the crack geometry.

In our proposal the design of shells is based on Eq. (4), were the allowable stress depends on the cracked critical stress (σ_{cr} (D)),

$$\sigma_{al}(D) = \sigma_{cr}(D)/c_D\,, \tag{10}$$

and the critical stress depends on the deterioration, *D*. Here $c_D > 1$ is the safety coefficient.

The paper deals with the correlation of the critical stress, $\sigma_{cr}(D)$, with the deterioration due to crack, *D*, taking account the crack geometry (depth, length and width). Deterioration of cracked tubular specimens statically or fatigue loaded were calculated.

3. Failure and allowable envelope

By superposing the stress σ (far away from the crack tip) and the crack effect in the stress intensity factor K_I, in the frame of Linear Elastic Mechanics (the critical stress is the yield stress), in the case stress σ opens the crack, based on the *principle of critical energy* [6;7;41;42], the following stress criterion was obtained [38],

$$\left(\sigma/\sigma_Y\right)^2 + \left(K_I/K_{I,c}\right)^2 = 1. \qquad (11)$$

On the basis of the Eq. (11) one obtained the generalized failure envelope (Fig. 1,a), a typical failure assessment diagram indicated safe and failure zones.

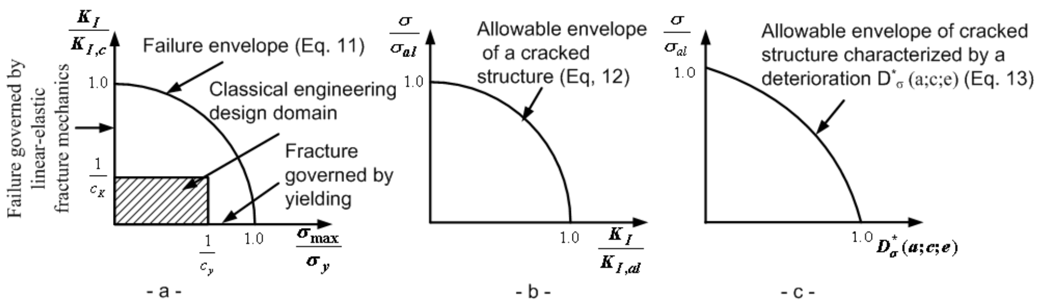


Fig. 1. Failure (a) and allowable (b) envelope taking into account the crack; c - allowable envelope taking into account the deterioration due to crack.

Taking into account the coefficients of safety (c_K and c_y) a classical engineering design domain has been defined.

If in Eq. (11) σ_y is replaced by σ_{al}, and $K_{I,c}$ is replaced by $K_{I,al}$ one obtains,

$$\left(\sigma/\sigma_{al}\right)^2 + \left(K_I/K_{I,al}\right)^2 = 1, \qquad (12)$$

which is *allowable envelope of the engineering design domain* of a cracked structure, as a result of principle of critical energy (Fig. 1, b).

For a cracked structure characterized by a deterioration *D*(a; c; e)* with respect to allowable state, on the basis of the principle of critical energy instead of Eq. (12), one obtains the following correlation,

$$\left(\sigma/\sigma_{al}\right)^2 = 1 - D_\sigma^*(a;c;e), \qquad (13)$$

which describes *the allowable envelope of a cracked structure* (Fig. 2) where *a* is the crack depth, *2c i*s the crack length while *e* is the crack width.

The classical calculation of strength with the general relation (4) and the calculation based on fracture mechanics concepts (7), as currently done, are distinct calculations.

4. Critical stress correlation with crack deterioration based on the Principle of Critical Energy (PCE)

The problem arises of carrying out the calculation of strength by considering the existing crack at a given time, irrespective of the concepts of fracture mechanics currently used for this purpose.

One correlates the fracture strength of the material with the crack deterioration with respect to critical state [35;36;47;48], written as *D (a, c, e)*.

Axial cracks are most dangerous in cylindrical and tronconical shells because they are perpendicular to σ_θ (*p*) (Table 1). If the total axial stress σ_z,, is the highest, in the same shell, the most dangerous cracks are the circumferential ones.

One takes up the general case of materials with non-linear behavior, power law, according to the laws [7],

$$\sigma = M_\sigma \cdot \varepsilon^k \text{ and } \tau = M_\tau \cdot \gamma^{k_1}, \tag{14}$$

where σ, τ is the normal and shear or tangential stress, respectively; ε - strain; γ- shear strain; M_σ, M_τ, k and k_1 are material constants, which are experimentally determined.

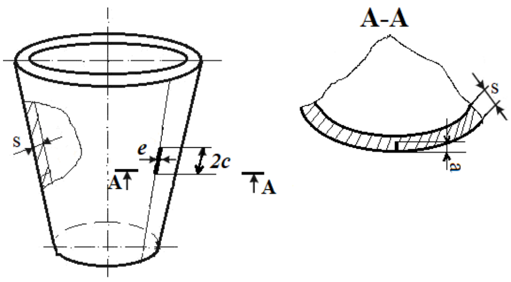


Fig. 2. Conical shell with outer axial crack: *a* – crack depth; 2*c* – crack length and *e* – crack width.

When the PE is *statically loaded*, the critical stresses (eg. fracture) of the cracked material feature the following expressions were obtained on the basis of PCE, starting with Eq. (13):

- critical normal stress,

$$\sigma_{cr}(a;c;e) = \sigma_{cr} \cdot \left[1 - D_\sigma(a;c;e)\right]^{\frac{1}{\alpha+1}}; \tag{15}$$

- critical shear stress,

$$\tau_{cr}(a;c;e) = \tau_{cr} \cdot \left[1 - D_\tau(a;c;e)\right]^{\frac{1}{\alpha_1+1}}, \tag{16}$$

where σ_{cr}; τ_{cr} is the critical normal and shear stress, respectively, of the specimen without cracks; $D_\sigma(a;c;e)$; $D_\tau(a;c;e)$ is the crack induced deterioration under the action of the normal or shearing stress, respectively; $\alpha=1/k$ and $\alpha_1=1/k_1$.

Deterioration D(a;c;e) is a dimensionless variable that ranges between zero and one [10;49].

Here the critical stresses and the deteriorations depend on the depth, length and *width* of the crack. In previous papers these material features were considered as dependent only on depth and length of failure.

With a PE, the actual stress state is calculated, as appropriately, with the relations in Table 1. If the PE is cracked, then the allowable stress is calculated with the Eq. (10), where $D = D(a;c;e)$.

The strength conditions (4) that take into consideration the crack deterioration, in the case of a cracked PE under normal stress loading becomes,

$$\sigma_{ech} \le \sigma_{al}(D). \quad (17)$$

In this way, one resorts to a single way of calculating fractured structures.

One uses *only the concept of equivalent stress* far from the crack area and one imposes the condition that it should be lower than the allowable stress: – *far out from the crack* the condition (4) must be fulfilled; – *at the tip of the crack*, the condition (17) must be fulfilled.

5. Deterioration calculation of cracked tubular specimens, statically loaded, on the basis of the Principle of Critical Energy

The practical use of relationships (13), (15) and (16) requires the assessment of the amount of deterioration caused by the crack. Under fatigue stress, deterioration D_σ increases with an increase in the number of loading cycles.

By laser beam welding (LBW) the fatigue deterioration evolution of the LBW joints was higher than the base material during cyclic loading [50].

One further calculates the deterioration D_σ(a;c;e) on the basis of literature data [50-52] and our own experiments [35;36;47].

There are two cases of calculating deterioration in some tubular specimens: - *under one load* (internal pressure or axial force); - *under two loads simultaneously* (internal pressure and bending moment or internal pressure and axial force).

5.1. Static single loading

For cracked tubular specimens, by neglecting the influence of crack width *e*, there have been proposed deterioration relationships in the case of circumferential cracks and axial cracks [35;36;47].

Deterioration of a shell with a circumferential semielliptical crack at the inner surface (Fig.3, a) under an axial force, F. For such a tubular specimen one showed the reported axial force $F_{cr}(a;\theta)/F_{cr}$ on the reported crack depth *a/s* (Fig. 3, b), where in F_{cr}; $F_{cr}(a;\theta)$ is the fracture axial force of crack free and cracked specimens, respectively, featuring dimensions *a* and *2θ*. On the basis of these data and using relation (15) one calculated the deterioration, $D_\sigma(a;\theta)$ in the case of linear-elastic behavior (k=1 and α=1) of the shell material with the relation,

$$D_\sigma(a;\theta) = 1 - \left(\sigma_{cr}(a;\theta)/\sigma_{cr}\right)^2 = 1 - \left(F_{cr}(a;\theta)/F_{cr}\right)^2 . \qquad (18)$$

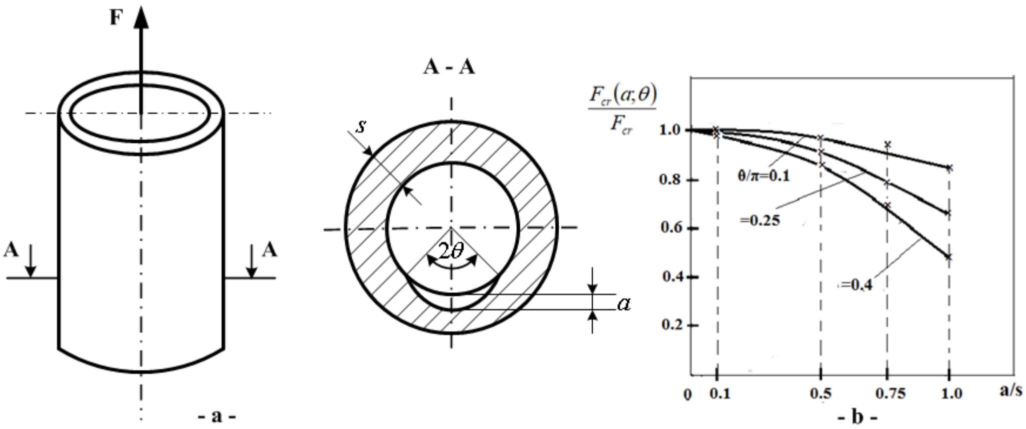


Fig. 3. Tubular specimens with a circumferential semielliptical crack at inner surface (*a*) and the dependence of the reported force, $F_{cr}(a;\theta)/F_{cr}$ on the reported crack depth (*a/s*) for three values of the angle θ/π (processed after [52]) (*b*).

With the Eq. (18) and the Fig. 3, *b* one obtains the deterioration variation represented in Fig. 4, for the tubular cracked specimen (Fig. 3, *a*).

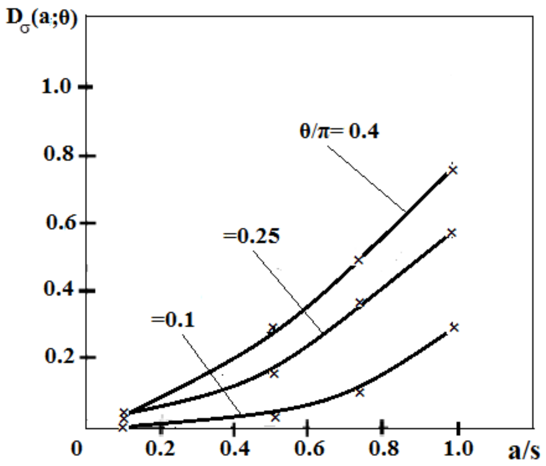


Fig. 4. The dependence of the deterioration of the tubular specimen (Fig. 3,*a*) on reported crack depth (*a/s*) and reported angle (θ/π).

5.2. Static double loading

Deterioration in this case is obtained on the basis of the principle of critical energy [6], by writing that the total participation due to the two loads is equal to the critical participation where in deterioration occurs,

$$P_T = P_{cr}(t) . \quad (19)$$

For behavior, under static loads, according to laws (14), in the case of linear-elastic materials, for more loads one writes,

$$\sum_i \left(S_{i,cr}(a;\theta)/S_{i,cr}\right)^2 = 1 - D(a;\theta) , \quad (20)$$

where $S_{i,cr}$ is the critical values of S_i in the case of a crack-free specimen, while $S_{i,cr}(a;\theta)$ is the critical value of S_i for a cracked specimen.

• *Loading under inner pressure, p, and bending moment M_b*

In this case Eq. (20) becomes,

$$\left(\frac{p_{cr}(a;\theta)}{p_{cr}}\right)^2 + \left(\frac{M_{b,cr}(a;\theta)}{M_{b,cr}}\right)^2 = 1 - D(a;\theta), \quad (21)$$

where p_{cr}, $M_{b,cr}$ are the critical values of the uncracked material; $p_{cr}(a;\ \theta)$, $M_{cr}(a;\ \theta)$, are the critical values of the cracked material.

The deterioration results from the relationship (21) written in the following form: $D(a;\theta) = 1 - \left[\left(p_{cr}(a;\theta)/p_{cr}\right)^2 + \left(M_{b,cr}(a;\theta)/M_{b,cr}\right)^2\right]$. (22)

Fig. 5 shows the dependence of the reported critical pressure $p_{cr}(a;\theta)/p_{cr}$ on the critical bending moment $M_{b,cr}(a;\theta)/M_{b,cr}$ for a tubular specimen with semi-elliptical circumferential crack at the inner surface (Fig. 3, a). The points (x) come from paper [52]. The curve contains these points. Due to the deterioration one obtains the curve 1, instead the curve 2 for crackless sample, using the same Eq. (21) with $D(a;\theta) = 0$.

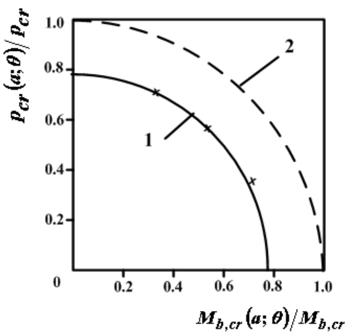
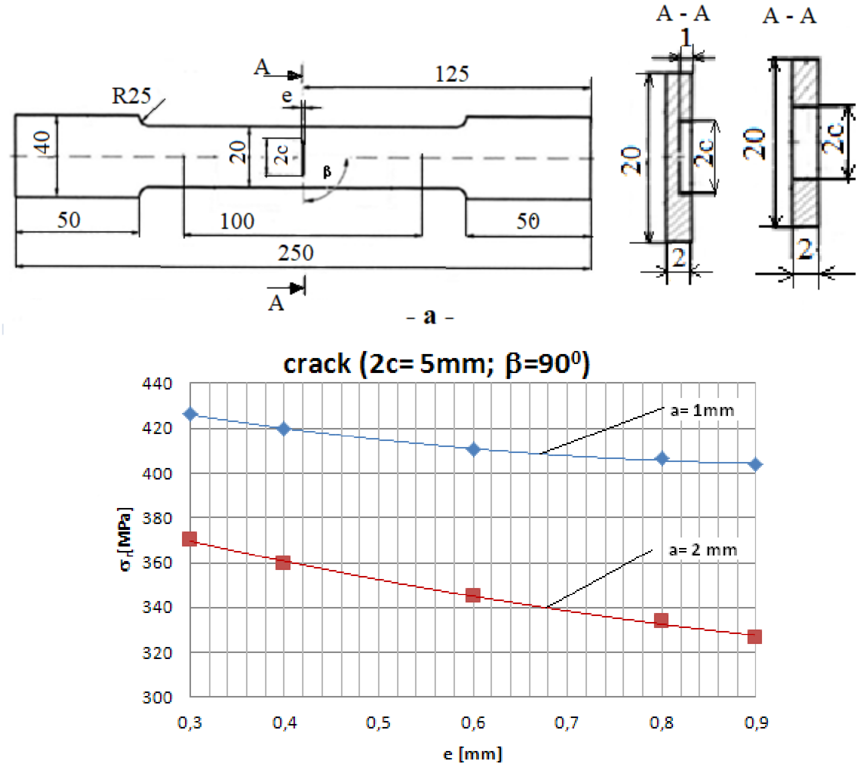


Fig. 5. The correlation between the reported critical pressure and the reported critical bending moment (according to relation (21)) for a tubular specimen: 1 - with a semi-elliptical circumferential crack at the internal surface, with a/s = 0.75 and θ/π = 0.4; ((x) come from the paper [52]); 2 - crackless specimen.

6. Influence of crack width on deterioration

Recent studies [48] on rectangular section specimens (Fig. 6) whose rectangular section features unpenetrated and penetrated cracks, have shown that with the increase in crack width, the material critical characteristics diminish, which means that deterioration increases.



- b -

Fig. 6. a – OL 304 specimen with *rectangular section* with a perpendicular rectangular crack to the direction of loading, unpenetrated (a=1*mm*) and penetrated (a=2*mm*), respectively; b – variation of specimen fracture strength, $\sigma_r = \sigma_u$ ($a;c;e$), *depending on crack width e*, for a=1*mm* and a=2*mm* (processed after [48]).

For example, for the specimen OL 304 with unpenetrated crack (a =1 mm), with e =0.3 mm, the total deterioration was $D_\sigma(a;c;e)$ = 0.885, while for e =0.9 mm, the total deterioration was higher, $D_\sigma(a;c;e)$ = 0.992. Higher the crack width, e, higher is the deterioration and less the critical and the allowable stress, respectively.

7. Fatigue deterioration

For example, the deterioration *D(n)* against the percentage of fatigue number of cycles *n/N* for the LBW joints and the base material are presented in Fig. 7 [50], where *n* is the number of cycles of loading, while *N* is the fatigue life time.

The deteriorations were near zero at steady-state when $n/N \leq 0.65$ and increased significantly after this value for welded joint, but only after $n/N = 0.80$ for base metal. Consequently, in relations (13), (15), (16) and (20) the deterioration does not influence the values of critical state if $n/N \leq 0.65$ - for welded joint and if $n/N \leq 0.80$ for base metal.

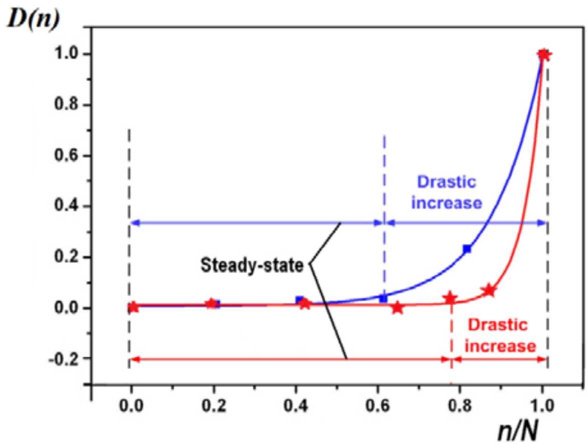


Fig. 7. Fatigue deterioration curves of *laser beam welded joint* (▪) and *base metal* (*) (processed after [50]).

8. Conclusions

The nowadays strength calculation of crackless and cracked shells on the basis of international codes were analyzed. Taking into account the principle of critical energy for cracked shells relationships and diagrams of failure envelope and allowable envelope have been derived.

Instead of calculation procedure recommended by the official design codes, a new strength calculation was proposed, taking into account the concept of critical stress which depends on the deterioration induced by cracks.

For linear - elastic behavior on the basis of principle of critical energy, relation (18) and (22) for deterioration calculation were obtained. With these relations taking into account some experimental results, the deterioration dependence on the crack geometry (depth, length or depth, length and width) were represented. The static single loading, static double loading and fatigue single loading were analyzed. One has found that the crack width affects the value of the deterioration and the critical stress.

The introduction in classical strength calculation of the critical stress concept based on the crack induced deterioration, as is done in this paper, allows the calculation based on the concept of equivalent stress, both away from the crack and at the tip of the crack. It is easier to determine the critical stress of a specimen featuring a certain crack, than to determine the expressions of actual stresses at the crack tip or to experimentally determine the fracture toughness.

Instead of the current method which calculates PE with classical strength relations and verifies the crack area with the concepts of fracture mechanics, we resorted to the evaluation of the crack zone strength by comparing the equivalent stress to the allowable stress, calculated by considering the deterioration caused by the crack.

Acknowledgements

This work was supported in part by University Politehnica of Bucharest, through the “Excellence Research Grants” Program, UPB – GEX. Identifier: UPB–EXCELENŢĂ–2017 Research project title, Contract number 45/25.09.2017 ME14-17-02, ID126.

R E F E R E N C E S

[1]. *V.V. Jinescu, V. I. Iordăchescu (Nicolof),* “Proiectarea echipamentelor sub presiune cu considerarea deteriorării produsă de fisuri”, Buletinul AGIR, Supliment **2**, 2015, pp.3-12.

[2]. *V.V. Jinescu*, Calculul şi Construcţia Utilajului Chimic, Petrochimic şi de Rafinării, vol.1, Editura Didactică şi Pedagogică, Bucureşti, 1983.

[3]. *** ASME Code, Boiler & Pressure Vessel Code, Division VIII, Section 3.

[4]. *** European Standard, EN 13445-3, 2009.

[5]. *** British Standard, PD 5500, 2009.

[6]. *V.V. Jinescu,* Tratat de Termomecanică, vol.1, Editura AGIR, Bucureşti, 2011.

[7]. *V.V. Jinescu*, Principiul energiei critice şi aplicaţiile sale, Editura Academiei Române, Bucureşti, 2005.

[8]. *J. F. Harvey*, Theory and Design of Pressure Vessels, Second Edition, Reinhold Comp., New York, 1974.

[9]. *E. E. Gdoutos*, Fracture Mechanics – An Introduction, Second Edition, Springer, Dordrecht, The Netherlands, 2005.

[10]. *L. M. Kachanov*, “On rupture time under condition of crack”, Izv. Akad. Nauk. SSSR, Otd. Techn. Nauk, vol. **8**, 1958, pp. 26 - 51.

[11]. *P. Gilormini, C. Licht, P. Siquet*, “Growth of voids in a ductile matrix - A review”, Arch. Mech., **40** (1), 1988, pp. 43 - 80.

[12]. *C. Smith, A. Kanvinde, G. Deierlein*, “A local criterion for ductile fracture under low-traxiality axisymmetric stress states”, Eng. Fracture Mech., **169**, 2017, pp. 321-335.

[13]. *T.W.J. Geus, R.H.J. Peerlings, M.G.D. Geers*, “Fracture in multi-phase materials: why sme microstructures are more critical than others”, Eng. Fracture Mech., **169**, 2017, pp. 354-370.

[14]. *S. Arun, A. Sherry, M.C. Smith, M. Skeikh*, “Simulations of the large - scale four point bending test using Rousselier model”, Eng. Fracture Mech., **178**, 2017, pp. 497-511.

[15]. *N. Prakash, G. Seidel,* "Computational electromechanical peridynamics modeling of strain and damage sensing in nanocomposite bonded explosive materials (NCBX)", Eng. Fracture Mech., **177**, 2017, pp. 180-202.

[16]. *C. Graci, M.H. Ali Abadi*, "Micromechanical boundary element modeling of transgranular and intergranular cohesive cracking in polycrystalline materials", Eng. Fracture Mech., **176**, 2017, pp. 351-374.

[17]. *H. Li, D. Yang, Z. Zhong, Y. Sheng, X. Liu*, "Experimental investigation on the micro damage evolution of chemical corroded limestone subjected to cyclic loads", Eng. Fracture Mech., **113**, 2018, pp. 23-32.

[18]. *A. Haghshenas, M.M. Khonsari*, "Damage accumulation and crack intiation detection based on the evolution of surface roughness parameters", Int. J. Fatigue, **107**, 2018, pp. 130-144.

[19]. *M. Kamaya*, "Fatigue crack tolerance design for stainless steel by crack growth analysis", Eng. Fracture Mech., **177**, 2017, pp. 14-32.

[20]. *D. Li, Q. Zhu, Z. Zhou, X. Li, P.G. Ranjith*, "Fracture analysis of marble specimens with a hole under uniaxial compression by digital image correlation", Eng. Fracture Mech., **183**, 2017, pp. 109-124.

[21]. *S. Pan, G. Yu, S. Li, X. He, R. Chen*, "Experimental and numerical study of crack damage under variable amplitude thermal fatigue for compacted graphite iron EN-GJV-450", Int. J. Fatigue, **113**, 2018, pp. 184-192.

[22]. *N. K. Cho, H. Chen, J.T. Boyle, F.Z. Xuan*, "Enhanced fatigue damage under cyclic thermo-mechanical loading at high temperature by structural creep recovery mechanism", Int. J. Fatigue, **113**, 2018, pp. 149-159.

[23]. *W. Zhang, W. Jiang, X. Zhao, S.T. Tu*, "Fatigue life of a dissimilar welded joint considering the weld residual stress: Experimental and finite element simulation", Int. J. Fatigue, **109**, 2018, pp. 182-190.

[24]. *C. Braccesi, G. Morettini, F. Cianetti, M. Palmieri,* "Development of a new simple energy method for life prediction in multiaxial fatigue", Int. J. Fatigue, **112**, 2018, pp. 1-8.

[25]. *J. Alfaiate, L.J. Sluys*, "On the modelling of mixed-mode discrete fracture: Part I - Damage models", Eng. Fracture Mech., **182**, 2017, pp. 157-186.

[26]. *M. Rezazadeh, V. Carvelli*, "A damage model for high-cycle fatigue behavior of bond between FRP bar and concrete", Int. J. Fatigue, **111**, 2018, pp. 101-111.

[27]. *K. Keerthana, C.M. Chandra Kishen,* "An experimental and analytical study on fatigue damage in concrete under variable amplitude loading", Int. J. Fatigue, **111**, 2018, pp. 278-288.

[28]. *B. Mohammadi, B. Fazlali,* "Off-axis fatigue behaviour of unidirectional laminates based on a microscale fatigue damage model under different stress ratios", Int. J. Fatigue, **106**, 2018, pp. 11-23.

[29]. *N.R. Petraš, J. Polák*, "Damage mechanism in austenitic steel during high temperature cyclic loading with dwells", Int. J. Fatigue, **113**, 2018, pp. 335-344.

[30]. *Z. Li, N. Limodin, A. Tandjaoui, P. Quaegebeur, J.F. Witz, D. Balloy,* "Influence of Fe content on the damage mechanism in A319 aluminum alloy: Tensile tests and digital image correlation", Eng. Fracture Mech., **183**, 2017, pp. 94-108.

[31]. *C. Horas, J.A.F.O. Correia, A.M.P. De Jesus, P. Kripakaran, R. Calcada*, "Application of the modal superposition technique combined with analytical elastoplastic approaches to assess the fatigue crack initiation on structural components", Eng. Fracture Mech., **185**, 2017, pp. 271-283.

[32]. *J.A.F.O. Correia, P. Raposo, M. Munitz-Calvente, S. Blasson, G. Lesiuk, A.M.P. De Jesus, P.M.G.P. Moreira, R.A.B. Calcada, A.F. Canteli*, "A generalization of the fatigue Kohout-Věchet model for several fatigue damage parameters", Eng. Fracture Mech., **185**, 2017, pp. 284-300.

[33]. *V.V. Jinescu*, "Fatigue Life of the Process Equipment Made of Nonlinear Materials", Rev. de Chimie, **63**, nr. 1, 2012, pp. 98-101.
[34]. *V.V. Jinescu*, "The Principle of Critical Energy, Consequences and Applications", Proceedings of the Romanian Academy, Series A, vol. **14**, nr. 2, 201, pp. 152-160.
[35]. *V.V. Jinescu, V.I. Iordăchescu, N. Teodorescu*, "Relations for the calculation of critical stress in pressure equipment with cracks", Rev. de Chimie, **64**, nr.8, 2013, pp. 858-863.
[36]. *V.V. Jinescu, V.I. Iordăchescu*, "Calculation of Deterioration Due to Cracks in Tubular Specimens", U.P.B. Sci. Bull, Series D, Mechanical Engineering, **76**, Iss.1, 2014, pp. 149-160.
[37]. *V.V. Jinescu, V.I. Nicolof, C. Jinescu, A. Chelu*, "Superposition of Effects in Calculating the Deterioration of Tubular Structures and in Non-newtonian Fluid Flow", Rev. de Chimie, **66**, nr. 5, 2015, pp. 698 – 702.
[38]. *V.V. Jinescu, V.I. Nicolof*, "Strength criteria at the tip of the crack", Eng. Fracture Mechanics, **142**, 2015, pp. 108-115.
[39]. *V.V. Jinescu, V.I. Nicolof, G. Jinescu, S.E. Manea*, "The Principle of Critical Energy is a Transdisciplinary Principle with Interdisciplinary Applications" Proceedings of the International Conference on Interdisciplinary Studies, INTECH ICIS, 2016, pp. 1-23.
[40]. *V.V. Jinescu, V.I. Nicolof, N. Teodorescu*, "Effects superposition under imposed deterioration and simultaneous fatigue in different regimes", Int. J. Damage Mech, **26**, Issue 5, 2017, pp. 633-650.
[41]. *V.V. Jinescu.*, "Principiul energiei critice", Rev. de Chimie, **35**, 1984, pp.858-861.
[42]. *V.V. Jinescu*, Energonica, Editura Semne, Bucureşti, 1997.
[43]. *H.R. Vanaei, A. Eslami, A. Egbewande*, "A review on pipeline corrosion, in-line inspection (ILI), and corrosion growth rate models", Int. J. Pressure Vessels and Piping, **149**, 2017, pp. 43-54.
[44]. *S. Hassan, L. Sweet, J. Huts, G. Valbuena, B. Singh*, "Corrosion risk-based subsea pipeline design", Int. J. Pressure Vessels and Piping, **159**, 2018, pp. 1-14.
[45]. *X. Gao, G. Koval, C. Chazallon*, "Energetical formulation of size effect law for quasi-brittle fracture", Eng. Fracture Mech., **175**, 2017, pp. 279-292.
[46]. *H.S. Zhao, S.T. Lie, Y. Zhang*, "Fracture assessment of mismatched girth welds in oval-shaped clad pipes subjected to bending moment", Int. J. Pressure Vessels and Piping, **160**, 2018, pp. 1-13.
[47]. *Vali-Ifigenia Iordăchescu (Nicolof)*, Cercetări asupra rezistenţei structurilor mecanice cu fisuri, cu aplicaţie la echipamentele sub presiune, PhD Thesis, University Politehnica of Bucharest, 2013.
[48]. *Simona-Eugenia Manea*, Cercetări în vederea atestării gradului de generalitate al principiului energiei critice, cu aplicaţii la suprapunerea şi/sau cumularea efectelor solicitărilor asupra structurilor mecanice şi la poluarea multiplă, PhD Thesis, University Politehnica of Bucharest, 2018.
[49]. *V.V. Jinescu*, "Cumulation of Effects in Calculation the Deterioration of Fatigue Loaded Structure", Int. J. Damage Mech., vol. **21**, 2012, pp. 671-675.
[50]. *J. Liu, X.L. Gao, L.J. Zhang, J.X. Zhang*, "A study of fatigue damage evolution on pulsed Nd: YAG Ti6Al4V laser welded joints", Eng. Fracture Mechanics, **117**, 2014, pp. 84-93.
[51]. *N.H. Kim, C.S. Oh, Y.J. Kim, J.S. Kim, D.W. Jerng, P.J. Budden*, "Limit loads and fracture mechanics parameters for thick-walled pipes", Int. J. Press. Vessels and Piping, **88**, 2011, pp. 403-414.
[52]. *Y.J. Kim, D.J. Shim, K. Nikbin, Y.J. Kim, S.S. Hwan, J.S. Kim*, "Finite element based plastic limit loads for cylinders with part-through surface cracks under combined loading", Int. J. Press Vessels and Piping, **80**, 2003, pp. 527-540.