ASSESSMENT OF THE ERRORS DUE TO NEGLECTING AIR COMPRESSIBILITY WHEN DESIGNING FANS

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Conform normelor recente, ventilatoarele sunt turbomaşini pneumatice ale căror rapoarte de comprimare nu depăşesc valoarea τ = 1,3. Proiectarea ventilatoarelor se face, de regulă, în ipoteza că aerul este incompresibil. În prezenta lucrare este prezentată o analiză a erorilor introduse de neglijarea compresibilităţii aerului şi este stabilită o dependenţă a acestor erori de raportul de comprimare.

According to recent norms, fans are pneumatic turbomachines, whose pressure ratios do not exceed the value τ = 1.3. When designing fans the assumption is usually made that the air is incompressible. In the present work an analysis of the errors resulting from neglecting the air compressibility is made and a relationship between these errors and the pressure ratio is established.

Keywords: compressibility, computing errors, fans

1. Introduction

According to ISO and EUROVENT norms, fans are turbomachines that raise the specific energy of a gas with up to 25 000 J/kg. For a fan handling air with a mean density within the fan of 1.2 kg/m^3, at an atmospheric pressure of about 100 000 Pa, the pressure rise can not exceed 30 000 Pa, or about 3 000 mm water. This limit of the pressure rise is higher than that considered until recently, of about 1200 mm water, above which turbomachines were called blowers. The ISO norms do not introduce anymore the denomination “blower”. Depending on the rise of the specific energy and on the pressure rise, respectively, fans are classified as follows [1]:

low pressure fans, up to 600 J/kg and up to 720 Pa, respectively;

medium pressure fans, between 600 J/kg and 3 000 J/kg and between 720 Pa and 3 600 Pa, respectively;

high pressure fans, between 3 000 J/kg and 25 000 J/kg and between 3 600 Pa and 30 000 Pa, respectively.

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Above 25 000 J/kg and 30 000 Pa, respectively, turbomachines are called turbocompressors.

It can be noticed that pressure ratios of fans do not exceed the value $\tau = 1.3$ (the pressure ratio being the ratio between the absolute pressure at outlet and the absolute pressure at inlet). When designing pneumatic turbomachines, up to this pressure ratio the air is usually considered an ideal and incompressible gas, i.e. its density is taken as constant. Under these conditions the question arises to what extent neglecting air compressibility influences the performances of a fan and its constructive characteristics. In the following, an assessment of the errors due to neglecting air compressibility when designing fan impellers is presented and a relationship between these errors and the pressure ratio is established.

2. Assessment of designing errors due to considering air as incompressible up to pressure ratios of 1.3

The analysis will be accomplished under the hypotheses that air is an ideal gas and during the flow through the impeller the heat exchange is negligible, i.e. the flow is adiabatic.

To prove the validity of the first hypothesis it is required to know the value of the compressibility factor of air. Air is actually a real gas, which fulfils the equation of state

$$\frac{p}{\rho} = Z\,R\,T\,, \tag{1}$$

where Z is the compressibility factor. This factor is a function of two independent state properties of the gas. However, the compressibility factor can be taken as constant and equal to one, $Z = 1$, if $T/T_{cr} > 2$ and $p/p_{cr} < 0.05$ [2], where, for the gas taken into consideration, T_{cr} is the critical temperature and p_{cr} is the critical pressure. The critical values of air are $T_{cr} = 132.45$ K and $p_{cr} = 37.9$ bar [3]. In fans air temperature is usually higher than 273.15 K and the absolute pressure does not exceed about 1.3 bar since the pressure ratio is smaller than 1.3. The following limiting values are obtained: $(T/T_{cr})_{\min} = 273.15/132.45 = 2.06$ and $(p/p_{cr})_{\max} = 1.3/37.9 = 0.034$. It can be noticed that these limiting values fulfil the aforementioned conditions, which means that the compressibility factor of air may be taken as constant and equal to 1 in the range of fan pressure ratios. By this, the hypothesis that considers air as being an ideal gas can be accepted.

The second hypothesis may be also accepted because the time required by air particles to travel through the fan impeller is very short and fans are not foreseen with coolers as turbocompressors are.

In the following the common notation will be used, with index 0 denoting a point located on the suction edge of an impeller blade, just before the entrance, and index 3 denoting a point located on the pressure edge of the blade, immediately after the exit of the impeller.

The dimensions of the impeller at exit are influenced by the volume flow rate at impeller outlet. Under the hypothesis of constant density, the continuity equation leads to the equality of the volume flow rates at inlet and outlet. These two volume flow rates are actually different, only the mass flow rates being equal. Because of this, the following relative error appears when computing the volume flow rate:

$$\Delta Q = \frac{Q_0 - Q_3}{Q_0}, \tag{2}$$

where Q is the volume flow rate. From the continuity equation

$$Q_{m_0} = Q_{m_3}, \tag{3}$$

where $Q_m = \rho Q$ is the mass flow rate and ρ is the air density, the following relationship between the volume flow rates at inlet and outlet is obtained:

$$Q_3 = Q_0 \frac{\rho_0}{\rho_3}. \tag{4}$$

At the outlet both the pressure and the temperature rise, but these two rises have opposite effects on the density, and, consequently, on the volume flow rate. The pressure rise leads to a density growth, while the temperature rise causes a decrease of the density. Whether eventually the density increases or decreases depends on which of the two parameters, pressure or temperature, has a stronger growth.

Keeping a constant air density corresponds to an isochoric change of state. But the change of state that air actually undergoes in a fan is a polytropic one, being described by an equation that can be written in the following form:

$$\frac{p}{\rho^n} = \text{const.}, \tag{5}$$

where p is the absolute pressure and n is the polytropic exponent. Considering this equation, the relationship between the volume flow rates becomes

$$Q_3 = Q_0 \left(\frac{p_0}{p_3}\right)^{1/n} = Q_0 \left(\frac{p_3}{p_0}\right)^{-1/n} = Q_0\, \tau^{-1/n}, \tag{6}$$

where $\tau = p_3 / p_0$ is the pressure ratio.

At this moment, assessing the error when computing the volume flow rate requires only the polytropic exponent n to be known. On the other hand, the polytropic exponent can be expressed in terms of the polytropic efficiency η_{pol} and the isentropic exponent k, since, at design moment, the polytropic efficiency can be estimated and the isentropic exponent is known.

The polytropic efficiency is given by the formula [4]

$$\eta_{pol} = \frac{l_{pol}}{h_3 - h_1}, \tag{7}$$

where l_{pol} is the polytropic specific compression work and h is the specific enthalpy of air. The polytropic specific compression work is given by the relationship [4, 5]

$$l_{pol} = \frac{n}{n-1}\frac{p_0}{\rho_0}\left[\left(\frac{p_3}{p_0}\right)^{(n-1)/n} - 1\right] = \frac{n}{n-1}R\left(T_3 - T_0\right). \tag{8}$$

Since air is considered an ideal gas, for which the specific heat at constant pressure c_p is constant, the difference of the enthalpies at outlet and inlet can be written in the form

$$h_3 - h_0 = c_p\left(T_3 - T_0\right) = \frac{k}{k-1}R\left(T_3 - T_0\right). \tag{9}$$

Introducing this relationship in the expression of the polytropic efficiency, one obtains

$$\eta_{pol} = \frac{n}{n-1}\frac{k-1}{k}, \tag{10}$$

from where the polytropic exponent results:

$$n = \frac{\eta_{pol}\, k}{1 - k\,(1 - \eta_{pol})}. \tag{11}$$

Replacing now in equation (6), the following relationship is obtained:

$$Q_3 = Q_0\, \tau^{\frac{\eta_{pol}\, k}{1-k\,(1-\eta_{pol})}}. \tag{12}$$

With this, the relative error of the volume flow as a function of the pressure ratio and the polytropic efficiency can be expressed as follows:

$$\frac{\Delta Q}{Q_0} = 1 - \frac{1}{\tau^{\frac{\eta_{pol}\, k}{1-k\,(1-\eta_{pol})}}}. \tag{13}$$

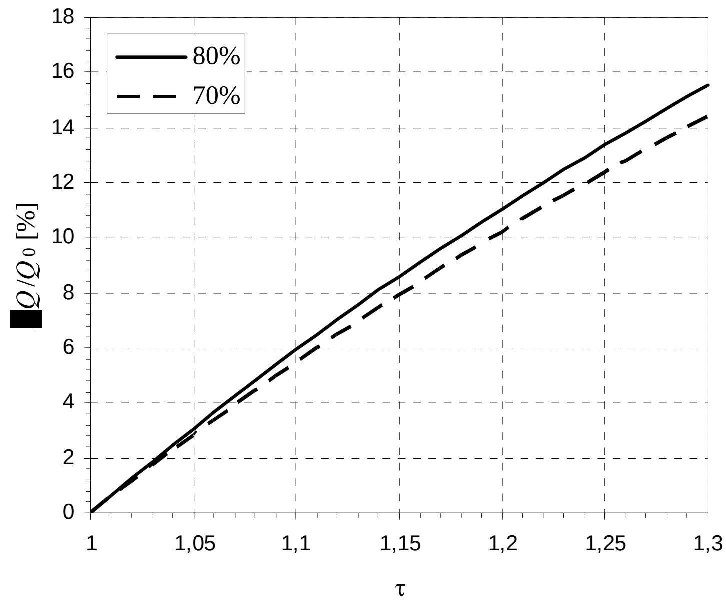

Fig. 1. Relative error of the volume flow rate as a function of the pressure ratio for polytropic efficiencies of 80 % and 70 %.

The relative error of the volume flow rate as a function of the pressure ratio for two values of the polytropic efficiency, η_{pol} = 80 % and η_{pol} = 70 %, is plotted in figure 1. It can be noticed that the error exceeds 2 % for pressure ratios higher than 1.03 and 5 %, respectively, for pressure ratios higher than 1.09, increasing up to about 15 % at a pressure ratio of 1.3. This means that for low pressure fans, which have pressure ratios smaller than τ = 1.0072, the errors can be considered negligible, since they have values below 1 %. In case of medium pressure fans, having τ = 1.0072…1.036, the computing error can grow above 2 %, so that when an increased accuracy is required for such fans, the air compressibility should be taken into account. Finally, neglecting the air compressibility when designing high pressure fans, having pressure ratios τ between 1.036 and 1.3, results in significant errors, the volume flow rate at the outlet being overestimated (since the design flow rate, which is considered constant throughout the impeller and equal to Q_0, is higher at the outlet than the real flow rate Q_3). This means that the impeller width b_2 at outlet, with which the volume flow rate is proportional, will be also overestimated, according to the flow rate. The impellers of high pressure fans and, generally, the entire machine result also over dimensioned when the air compressibility is neglected. Because of this, the production cost increases and the energy consumption of the fan, when operated, will be higher. As a result, when designing high pressure fans it is advisable to consider always the change of the air density in the impeller.

3. Conclusions

In this work the errors that arise when designing fan impellers under the assumption of the air incompressibility were analyzed. The results obtained show that for low pressure fans the errors are negligible but they can increase above 2 % in case of medium pressure fans. For high pressure fans the errors become significant, growing up to about 15 %, depending on the pressure ratio, and thus causing an over dimensioning of the fan. As a result, when the economical aspects related to production and operating costs are of the first importance, a proper design should consider the variation of the air density in the fan impeller.

R E F E R E N C E S

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