ROBOT PATH PLANNING WITH A HYBRID ARITHMETIC OPTIMIZATION ALGORITHM

Qiang SONG[1]

To address the premature convergence and search stagnation of arithmetic optimization algorithm (AOA), the paper proposes a hybrid arithmetic optimization algorithm (HAOA) and applies it to the practical robot path planning (RPP) problem. In initialization phase, the chaotic mechanism is used to create high-quality individuals. In evolution phase, the stochastic disturbance method is applied to enhance the exchange of information between different individuals. The solution space information around the candidate solution was integrated by Levy flight to enhance gene diversity and enhance search performance. In order to better solver the RPP problem, the Spline interpolation method is adapted to the HAOA algorithm for path optimization, so as to smooth the path curve and ensure the planning accuracy. Finally, numeric experiments of benchmark functions and RPP instances verify HAOA's excellent performance in accuracy and robustness. This research shows arithmetic optimization algorithm can quickly find the global optimal solution, and has fast convergence speed and accuracy.

Keywords: robot path planning; optimization; arithmetic optimization algorithm; chaos; levy flight

1. Introduction

The core contents of robot navigation include perception, positioning, control and path planning, among which path planning plays an extremely significant role [1]. The path planning task aims at optimizing travel distance and mobile energy consumption and determines an optimized movement route from the starting position to the destination position that meets the obstacle avoidance requirements according to the pre-set position requirements [2]. In recent years, automation and intelligent technology have promoted the rapid upgrading of the smart industry, which makes intelligent mobile robots widely used in many fields, including smart logistics, underwater operations and abnormal environment detection [3]. Therefore, the research on RPP problem decision algorithm is of great significance.

RPP has attracted the research enthusiasm of many scholars at home and aboroad, and its decision-making methods and research results have been fruitful [4]. Classical RPP decision-making methods include A* algorithm, view method

[1] Professor, Dept. of Computer Science and Software Engineering, Zhaoqing University, Zhaoqing, China, e-mail: gdpcit@163.com

and free space method, etc. The algorithms are simple and practical, and have high engineering value in specific business scenarios, but at the same time, there are certain defects [5]. For example, the A* algorithm has the defects of too many node expansion times and too many search nodes, so the overall optimization efficiency of the algorithm is low. The computational amount of free space method is positively correlated with the number of obstacles, and it is difficult to solve business scenarios with complex obstacle space [6]. In recent years, the evolutionary algorithm is rapid, and the adaptation and application of a number of improved versions of the evolutionary algorithm to the RPP problem have achieved good results [7,8]. For example, Ma [9] built an improved algorithm based on sine and cosine algorithm, coupled meme-grouping and reverse learning, and verified the effectiveness of the improved sine and cosine algorithm through numerical experiments. Based on relevant academic studies, it can be seen that evolutionary algorithms should focus on the following problems to solve the RPP problem: (1) improve the efficiency and performance of the basic evolutionary algorithm; (2) adapt the algorithm to the RPP problem [10]. Based on the above description, this paper proposes a new RPP problem decision method based on arithmetic optimization algorithm.

AOA is a new evolutionary algorithm constructed in 2021, which has the advantages of compact architecture, fewer control parameters and high search efficiency. Studies have shown that AOA has more advantages than particle swarm optimization, bee swarm algorithm, firefly algorithm and other biogeographic optimization algorithm [11]. AOA algorithm has been successfully applied and achieved remarkable results in many practical engineering problems, such as feature extraction, text clustering and image classification, etc., the relevant content can be referred to the review literature [12]. For example, Lan [13] built a multi-strategy fusion arithmetic optimization algorithm by integrating Sobol sequence initialization method, reconstruction optimizer acceleration function and introducing chaotic elite mutation method, and successfully applied it to mechanics problems. In view of the excellent performance of AOA algorithm, this paper applies it to RPP problem.

AOA algorithm uses random method to initialize the solution set, which makes the initial solution uneven in the solution space distribution and the quality of the solution is relatively ordinary, which weakens the optimization efficiency of the algorithm in some degree. Similarly, the evolution process of AOA algorithm mainly relies on the optimal solution for the operation of four types of rules to generate candidate solutions, which fails to effectively use the information of all solution individuals and lacks information exchange among different individuals. In addition, considering that the algorithm is prone to stagnation in the late iteration, it is necessary to add a more efficient solution space exploration mechanism. Based on the above analysis, the HAOA algorithm was constructed in this paper. In the initialization stage, chaos mechanism was used to generate an initial solution set with better quality, random disturbance was used to strongly resolve information exchange among individuals in the population evolution stage, and solution space information around candidate solutions was explored by Levy flight, aiming to enhance gene diversity and enhance search performance. To solve RPP problem, this paper adapts Spline interpolation method with HAOA algorithm to optimize the path more efficiently and tries to obtain smooth and high-quality path curves.

2. AOA algorithm

AOA algorithm uses random methods to build initial populations, and updates population locations with the help of four-rule operation. The relevant mathematical mechanisms are summarized as follows [11] :

(1) Population initialization

The algorithm represents the candidate solution with a D-dimensional array $x=(x_1,\cdots,x_d,\cdots)$, $\forall d\in\{1,2,\cdots,D\}$, the coding range is set as $[x_d^l, x_d^u]$, and the population initialization formula is defined as follows:

$$x_d = x_d^l + rand(0,1)\cdot(x_d^u - x_d^l) \tag{1}$$

Where, rand (0,1) represents a random number in the interval [0,1].

(2) Population location update

AOA algorithm adopts multiplication and division operators for global development, and addition and subtraction operators for local mining. The position update formula is as follows:

$$x_d \leftarrow \begin{cases} x_d^* \div (MOP+\varepsilon)\cdot((x_d^u - x_d^l)\cdot\mu + x_d^l) & rand(0,1)\le 0.5 \\ x_d^* \times MOP\cdot((x_d^u - x_d^l)\cdot\mu + x_d^l) & \text{otherwise} \end{cases} \tag{2}$$

$$x_d \leftarrow \begin{cases} x_d^* - MOP\cdot((x_d^u - x_d^l)\cdot\mu + x_d^l) & rand(0,1)\le 0.5 \\ x_d^* + MOP\cdot((x_d^u - x_d^l)\cdot\mu + x_d^l) & \text{otherwise} \end{cases} \tag{3}$$

$$MOP = 1 - \frac{g^{1/\alpha}}{G^{1/\alpha}} \tag{4}$$

Where, x_d^* is the encoded value of the current optimal solution x^* in the dimension d . Parameter μ is used to coordinate the optimization process. The recommended value is 0.499. ε is a minimal positive number used to avoid zero division. In the Math Optimizer probability (MOP) update formula, the parameter g and G are respective the quantity of algorithm iterations and the maximum value of iterations, the recommended sensitivity coefficient α is 5.

AOA algorithm uses an adaptive mechanism to generate parameter MOA (Math optimizer accelerated) to compare the value sizes of MOA with random quantities in the range of [0,1]. If the random quantity is less than MOA, global development is carried out; otherwise, local mining side rate is selected. The adaptive update formula of MOA parameters is defined as follows:

$$MOA \leftarrow MOA^{lb} + \frac{g}{G} \cdot (MOA^{ub} - MOA^{lb}) \tag{5}$$

Among them, the recommended values of MOA^{lb} and MOA^{ub} are 0.2 and 1.0 respectively.

Combined with the above elaboration, the operation process of AOA algorithm is summarized as follows:

Step1. Set the coding length and value domain of the problem according to the problem characteristics and combine the evaluation function of the optimization goal setting algorithm.

Step2. Generate the coding value of the initial solution by random method, evaluate the population and determine the current

optimal solution according to it.

Step3. Update adaptively the parameters of MOA and MOP in the algorithm.

Step4. Update the coding values of the population according to formulas (2)~(5), and evaluate the fitness of the new solutionvector accordingly.

Step5. Reset the former optimal solution.

Step6. If the set conditions are reached, output the optimal solution; otherwise go to Step3.

3. HAOA algorithm

Based on the operation architecture of AOA algorithm, this paper constructs HAOA algorithm with the help of chaos mechanism, random disturbance and Levy flight. Specifically, Fuch mechanism is adopted to build the initial solution to enhance the performance of individual populations in the initial stage. In the process of population evolution, new candidate child solutions are generated by the positional difference component traction between solution vectors and Levy random walk, so as to improve the genetic diversity of the population and enhance the optimization performance of the algorithm.

3.1 Chaos initialization

According to the description of the principle of AOA algorithm, the optimization effect depends on the performance of the initial candidate solution to a certain extent, and the algorithm only uses random methods to generate the initial solution vector, which weakens the performance of the initial solution in some degree. In view of this, this paper introduces chaos mechanism in the initialization stage of AOA algorithm, aiming to build an initial solution with better quality and balanced distribution.

The randomness and ergodicity of chaos mechanism have significant [14] effect on improving population diversity. In particular, Fuch chaotic mapping converges quickly and is ergodically balanced, and its overall performance is outstanding. The initial solution construction method based on Fuch is constructed here, and the expression is as follows:

$$\theta_{i+1} = \cos\left(\frac{1}{\theta_i^2}\right) \tag{6}$$

$$x_d = x_d^l + \mathrm{mod}(\theta_{i+1}, 1) \cdot (x_d^u - x_d^l) \tag{7}$$

Where, θ_0 take the random quantity on the interval (0,1), $\mathrm{mod}(\cdot)$ is the complementary function used to map θ_{i+1} to the interval [0,1].

3.2 Random perturbation

In the stage of population evolution, AOA algorithm generates progeny populations only by means of the current optimal solution, ignoring the genetic information of other individuals and the information exchange among different individuals. In order to overcome the above shortcomings, this paper uses the evolutionary strategy of artificial bee colony algorithm to add random disturbance in the design of HAOA algorithm, that is, comprehensively compares the genetic differences between the current solution and other individuals to generate new progeny solutions, so as to enhance the genetic diversity [15] of the population.

Given the candidate solution x, $\forall d \in \{1, 2, \cdots, D\}$, the random disturbance generates the progeny candidate solution with the help of the following formula:

$$x_d \leftarrow x_d + \varphi \cdot (x_d - x_d^r) \tag{8}$$

Where, φ is a random quantity belonging to [0,1], x_d^r is the encoding of different individuals in dimension d. The power law distribution is used to select the probability values, and the process is as follows: First, all the parent solutions are ranked according to the optimization objective; Secondly, the following formula is used to determine the probability value $p(x)$ of the candidate solution x being selected. The calculation formula is as follows:

$$p(x) = \frac{\exp(-rank(x))}{\sum_{j=1}^{n} \exp(-rank(x^j))} \tag{9}$$

In the expression (9), $rank(x)$ denotes the superior or inferior order of candidate solutions x. The above formula can effectively reconcile the probability of different candidate solutions being selected, and at the same time ensure that high-quality solutions are selected with a larger probability value, so as to ensure that the gene information of high-quality solutions plays a positive role in population evolution.

3.3 Levy Flight

In order to further expand the search coverage and increase the gene diversity of the population, Levy flight [16] is introduced in the evolution process of HAOA algorithm. Relevant studies show that updating population location information by Levy flight has positive significance for enhancing the search performance of the algorithm.

The Mantegna algorithm was used to simulate Levy flight, and the formula was as follows:

$$L(\lambda) = \frac{\mu}{|\nu|^{1/\delta}} \tag{10}$$

$$\mu \sim N\left(0, \sigma_\mu^2\right) \tag{11}$$

$$\nu \sim N\left(0, \sigma_\nu^2\right) \tag{12}$$

$$\sigma_\mu = \left(\frac{\Gamma(1+\delta) \cdot \sin(\pi \cdot \delta / 2)}{2^{(\delta-1)/2} \cdot \Gamma\left((1+\delta)/2\right) \cdot \delta} \right)^{1/\delta} \tag{13}$$

In the espression, $\delta = 1, \sigma_\nu = 1, \Gamma(\cdot)$ is the gamma function.

Based on the above formula description, the mathematical formula of Levy flight disturbance solution is added, and the expression is defined as follows:

$$x_d \leftarrow x_d + \beta \cdot L(\lambda) \tag{14}$$

Where, parameter $\beta > 0$ is used to control step length.

3.4 HAOA algorithm flow

Combined with the above algorithm mechanism description, Fig. 1 combs and plans the complete operation flow of HAOA algorithm.

4 Path Planning

4.1 Environmental Model

As shown in Fig. 2, environmental modeling of RPP problem is carried out, and a series of obstacle sets are distributed around the beginning and end points.

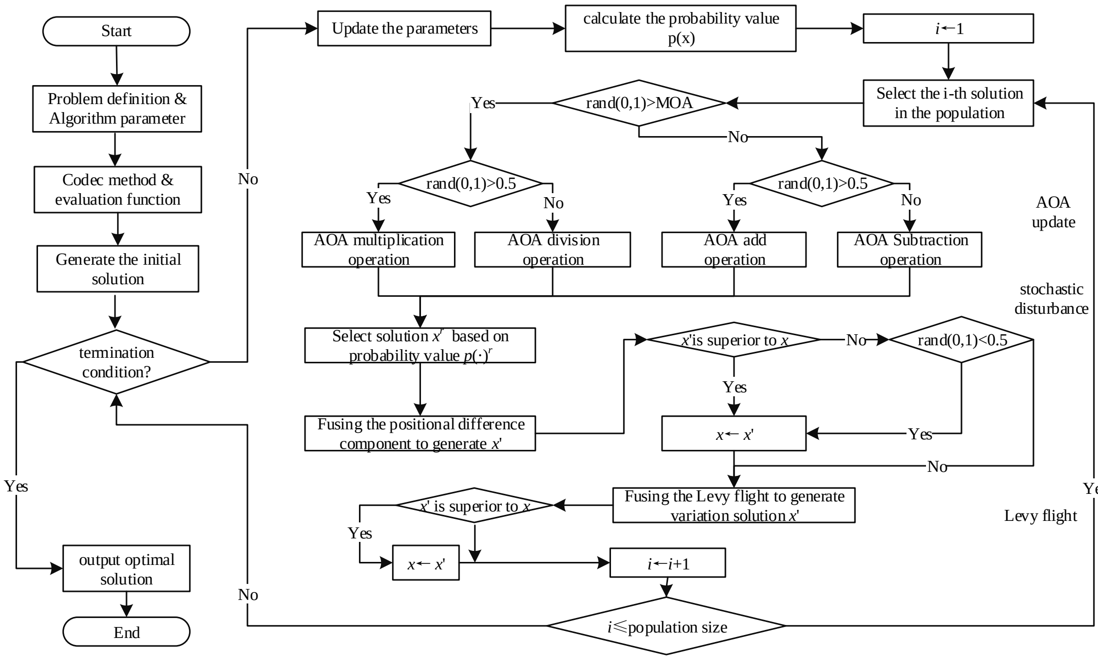

Fig. 1. Flow of HAOA algorithm

Rectangular coordinate system is adopted for modeling, and the mathematical description of each obstacle is as follows:

$$(x-a_k)^2+(y-b_k)^2=r_k^2 \tag{15}$$

In the expression, (a_k, b_k) and r_k respectively represent the central position and radius of the obstacle.

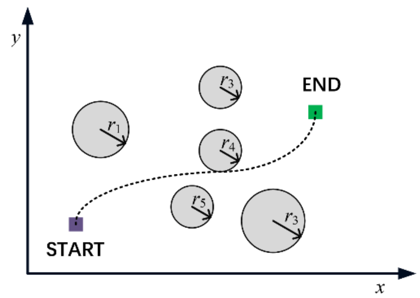

Fig. 2. Environment model of RPP problem

In order to characterize the path characteristics, this paper uses the node set $\pi=\{(x_0,y_0),(x_1,y_1),\cdots,(x_n,y_n),(x_{n+1},y_{n+1})\}$ of two-dimensional space to characterize the path. Specifically, (x_0,y_0) and (x_{n+1},y_{n+1}) represents the starting coordinates and the ending coordinates, and the other nodes are the position coordinates that the algorithm needs to optimize. Obviously, the current problem dimension is uniquely determined by the number n of nodes. Based on relevant studies, this paper introduces Spline spline interpolation method to generate path curves to avoid obvious line edges [17] caused by direct optimization of n node coordinates, and to ensure optimization accuracy, reduce problem dimension and smooth path curves (see Fig. 3).

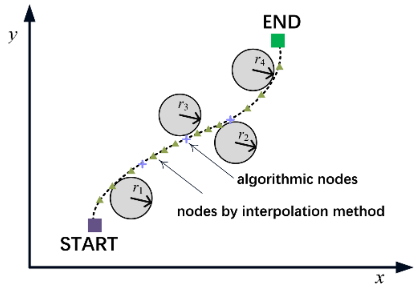

Fig. 3. Diagram of interpolation method

4.2 Path evaluation function

In this paper, minimizing the path length is adopted as the decision goal of the RPP problem currently studied, and in order to effectively deal with obstacle avoidance constraints, the penalty function method is introduced here to construct the path evaluation function. The formulas are as follows:

$$L(\pi)=\sum_{i=1}^{n+1}\sqrt{(x_i-x_{i-1})^2+(y_i-y_{i-1})^2}+\lambda\cdot\sum_{k=1}^{K}\sum_{i=1}^{n}\kappa_{i,k} \tag{16}$$

$$\kappa_{i,k}=\max\left(0,1-\frac{\sqrt{(x_i-a_k)^2+(y_i-b_k)^2}}{r_k}\right) \tag{17}$$

In the expression, the penalty coefficient λ is a larger integer, K represents the total number of obstacles. The formula $\kappa_{i,k}$ is defined in a normalized way, aiming to deal with the obstacle avoidance constraints and to reconcile the constraint violation number of obstacles of different sizes.

5. Experimental analyses

In order to verify the optimization performance of the HAOA algorithm proposed in this paper, the algorithm was applied to Benchmark function test and RPP simulation respectively. The programming platform was selected Matlab2018a. The computer parameters are 2.4GHz, the memory is 4GB and the CPU is Intel(R) Core (TM) i5-2430M.

5.1 Benchmark function test

Benchmark function in literature [11] is selected for experiment. The function information is shown in Table 1, where n is the dimension of the problem. Functions F1~F3 have unimodal characteristics, and functions F4 and F5 have multimodal characteristics. The above benchmark functions have extremely complex solution space characteristics, and the optimization is difficult.

In order to verify the optimization performance of the algorithm constructed in this article, the comparative experiment between HAOA and AOA algorithms is first carried out to verify the effectiveness of the chaos mechanism, random disturbance and Levy flight coupled in this article. The parameters of the simulation experiment are set as follows: the independent variable dimension of the test problem is set to 20 and 40, and the population specification and iteration times of the algorithm are respectively set to 50 and 2000. For each benchmark function, each AOA algorithm is solved independently for 20 times, and the experimental results are collected in Table 2.

Table 1

Basic test functions

Function identification	Function expression	Domain of definition
F1	$y = 10^6 \cdot x_1^2 + \sum_{i=2}^{n} x_i^2$	(-100,100)
F2	$y=\sum_{i=1}^{n} x_i^2$	(-100,100)
F3	$y = \sum_{i=1}^{n-1} (100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2)$	(-50,50)
F4	$y = \sum_{i=1}^{n} (x_i^2 - 10\cos(2\pi x_i) + 10)$	(-5.12,5.12)
F5	$y = \frac{1}{4000}\sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n} \cos(\frac{x_i}{\sqrt{i}}) + 1$	(-100,100)

Table 2

Experimental results of AOA vs HAOA

Functions	Dimensions	Algorithms	Average Standard	Deviation
F1	20	AOA	8.13e-14	2.63e-15
		HAOA	2.84e-20	7.68e-20
	40	AOA	2.08e-08	4.56e-09
		HAOA	5.07e-16	2.17e-16
F2	20	AOA	6.62e-16	6.62e-17
		HAOA	4.93e-22	8.44e-22
	40	AOA	1.30e-12	8.56e-12
		HAOA	4.96e-20	4.68e-20
F3	20	AOA	1.03e-01	8.60e-02
		HAOA	2.27e-04	5.17e-04
	40	AOA	3.39e-01	3.62e-01
		HAOA	1.75e-03	5.42e-03
F4	20	AOA	8.11e-16	7.86e-16
		HAOA	1.95e-19	5.36e-20
	40	AOA	3.73e-13	3.23e-13

		HAOA	1.64e-17	4.57e-17
F5	20	AOA	5.65e-15	4.20e-15
		HAOA	1.69e-17	2.91e-18
	40	AOA	9.99e-12	5.30e-12
		HAOA	2.23e-15	6.01e-16

Based on the above test results, it shows that compared with the basic version of AOA algorithm, HAOA proposed in this paper obtains better mean value index for each benchmark function problem, and the mean value is updated, which shows that the overall search performance of HAOA algorithm is better. Meanwhile, compared with the standard deviation index obtained by 20 independent runs, it indicates that the standard deviation value obtained by the improved algorithm is smaller, that is, the operation results of the HAOA algorithm are more stable than those of the AOA algorithm.

In order to further evaluate the optimization performance of the HAOA algorithm proposed in this article, it is compared with three other improved AOA algorithms, including AAOA [18], ESAOA [19] and CAOA [20]. The independent variable dimension of the test problem is set to 40, the population specifications and iteration times of the four AOA algorithms are set to 50 and 2000 respectively, and the other parameters are set to values in the references. For each benchmark function, each AOA algorithm is solved independently for 20 times. Table 3 indicates the experimental results.

Table 3

Experimental results of four AOA algorithms

Functions	Dimensions	Algorithms	Average Standard	Deviation
F1	40	AAOA	5.38e-13	7.71e-13
		ESAOA	1.09e-14	3.14e-14
		CAOA	7.51e-15	8.54e-15
		HAOA	4.05e-16	3.17e-16
F2	40	AAOA	8.14e-17	7.61e-17
		ESAOA	4.34e-16	2.43e-16
		CAOA	5.72e-18	2.84e-19
		HAOA	5.23e-20	4.97e-21
F3	40	AAOA	1.80e-02	7.95e-02
		ESAOA	2.70e-03	7.80e-02
		CAOA	5.93e-03	3.88e-03
		HAOA	2.95e-03	8.24e-04
F4	40	AAOA	8.43e-14	2.86e-14
		ESAOA	5.65e-17	2.65e-16
		CAOA	8.42e-16	8.92e-17
		HAOA	8.30e-17	6.10e-17
F5	40	AAOA	1.72e-13	6.60e-13
		ESAOA	3.20e-12	7.02e-13
		CAOA	7.20e-14	5.88e-14
		HAOA	7.66e-15	7.33e-15

According to the test results in table 3, it shows that the mean solution index obtained by HAOA algorithm has more advantages than that obtained by CAOA, AAOA and ESAOA algorithms, that is, the mean solution index obtained by HAOA algorithm is smaller than that of its comparison algorithm, which means that the chaos mechanism, random disturbance and Levy flight embed significantly enhance the optimization performance of the basic version of AOA algorithm. At the same time, the smaller standard deviation value index of the HAOA algorithm indicates that it has better robustness and can effectively adapt and solve various complex benchmark function optimization problems.

5.2 Simulation of RPP problem

In order to carry out the simulation optimization experiment of RPP problem, and CAOA, AAOA, ESAOA and HAOA proposed in this paper are adopted to solve it. Two kinds of obstacle environment are set up: Environment 1 is relatively simple, the total obstacle is 6, and the set of obstacle radius values is {7,8,9}; Environment 2 is more complex, the total number of obstacles is 12, and the set of obstacle radius values is {5,6,7,8,9}. The population specifications and iteration times of the four AOA algorithms are set to 30 and 500, respectively, and the other parameters are set to reference values. For each obstacle environment, each AOA algorithm is solved independently for 20 times, and the results are summarized as follows:

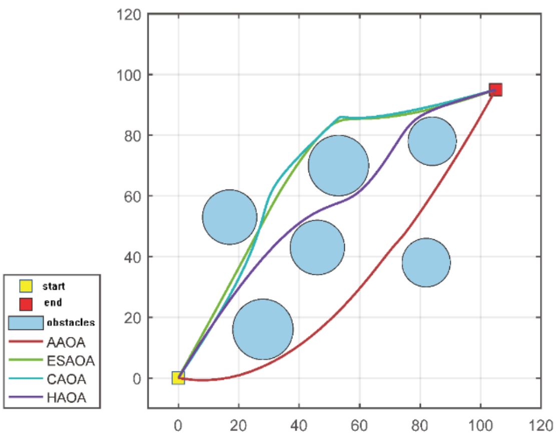

Fig. 4. Comparison of optimal paths in environment 1

Fig. 4 and Fig. 5 respectively show the optimal path curves obtained by each algorithm in solving corresponding examples. According to these, it shows that the optimal path curves obtained by the HAOA algorithm are more advantageous, that is, the optimal, worst and average path length obtained by the algorithm after multiple independent operations are superior to the comparison algorithms.

Meanwhile, Fig. 6 shows the optimal evolution curves of different algorithms for solving each example.

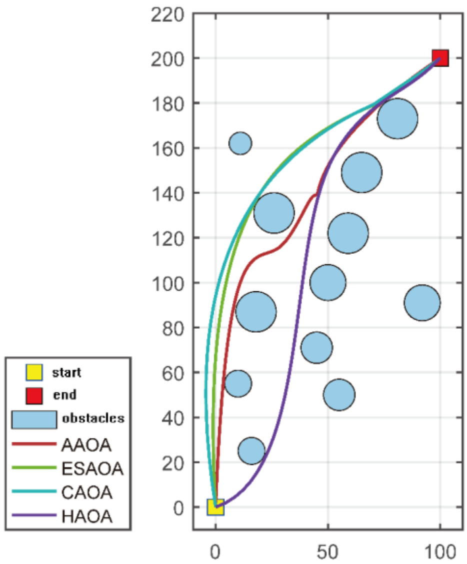

Fig. 5. Optimal path comparison for environment 1

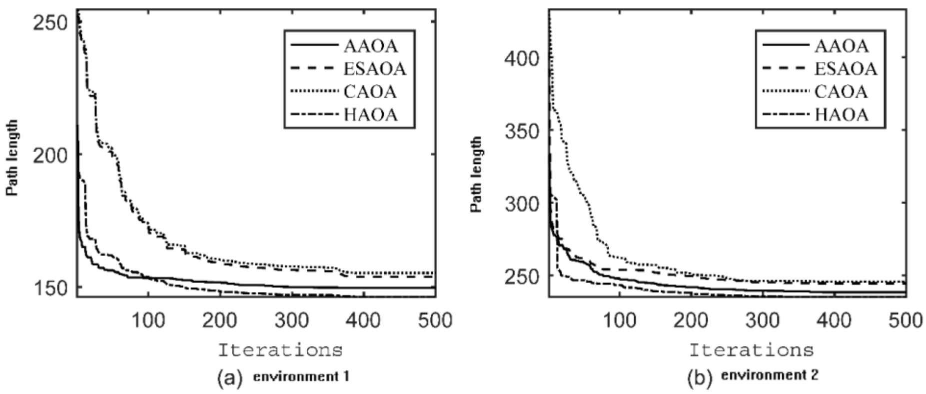

Fig. 6. Comparison of optimal evolution curves

It shows that the embedding of chaos mechanism makes HAOA algorithm to have an initial population of superior quality, while random perturbation and Levy flight further enhance the performance of global exploration and local mining of the algorithm, enabling the algorithm to continuously explore solutions with better quality. Further, Table 4 compares the performance of the four improved algorithms from the three index dimensions of optimal, worst and mean value, and it indicates that the HAOA algorithm constructed in this article is more competitive to some extent.

Table 4

Optimization results of RPP problem

Example	Algorithm	Optimal	Worst	Mean
Environment 1	AAOA	238.47	266.31	254.82
	ESAOA	244.63	268.74	259.77
	CAOA	245.87	273.92	264.29
	HAOA	235.49	263.04	249.99
Environment 2	AAOA	149.73	164.17	156.75
	ESAOA	153.87	165.91	160.03
	CAOA	155.22	169.03	162.87
	HAOA	146.33	161.80	154.47

6. Conclusion

In this paper, HAOA algorithm is constructed and adapted to solve the RPP problem. Based on the AOA algorithm architecture, HAOA algorithm is improved from three aspects: improving the initial population quality, resolving the information exchange between individuals, and enhancing the diversity of its own genes. Compared with the comparison algorithm, numerical experiments show that HAOA algorithm has strong competitiveness in the mean value solution, optimal solution and solution stability.

Subsequently, the research in this paper is expanded to adapt to robot path planning in more specific business scenarios, such as task assignment and path planning scheduling for material handling robots in manufacturing workshops and optimal escape route planning for underwater robots. Meanwhile, it is also of great research value to improve the HAOA algorithm constructed in this paper to adapt to multi-objective decision in complex industrial environment, such as the production scheduling and supply chain optimization of manufacturing enterprise.

Acknowledgement

This work was supported by 2021 Industry-University Cooperative Education Project of Ministry of Education (No.: 202101024034) in China.

R E F E R E N C E S

[1]. *Ab Wahab M N, Nefti-Meziani S and Atyabi A.*, “A comparative review on mobile robot path planning: Classical or meta-heuristic methods”, in Annual Reviews in Control, **vol. 50**, Oct. 2020, pp. 233-252.

[2]. *Mao Jianxu, He Zhenyu,Wang Yaonan, Zhang Hui and Zhong Hang,* “Path planning Technology and Application Review of Power inspection Robot”, in Control and Decision, **vol 38,** no, 11, 2023, pp. 3009-3024.

[3]. *Patle, B. K., Ganesh, B. L., Pandey, A., Parhi, D. R. K. and Jagadeesh, A.*, “A review: On path planning strategies for navigation of mobile robot”, in Defence Technology, **vol 15**, no 4, 2019, pp. 582-606.

[4]. *Koubaa, A., Bennaceur, H., Chaari, I., Trigui, S. and Javed, Y.*, "Introduction to mobile robot path planning", in Robot path planning and cooperation: foundations, algorithms and experimentations, Springer, Cham. **vol 772**. Apr.2018, pp. 3-12.
[5]. *Mac, T. T., Copot, C., Tran, D. T. and Keyser R. D*., "Heuristic approaches in robot path planning: A survey", in Robotics and Autonomous Systems, vol 86, 2016, pp. 13-28.
[6]. *Orozco-Rosas, U., Montiel, O., and Sepúlveda, Roberto.*, "Mobile robot path planning using membrane evolutionary artificial potential field", in Applied Soft Computing, **vol. 77**, 2019, pp. 236-251.
[7]. *Tian, S., Li, Y., Kang, Y. and Xia, J.*, "Multi-robot path planning in wireless sensor networks based on jump mechanism PSO and safety gap obstacle avoidance", in Future Generation Computer Systems, **vol. 11**, 2021, pp. 37-47.
[8]. *Rahmaniar W. Rakhmania A. E.*, "Mobile robot path planning in a trajectory with multiple obstacles using genetic algorithms", in Journal of Robotics and Control (JRC), **vol. 3**, no. 1, 2022, pp. 1-7.
[9]. *Ma Yingying, Du Nuannan*, "Path planning of robot based on improved sine-cosine algorithm", in Journal of Chongqing Jiaotong University (Natural Science Edition), **vol. 40**, no. 9, 2021, pp. 17-23.
[10]. *Wang Yongqi, Jiang Xiaoxiao,* "Path planning of robot based on Hybrid Gray Wolf Algorithm", in Computer Engineering and Science, **vol. 42,** no. 7, 2020, pp, 1294-1301.
[11]. *Abualigah, L., Diabat, A., Mirjalili, S. and Abd Elaziz, M.*, "The arithmetic optimization algorithm", in Computer methods in applied mechanics and engineering, **vol. 376**, Apr. 2021, pp. 113609.1-38.
[12]. *Dhal K. G., Sasmal B. and Das A.*, "A comprehensive survey on arithmetic optimization algorithm", in Archives of Computational Methods in Engineering, **vol. 30**, no. 5, 2023, pp. 3379-3404.
[13]. *Lan Zhouxin, He Qing*, "Multi-strategy fusion arithmetic optimization algorithm and its engineering Optimization", in Application Research of Computers, **vol. 39**, no. 3, 2022, pp. 758-763.
[14]. *Jin Q, Lin N. and Zhang Y.,* "K-Means Clustering Algorithm Based on Chaotic Adaptive Artificial Bee Colony", in Algorithms, **vol. 14**, no. 53. Feb. 2021.
[15]. *Gao H., Shi Y., Pun C. M. and Kwong S.*, "An improved artificial bee colony algorithm with its application", in IEEE Transactions on Industrial Informatics, **vol.15**, no. 4, 2018, pp. 1853-1865.
[16]. *Liu Y, Cao B.*, "A novel ant colony optimization algorithm with Levy flight", in IEEE Access, **vol. 8**, 2020, pp. 67205-67213.
[17]. *Elbanhawi, M., Simic, M. and Jazar, R. N.*, "Continuous path smoothing for car-like robots using B-spline curves", in Journal of Intelligent & Robotic Systems, **vol. 80**, 2015, pp. 23-56.
[18]. *Abualigah L., Elaziz M. A., Yousri D., Al-Qaness M. A., Ewees A. and Zitar R. A.*, "Augmented arithmetic optimization algorithm using opposite-based learning and lévy flight distribution for global optimization and data clustering", in Journal of Intelligent Manufacturing, **vol. 34**, no. 8, 2022, pp. 1-39.
[19]. *Elkasem A. H. A., Kamel S., Hassan M. H., Khamies M. and Ahmed E. M.*, "An eagle strategy arithmetic optimization algorithm for frequency stability enhancement considering high renewable power penetration and time-varying load", in Mathematics, **vol. 10**, no. 6, 2022, pp.1-38.
[20]. *Aydemir S. B.*, "A novel arithmetic optimization algorithm based on chaotic maps for global optimization", in Evolutionary Intelligence, **vol. 16,** no. 3, 2023, pp. 981-996.