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MATHEMATICAL MODELING AND NUMERICAL SIMULATION OF SLUDGE THICKENING PROCESSES IN GRAVITATIONAL SLUDGE THICKENERS

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Aspectele teoretice de mişcare ale nămolului prin instalaţiile din gospodăria de nămol nu au fost elucidate. În literatura de specialitate, problemele care apar în gospodăria de nămol, referitoare la curgerea acestuia, precum şi la separarea fazelor, sunt rezolvate numai din punct de vedere tehnic. Lucrarea prezintă modelarea şi simularea procesului de concentrare a sistemului bifazic format din apă – mediu de dispersie şi particule solide în decantoarele gravitaţionale, care este deosebit de importantă pentru studiul formării păturii de nămol îngroşat, pentru precizarea dimensiunii acesteia, precum şi pentru stabilirea formei geometrice optime pentru proces.

The theoretical aspects of sludge movement through the sludge handling equipments have not been solved entirely until now. In the literature, the sludge handling problems (flow and phases separation) have been solved only from the technical point of view. This paper presents the mathematical modeling and numerical simulation of the thickening process of the biphasic system composed from water (the dispersion medium) and the solid particles in gravitational thickeners. This simulation is important for the formation of thickener sludge laps, to determine the laps dimensions and also to establish the optimum geometric dimensions of the thickening equipments.

Keywords: sludge, thickening, mathematical modeling, numerical simulation, thickener.

1. Introduction

The sludge concentration is a unitary process needed for the reduction at 50 per cent of sludge volume that will enter into the sludge handling [1]. In this way the thickeners design and maintenance costs used in the sludge treatment will be reduced. For sludge concentration, physical processes based on the specific weight difference of sludge phases are used.

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In this way, the following processes can be used [2]:

- ✓ Sedimentation processes in gravitational masic forces field – sludge thickener;
- ✓ Sludge concentration by flotation, which accelerate the separation of the phase. This process needs extra energy for the formation of gas bubbles into the sludge;
- ✓ Accelerate separation processes by using the centrifugal masic forces field – solid bowl conveyor centrifuge for sludge separation.

The concentration process of aqueous solutions (the concentration in dry solid substances is between 1 and 2 %) is insufficiently studied theoretically. The first steps of thickening are the granular particles sedimentation and sludge concentration.

2. Sludge movement equations

Sludge thickener is present in sludge handling in all wastewater treatment plants. The development of this equipment is similar to the circular settling tank; the difference between them is that the retention time is bigger [3]. The necessity to increase the retention time appears because the particles sedimentation is difficult due to the multiphase medium concentration increment.

For the process study, the following hypotheses are considered:

- ✓ The initial sedimentation speed of sludge bed particles is constant.
- ✓ On the way of particle settling, the sludge bed concentration increases so the sedimentation speed continuously decreases.
- ✓ The sedimentation speed varies with concentration increase, with holes fraction decrease, so it is correctly to consider the particle final velocity as $w_p = \frac{gd^2}{18\upsilon} f(c)$, where g is the gravity, d is the particle diameter, υ is the kinematic viscosity of multiphase medium, and c is the holes fraction.
- ✓ The suspense solid particle sedimentation is a displacement movement.
- ✓ In the sedimentation process, the particles fusion through flocculation appears, and the result of this process is the formation of bigger size particles.
- ✓ The influence of thickener walls is neglected.
- ✓ The sedimentation speed near the thickener bottom area is reduced until it becomes zero.

For the simplification of the equations system, the multiphase fluid has two constituents – the biphasic medium composed of the basic fluid medium (water) and the dispersed solid particles into the liquid. From the point of view of the particle shape, the shape and dimensions of all particles are identical.

Since the sludge thickener movement is reproduced identical in all positions, the movement equations are reduced to the bi-dimensional form:

a) Movement equations for x and y axis

$$\frac{\partial}{\partial t}(\rho_i C_i u_i) + u_i \frac{\partial}{\partial x}(\rho_i C_i u_i) + v_i \frac{\partial}{\partial x}(\rho_i C_i u_i) = - C_i \frac{\partial p_i}{\partial x} + \rho_i C_i X + v_i \Delta(\rho_i C_i u_i) + n_{xi} + (p_d - p)\frac{\partial C_i}{\partial x} \tag{1}$$

$$\frac{\partial}{\partial t}(\rho_i C_i v_i) + u_i \frac{\partial}{\partial x}(\rho_i C_i v_i) + v_i \frac{\partial}{\partial x}(\rho_i C_i v_i) = - C_i \frac{\partial p_i}{\partial y} + \rho_i C_i Y + v_i \Delta(\rho_i C_i v_i) + n_{yi} + (p_d - p)\frac{\partial C_i}{\partial y}$$

b) Continuity equation

$$\frac{\partial(\rho_i C_i)}{\partial t} + \frac{\partial(\rho_i C_i u_i)}{\partial x} + \frac{\partial(\rho_i C_i v_i)}{\partial y} = h_i \tag{2}$$

in which $i = 1, 2$ for the case of two constituents (fluid f and solid s), p_d is the diffusive pressure, p is the hydrostatic pressure, n is the interaction forces between constituents, C is the concentration, u and v are the velocity components and ρ is the density.

Because the solid particles are dispersed into the fluid, the solid phase viscosity terms are neglected. So, the equation for the holes fraction evolution becomes:

$$\frac{\partial}{\partial t}\left[\rho_f c u_f + \rho_s(1-c)u_s\right] + u_f \frac{\partial}{\partial x}(\rho_f c u_f) + u_s \frac{\partial}{\partial x}\left[\rho_s(1-c)u_s\right] = + \nu_f\left[\frac{\partial^2}{\partial x^2}(\rho_f c u_f) + \frac{\partial^2}{\partial y^2}(\rho_f c u_f)\right] \tag{3}$$

On the Oy axis, with the neglection of solids cinematic viscosity terms, the movement equation of the two phases becomes

$$\frac{\partial}{\partial t}\left[\rho_s(1-c)v_s\right] + u_s \frac{\partial}{\partial x}\left[\rho_s(1-c)v_s\right] + v_s \frac{\partial}{\partial y}\left[\rho_s(1-c)v_s\right] = -\frac{\partial p}{\partial y} - \rho_f c g - \rho_s(1-c)g \tag{4}$$

Because the expression of the pressure partial derivative is $\frac{\partial p_f}{\partial y} = -\rho_f g$, by replacing it in equation (4), the following equation results:

$$\frac{\partial}{\partial t}\left[\rho_s(1-c)v_s\right]+u_s\frac{\partial}{\partial x}\left[\rho_s(1-c)v_s\right]+v_s\frac{\partial}{\partial y}\left[\rho_s(1-c)v_s\right]= \\ =\rho_f(1-c)g-\rho_s(1-c)g \tag{5}$$

3. Equations solutions with variable holes fraction

The solution can be obtained through numerical integration of equations (3) and (5) in the following hypothesis and conditions:

- ✓ The movement in the sludge thickener is permanent;
- ✓ The sludge sedimentation speed depends on holes fraction, $v_s = w_0\, c^n\,(1-c)$, where n = 2.4 ... 4.6 for spherical particle, w_0 is the terminal velocity of the granular particles; in this condition the sedimentation speed decreases from 6 to 1.8 m/h.

In the circular settling tank, due to the central alimentation, the speed of fluid medium decreases according to the expression $u_f = \frac{Q_n}{2\pi r\, h}$, where Q_n is the sludge flow rate introduced into the tank, r is the vector radius dimension, and h is the vertical height. If the surface loading rate is w_s = 0.2 ...0.4 $m^3/m^2\cdot h$ the sludge flow rate becomes $Q_n = (0.2...0.4)\frac{\pi D^2}{4}$, in which D = 10...50 m is the tank diameter.

So, the horizontal water speed is

$$u_f = \frac{Q_n}{2\pi r\, h} = \frac{(0{,}2\ldots 0{,}4)\pi D^2}{8\pi x h} = \frac{K}{x},$$

in which x is the variable radius of thickener tank. The horizontal speed of the solid particle is $u_s = u_f - v_r$, in which v_r is the relative speed between the two constituents.

Introducing all these specifications in equation (3), the following relationship will result:

$$\frac{\partial}{\partial t}\left[\rho_f c\frac{K}{x}+\rho_s(1-c)\left(\frac{K}{x}-v_s\right)\right]+\frac{K}{x}\frac{\partial}{\partial x}\left(\rho_f c\frac{K}{x}\right)+ \\ +\frac{K}{x}\frac{\partial}{\partial x}\left[\rho_s(1-c)\left(\frac{K}{x}-v_s\right)\right]=v_f\left[\frac{\partial^2}{\partial x^2}\left(\rho_f c\frac{K}{x}\right)+\frac{\partial^2}{\partial y^2}\left(\rho_f c\frac{K}{x}\right)\right] \tag{6}$$

Equation (6) represents the differential equation of the holes fraction repartitions into the sludge thickener.

4. Numerical simulation

For obtaining the optimum solution, the equation (6) will be numerical integrated with the FlexPDE software [4, 5]. The thickener has a cylinder-conic shape. The holes fraction is denoted with *c.*

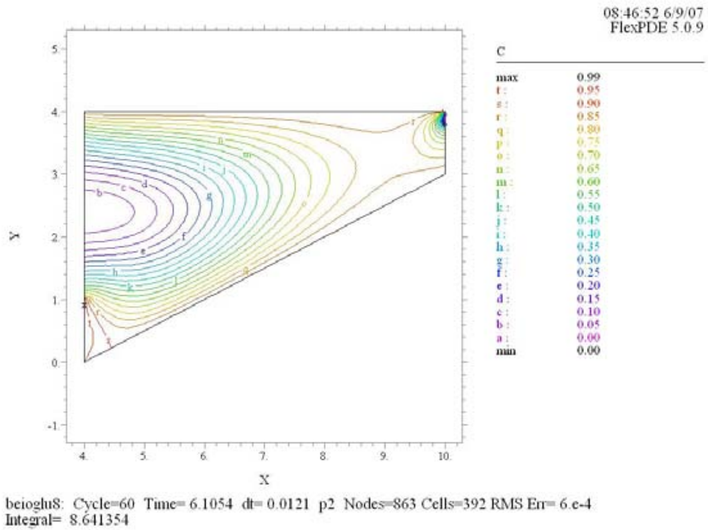

Fig. 1. The holes fraction evolution for fines particles.

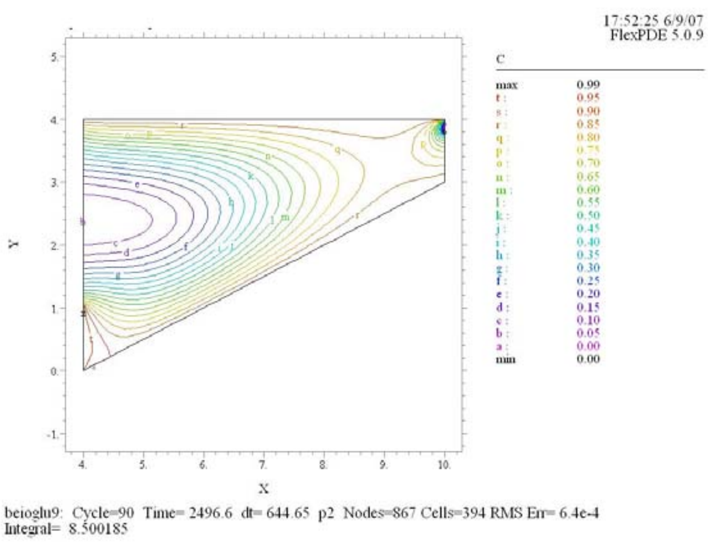

Fig. 2. The equal concentration curves for the holes fraction. The diameter of sedimentation tank is 20 meters.

In figures 1 and 2, it can be observed the equal concentration curves for the holes fraction. In the centre of the settling tank, the holes fraction *c* is equal to 0.75. For the gravitational thickening process simulations in FlexPDE, the initial and boundary conditions were formulated on the solid edge of the settling.

5. Conclusions

For the gravitational thickening process, the mathematical modeling of movement equations is combined into a multi-component system model. Considering the hypotheses formulated for the geometry of a circular settling tank, a differential equation with partial derivates results, in which the holes fraction is unknown. Through the numerical simulation in the FlexPDE, the holes fraction *c* repartition and the sludge bed thickness are obtained.

The solution presented in the present paper allows to obtain a quick response to the vertical repartition of the holes fractions in a given domain.

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