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# CALIBRATION OF SPHERICAL LENSES USED FOR FOCIMETERS' TESTING UNCERTAINTY BUDGET

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*Articolul descrie metoda de etalonare a lentilelor sferice etalon utilizate pentru verificarea dioptrimetrelor şi evaluarea incertitudinii de măsurare a acestora în laboratorul Lungimi al Institutului Naţional de Metrologie, Bucureşti, România. Contribuţia autorului constă în elaborarea unui program de calcul al incertitudinii de măsurare pentru lentilele etalon.*

*The article describes the method of calibration of the standard lenses used for focimeters' testing in Length Laboratory of National Institute of Metrology, Bucharest, Romania. The contribution of the author consists in elaboration of a special program for computation of lenses uncertainty measurement.*

**Key words**: lens, focimeters, uncertainty budget

## 1. Introduction

Focimeters are ones of the basic clinical instruments used in every optometric practice, and verification of the accuracy and calibration of the instrument are of the utmost importance. Calibration is accomplished using special test lenses. These lenses must be of high quality and of nominal ophthalmic power that is known with high accuracy. The standard ISO 9342:1996 Optics and Optical Instruments: Test Lenses for Calibration of Focimeters imposes stringent requirements on permitted deviation from nominal ophthalmic power of test lenses.

In this article, the influence of tolerance and uncertainties in design and production of standard test lenses for calibration of focimeters was analyzed. For the calibration of focimeters is used a set of special lenses composed from 5 positive spherical lenses of nominal power equal to +5, + 8, + 10, + 15, +20, dioptres, 5 negative spherical lenses of nominal power equal to – 5, - 8, -10, -15, -20, dioptres, 1 cylindrical lens of nominal power + 5 dioptres and 1 prismatic lens of nominal power equal to 5 prismatic dioptres. The spherical lenses are made of K5 glass having a diameter equal to 17 mm and are mounted in a plastic frame. The cylindrical lens is made of K5 glass with a surface equal to (40 x 40) $mm^2$. The prismatic lens is made of K5 glass too and has a surface equal to (40 x 40) $mm^2$. Calibration lens must have very closed characteristics (refractive index, curvature radii) with the spectacular glasses.

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## 2. Method and measuring instruments for spherical lenses

The ophthalmic power is obtained from the geometrical and physical characteristics of the lenses. In the description of the measuring method are used the following symbols: $N$ – refractive index of the glass; $R_1$ – first radius of curvature; $R_2$ – second radius of curvature; $T$ – axial thickness of the length; $f$ – focal length of the lens; $\phi$ - power of the lens ($\phi$= 1/f); $BFL$ – back focal length; $\Phi$ - ophthalmic power of the lens ($\Phi$ = 1/BFL)

*Note: it is assumed by convention that the light travels from left to right; the radii of curvature are named first and second according to their order along the optical path; a radius is positive if the center of curvature lies to the right.*

The basic equations for power of the lenses and BFL are the following:

$$\phi = \frac{N-1}{R_1} - \frac{N-1}{R_2} + \frac{(N-1)^2}{N} \cdot \frac{T}{R_1 R_2} \tag{1}$$

$$BFL = \frac{1}{\phi}\left(1 - \frac{N-1}{N} \cdot \frac{T}{R_1}\right) \tag{2}$$

thus:

$$\Phi = \frac{N(N-1)}{NR_1 - (N-1)T} - \frac{N-1}{R_2} \tag{3}$$

$$\frac{\delta\Phi}{\delta R_1} = -\frac{N^2(N-1)}{[NR_1 - (N-1)T]^2} \tag{4}$$

$$\frac{\delta\Phi}{\delta R_2} = \frac{N-1}{R_2^2} \tag{5}$$

$$\frac{\delta\Phi}{\delta T} = \frac{N(N-1)^2}{[NR_1 - (N-1)T]^2} \tag{6}$$

$$\frac{\delta\Phi}{\delta N} = \frac{(2N-1)[NR_1 - (N-1)T] - N(N-1)(R_1 - T)}{[NR_1 - (N-1)T]^2} \tag{7}$$

In table 1 is given performing scheme of the computation, where: capital is the magnitude, lowercase - the measured values and the overlines indicate the operation of average.

*Table 1*

**Scheme of computation**

| Magnitude | Estimate | Standard uncertainty | Sensitive coefficient | Uncertainty contribution |
|---|---|---|---|---|
| $R_1$ | $\bar{r}_1$ | $\sigma(r_1)$ | $\left.\frac{\delta\Phi}{\delta R_1}\right\vert_{R_1=\bar{r}_1}$ | $\left\vert\frac{\delta\Phi}{\delta R_1}\right\vert\sigma(r_1)$ |
| $R_2$ | $\bar{r}_2$ | $\sigma(r_2)$ | $\left.\frac{\delta\Phi}{\delta R_2}\right\vert_{R_2=\bar{r}_2}$ | $\left\vert\frac{\delta\Phi}{\delta R_2}\right\vert\sigma(r_2)$ |
| $T$ | $\bar{t}$ | $\sigma(t)$ | $\left.\frac{\delta\Phi}{\delta T}\right\vert_{T=\bar{t}}$ | $\left\vert\frac{\delta\Phi}{\delta T}\right\vert\sigma(t)$ |
| $N$ | n | $\sigma(n)$ | $\left.\frac{\delta\Phi}{\delta N}\right\vert_{N=n}$ | $\left\vert\frac{\delta\Phi}{\delta N}\right\vert\sigma(n)$ |

Using usual instruments can be measured the following parameters: $R_1$, $R_2$ and $T$. To measure the central thickness of the lens it is used a digital micrometer with 0.001 mm resolution. The values for $N$ are taken from catalogue of the firm that produces the glass. As example we have:

$n = 1.5236 \pm 0.0001$

For the uncertainty on $n$, it is assumed a standard deviation $\sigma$(n) = 0.0003.

To measure the radii of curvature of the spherical lenses, it is used a spherometer. A spherometer is an instrument that measure the sag of a surface with great precision. A common spherometer is the Aldis spherometer in which three small balls are arranged to form an equilateral triangle. In the center of the triangle there is a probe mounted on a micrometer. In use, the surface to be measured is placed on the balls and the probe is brought into contact with the surface. The sag of the surface is measured using a micrometer. If we denote the sag of the surface by $h$, the distance between the center of the balls $d$, and the radii by $r$, then the radius $R$ of the curvature of the unknown surface is given by:

$$R = \frac{d^2}{6h} + \frac{h}{2} \pm r$$

Where the positive sign is taken when the unknown surface in concave and the negative sign when convex.

The corresponding uncertainties are of the type *A*. The value for $N$ is taken for the specifications of the glass and the uncertainty for $N$ is of the type *B*. The ophthalmic power is computed from formula (3). Formula (4) to (7) are the sensitivity coefficients.

The result is expressed in the following form:

$$\Phi = \Phi(\bar{r}_1, \bar{r}_2, \bar{t}, n) \pm 2\sigma(\Phi) \tag{8}$$

where $\sigma(\Phi)$ is the combined uncertainty given by

$$\sigma(\Phi) = [\left|\frac{\delta\Phi}{\delta R_1}\right|^2 \sigma^2(r_1) + \left|\frac{\delta\Phi}{\delta R_2}\right|^2 \sigma^2(r_2) + \left|\frac{\delta\Phi}{\delta T}\right|^2 \sigma^2(t) + \left|\frac{\delta\Phi}{\delta n}\right|^2 \sigma^2(n)]^{1/2} \tag{9}$$

In table 2 are given the values of the radii and the permissible limits in the computation of the radii for a refractive index of 1.5236.

*Table 2*

**Permissive limits of the radii and thickness**

| Vertex power (dpt) | $R_1$ (mm) | $R_2$ (mm) | Thickness of the center (mm) |
|---|---|---|---|
| - 20 | ∞ | 26.18 | 1 ± 0,1 |
| - 15 | 505.8 | 32.65 | 1 ± 0,1 |
| - 10 | 261.8 | 43.65 | 1.2 ± 0.02 |
| - 8 | 212.3 | 50.0 | 1 ± 0.02 |
| - 5 | 141.5 | 60.12 | 1 ± 0.02 |
| - 2 | 105.44 | 74.89 | 1 ± 0.02 |
| + 2 | 84.875 | 124.45 | 1.52 ± 0.05 |
| + 5 | 58.996 | 132.6 | 1.4 ± 0.02 |
| + 8 | 54.54 | 308.3 | 1.6 ± 0.02 |
| + 10 | 46.345 | 363.2 | 1.7 ± 0.05 |
| + 15 | 33.04 | 435.851 | 2.1 ± 0.05 |
| + 20 | 26.18 | ∞ | 2.4 ± 0.02 |

The calibration conditions are verified to each series of measurements and the standard deviation is calculated for 11 measurements.

*Table 3*

**Experimental data**

| Lens (dpt) | $\bar{r}_1$ mm | $\sigma(r_1)$ mm | $\bar{r}_2$ mm | $\sigma(r_2)$ mm | $\bar{t}$ mm | $\sigma(t)$ mm |
|---|---|---|---|---|---|---|
| - 20 | ∞ | - | 26.18 | 0.29 | 1.05 | 0.02 |
| - 15 | 505.8 | 11.56 | 32.65 | 1.16 | 1.02 | 0.03 |
| - 10 | 261.8 | 4.05 | 43.65 | 1.73 | 1.20 | 0.01 |
| - 8 | 212.3 | 3.98 | 50.0 | 2.01 | 1.01 | 0 |
| - 5 | 141.5 | 2.55 | 60.12 | 2.23 | 1.01 | 0.01 |
| - 2 | 105.44 | 2.05 | 74.89 | 2.29 | 1.00 | 0 |
| + 2 | 84.875 | 1.95 | 124.45 | 2.30 | 1.50 | 0.02 |
| + 5 | 58.996 | 1.92 | 132.6 | 2.85 | 1.40 | 0.01 |
| + 8 | 54.54 | 1.89 | 308.3 | 2.89 | 1.61 | 0.01 |
| + 10 | 46.345 | 1.75 | 363.2 | 3.47 | 1.72 | 0.03 |
| + 15 | 33.04 | 1.08 | 435.851 | 8.67 | 2.13 | 0.02 |
| + 20 | 26.18 | 0.22 | ∞ | - | 2.41 | 0.01 |

Considering the formula from point 3) to point 9), in the annex *A* I gave a mathematic model for calculating the measurement uncertainties for lenses with nominal field between – 20 $m^{-1}$ to + 20 $m^{-1}$, using the characteristics of the lenses from our laboratory.

## 3. Conclusions

As we can see from annex *A*, the design and the manufacture of the lenses are the most important parameters for obtaining a better accuracy in calibration of focimeters with a good measurement uncertainty.

The scope of this article is to give an easy way to calculate the measurement uncertainty of every laboratory which owns such kind of lenses, knowing the parameters of the lenses as curvature radii, thickness and refractive index of material of the glasses.

Applying this method of calibration it can be obtain a closed value of measurement uncertainty provided by the requirements of Standard ISO 9342:1996 Optics and Optical Instruments: Test Lenses for Calibration of Focimeters.

## R E F E R E N C E S

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**Annex A – Mathematic model for calculation of measurement uncertainty for lenses , having nominal range between - 20 dpt…+ 20 dpt**

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *N* | 1.5236 | 1.5236 | 1.5236 | 1.5236 | 1.5236 | 1.5236 | 1.5236 | 1.5236 | 1.5236 | 1.5236 | 1.5236 | 1.523 |
| *R*1 (mm) | 505.8 | 261.8 | 212.3 | 141.5 | 105.44 | 84.88 | 58.99 | 54.54 | 46.34 | 33.04 | 26.18 | 1000 |
| *R*2 (mm) | 32.65 | 43.65 | 50 | 60.12 | 74.89 | 124.45 | 132.6 | 308.3 | 363.2 | 435.851 | 10000 | 26.1 |
| *T*(mm) | 1.02 | 1.2 | 1.01 | 1.01 | 1.01 | 1 | 1.5 | 1.4 | 1.61 | 1.72 | 2.13 | 1.0 |
| **$\Phi$ ($m^{-1}$)** | **-15** | **-10** | **-8** | **-5** | **-2** | **2** | **5** | **8** | **10** | **15** | **20** | **-2** |
| $\sigma$(r1) (mm) | 11.56 | 4.05 | 3.98 | 2.55 | 2.05 | 1.95 | 1.92 | 1.89 | 1.75 | 1.08 | 0.22 | 1000 |
| $\sigma$(r2) (mm) | 1.16 | 1.73 | 2.01 | 2.23 | 2.29 | 2.3 | 2.85 | 2.89 | 3.47 | 8.67 | 100000 | 0.2 |
| $\sigma$(t) (mm) | 0.03 | 0.01 | 0 | 0.05 | 0 | 0.02 | 0.01 | 0.01 | 0.03 | 0.02 | 0.01 | 0.0 |
| $\sigma$(n) | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.000 |
| **$\delta\Phi/\delta R1$** | | | | | | | | | | | | |
| $N \cdot N(N-1)$ | 1.215462504 | 1.215462504 | 1.215462504 | 1.2154625 | 1.2154625 | 1.2154625 | 1.2154625 | 1.2154625 | 1.2154625 | 1.2154625 | 1.2154625 | 1.215462 |
| $NR1-(N-1)T$ | 770.102808 | 398.25016 | 322.931444 | 215.06056 | 160.11955 | 128.79957 | 89.091764 | 82.364104 | 69.760628 | 49.439152 | 38.77258 | 15235.4 |
| | 593058.3349 | 158603.1899 | 104284.7175 | 46251.046 | 25638.27 | 16589.329 | 7937.3424 | 6783.8456 | 4866.5452 | 2444.2298 | 1503.313 | 23211894 |
| | **2.04948E-06** | **7.66354E-06** | **1.16552E-05** | **2.63E-05** | **4.74E-05** | **7.33E-05** | **0.000153** | **0.000179** | **0.00025** | **0.000497** | **0.000809** | **5.24E-0** |
| **$\delta\Phi/\delta R2$** | | | | | | | | | | | | |
| $N-1$ | 0.5236 | 0.5236 | 0.5236 | 0.5236 | 0.5236 | 0.5236 | 0.5236 | 0.5236 | 0.5236 | 0.5236 | 0.5236 | 0.523 |
| $R2 \cdot R2$ | 1066.0225 | 19053.225 | 25000 | 36144.144 | 56085.121 | 154878.03 | 17582.76 | 95048.89 | 131914.24 | 189966.09 | 100000000 | 685.392 |
| | **0.000491172** | **2.74809E-05** | **0.000020944** | **1.45E-05** | **9.34E-06** | **3.38E-06** | **2.98E-05** | **5.51E-06** | **3.97E-06** | **2.76E-06** | **5.24E-09** | **0.00076** |
| **$\delta\Phi/\delta T$** | | | | | | | | | | | | |
| $N(N-1)(N-1)$ | 0.417705544 | 0.417705544 | 0.417705544 | 0.4177055 | 0.4177055 | 0.4177055 | 0.4177055 | 0.4177055 | 0.4177055 | 0.4177055 | 0.4177055 | 0.417705 |
| $NR1-(N-1)T$ | 770.102808 | 398.25016 | 322.931444 | 215.06056 | 160.11955 | 128.79957 | 89.091764 | 82.364104 | 69.760628 | 49.439152 | 38.77258 | 15235.4 |
| | 593058.3349 | 158603.1899 | 104284.7175 | 46251.046 | 25638.27 | 16589.329 | 7937.3424 | 6783.8456 | 4866.5452 | 2444.2298 | 1503.313 | 23211894 |
| | **7.04325E-07** | **2.63365E-06** | **4.00543E-06** | **9.03E-06** | **1.63E-05** | **2.52E-05** | **5.26E-05** | **6.16E-05** | **8.58E-05** | **0.000171** | **0.000278** | **1.8E-0** |
| **$\delta\Phi/\delta N$** | | | | | | | | | | | | |
| $(2N-1)$ | 2.0472 | 2.0472 | 2.0472 | 2.0472 | 2.0472 | 2.0472 | 2.0472 | 2.0472 | 2.0472 | 2.0472 | 2.0472 | 2.047 |
| $NR1-(N-1)T$ | 770.102808 | 398.25016 | 322.931444 | 215.06056 | 160.11955 | 128.79957 | 89.091764 | 82.364104 | 69.760628 | 49.439152 | 38.77258 | 15235.4 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $N(N\text{-}1)(R1\text{-}T)$ | 402.6917583 | 207.8954638 | 168.5580681 | 112.07688 | 83.309759 | 66.915854 | 45.863048 | 42.392805 | 35.683669 | 24.985748 | 19.186055 | 7976.73 |
| | 1173.86271 | 607.4022638 | 492.5471841 | 328.19511 | 244.48698 | 196.76262 | 136.52561 | 126.22299 | 107.13029 | 76.226084 | 60.189171 | 23213.28 |
| $NR1\text{-}(N\text{-}1)T$ | 770.102808 | 398.25016 | 322.931444 | 215.06056 | 160.11955 | 128.79957 | 89.091764 | 82.364104 | 69.760628 | 49.439152 | 38.77258 | 15235.4 |
| | 593058.3349 | 158603.1899 | 104284.7175 | 46251.046 | 25638.27 | 16589.329 | 7937.3424 | 6783.8456 | 4866.5452 | 2444.2298 | 1503.313 | 23211894 |
| | 0.001979338 | 0.003829698 | 0.0047231 | 0.007096 | 0.009536 | 0.0118608 | 0.0172004 | 0.0186064 | 0.0220136 | 0.0311861 | 0.0400377 | 0.000 |
| $1/R2$ | 0.030627871 | 0.022909507 | 0.02 | 0.0166334 | 0.0133529 | 0.0080354 | 0.0075415 | 0.0032436 | 0.0027533 | 0.0022944 | 0.0001 | 0.038197 |
| | **-0.02864853** | **-0.01907981** | **-0.0152769** | **-0.00954** | **-0.00382** | **0.003825** | **0.009659** | **0.015363** | **0.01926** | **0.028892** | **0.039938** | **-0.038** |
| $[\delta\Phi/\delta R1\cdot\sigma(\mathrm{r1})]^2$ | 5.61312E-10 | 9.63317E-10 | 2.15183E-09 | 4.491E-09 | 9.445E-09 | 2.041E-08 | 8.644E-08 | 1.147E-07 | 1.91E-07 | 2.884E-07 | 3.164E-08 | 2.742E-0 |
| $[\delta\Phi/\delta R2\cdot\sigma(\mathrm{r2})]2$ | 3.24625E-07 | 2.26024E-09 | 1.77219E-09 | 1.044E-09 | 4.571E-10 | 6.046E-11 | 7.203E-09 | 2.535E-10 | 1.897E-10 | 5.711E-10 | 2.742E-07 | 4.908E-0 |
| $[\delta\Phi/\delta T\cdot\sigma(\mathrm{t})]^2$ | 4.46466E-16 | 6.93612E-16 | 0 | 2.039E-13 | 0 | 2.536E-13 | 2.769E-13 | 3.791E-13 | 6.63E-12 | 1.168E-11 | 7.72E-12 | 1.295E-2 |
| $[\delta\Phi/\delta N\sigma(\mathrm{N})]^2$ | 7.38665E-11 | 3.27635E-11 | 2.10045E-11 | 8.187E-12 | 1.311E-12 | 1.317E-12 | 8.397E-12 | 2.124E-11 | 3.339E-11 | 7.513E-11 | 1.436E-10 | 1.306E-1 |
| $\sum$ | 3.25261E-07 | 3.25632E-09 | 3.94503E-09 | 5.543E-09 | 9.904E-09 | 2.047E-08 | 9.366E-08 | 1.149E-07 | 1.913E-07 | 2.891E-07 | 3.059E-07 | 5.195E-0 |
| $u_c$ | 0.000570316 | 5.70642E-05 | 6.28095E-05 | 7.445E-05 | 9.952E-05 | 0.0001431 | 0.000306 | 0.000339 | 0.0004373 | 0.0005377 | 0.0005531 | 0.000227 |
| $U=2u_c$ | 0.001140632 | 0.000114128 | 0.000125619 | 0.0001489 | 0.000199 | 0.0002862 | 0.0006121 | 0.0006781 | 0.0008747 | 0.0010753 | 0.0011063 | 0.000455 |

$\Phi=N(N\text{-}1)/\{(NR1\text{-}(N\text{-}1)T)(NR1\text{-}(N\text{-}1)R1]\}$

**$\Phi$ (mm$^{-1}$)**

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $N(N\text{-}1)$ | 0.79775696 | 0.79775696 | 0.79775696 | 0.797757 | 0.797757 | 0.797757 | 0.797757 | 0.797757 | 0.797757 | 0.797757 | 0.797757 | 0.79775 |
| $NR1\text{-}(N\text{-}1)T$ | 770.102808 | 398.25016 | 322.931444 | 215.06056 | 160.11955 | 128.79957 | 89.091764 | 82.364104 | 69.760628 | 49.439152 | 38.77258 | 15235.4 |
| | 0.00103591 | 0.002003155 | 0.00247036 | 0.0037095 | 0.0049823 | 0.0061938 | 0.0089543 | 0.0096857 | 0.0114356 | 0.0161361 | 0.0205753 | 5.236E-0 |
| $(N\text{-}1)/R2$ | 0.016036753 | 0.011995418 | 0.010472 | 0.0087092 | 0.0069916 | 0.0042073 | 0.0039487 | 0.0016983 | 0.0014416 | 0.0012013 | 5.236E-05 | 0.0 |
| | - | - | | - | - | | | | | | | |
| | 0.015000844 | 0.009992263 | -0.00800164 | 0.0049998 | 0.0020093 | 0.0019865 | 0.0050056 | 0.0079874 | 0.009994 | 0.0149348 | 0.0205229 | 0.019947 |
| **$\Phi$(m$^{-1}$)** | -15 | -10 | -8 | -5 | -2 | 2 | 5 | 8 | 10 | 15 | 20 | -2 |