U.P.B. Sci. Bull., Series D, Vol. 73, Iss. 2, 2011 ISSN 1454-2358

MODELING AND SIMULATION OF THE EYEBALL OPTICAL SYSTEM

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Lucrarea prezintă rezultatele legate de modelarea şi simularea sistemului optic al globului ocular şi propune o metodă de calcul a unui polinom de interpolare care să permită modelarea şi simularea modificării indicelui de refracţie al cristalinului pentru ochiul aflat în repaus cât şi pentru ochiul acomodat. Acest lucru devine obligatoriu la calculul aberaţiilor sistemului optic al globului ocular şi formează baza a cursului Modelarea Sistemului Vizual predat studenţilor din anul 1 de master.

The paper presents the results of modeling and simulation of the eyeball optical system and proposes a method of computing an interpolation polynomial that would allow modeling and simulation of the variation of the crystalline lens refraction index for the eye at rest and for the fully accommodated eye. This stage is compulsory for the calculus of aberrations of the eyeball optical system and represents the basis of the course "Visual System Modeling", taught to the students of the 1st year of Master.

Key words: Technical optical system, Biological optical system, Interpolation polynomial.

1. Introduction

The study of the eyeball optical system involves the same relations and numerical methods used in the case of technical optical systems. Biological optical systems can be thus attached to the class of technical optical systems. The difference between the two classes defined this way consists in the engineering solutions that make technical optical systems function and the solutions chosen by nature for the functioning of biological systems. For a common language, it will be defined onwards the notion of image surface that is represented by a plane in

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the case of technical optic systems and by a sphere or by a similar shape for the biological optical systems.

The main difference between the functioning of a technical optical system and a biological optical system consists in the manner that the two optical systems adapt to various object abscissas. Therefore, in the case of a technical optical system, its position between the object plane and the image plane is modified or the image abscissa is modified correspondingly. In the case of a biological optical system, nature imagined a very complex and efficient solution that involved the simultaneous modification of its main constructive parameters (radii, separations, refraction indexes).

Another major distinction between the two classes is that the biological optical system is affected by aging while the technical optical system is stable over time.

The optics of the human eye visual system can be studied on a mathematical model that is correlated with the statistically established measured anatomic parameters. This model has to be completed with a set of mathematical relations that simulate human eye aging, adaptation and correction.

2. Human eye parameters established by measurements

The start statistical parameters are taken from specialty literature. Thus, figure 1 presents a section through the right human eyeball and table 1 lists the range of parameters, measured and statistically classified [1].

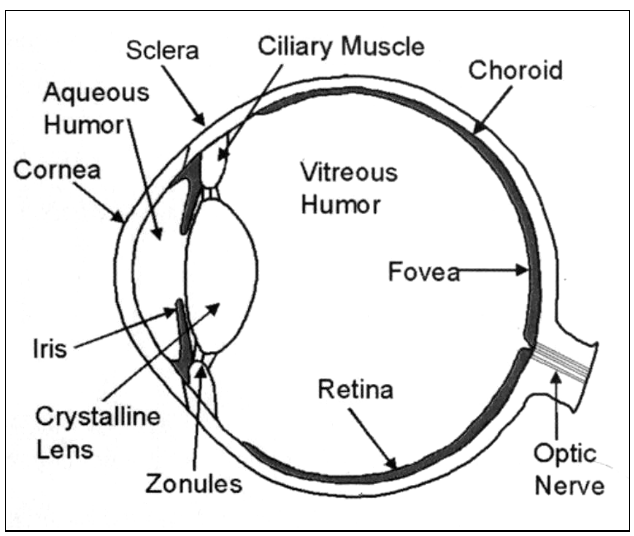

Fig. 1. Section through the right human eyeball.

The ophthalmic specialty literature offers few information on the optical characteristics of the eye optical system components. Besides, the data extracted from different works do not coincide. These inaccuracies cannot be accepted for the calculation of the optical performances of the eye, due to the fact that optics involve high accuracy, thus complete and exact knowledge of all discussed values, mainly of the refraction indexes, is compulsory.

Because practical ophthalmology does not emphasize the accurate knowledge of the optical features of the eye optical system components, inaccuracies are good tolerated. Nevertheless, it is a fact that cannot be accepted if a mathematical model that would simulate the functioning of the eye optical system is needed. The mathematical model used for the simulation of the functioning of the eyeball optical system helps to know better the human eye, therefore to develop the field of practical ophthalmology.

Table 1

The statistical limits of the measurements of the eyeball optical system

Name	Range
Cornea anterior radius	7.00 - 8.65 mm
Cornea posterior radius	6.20 - 6.60 mm
Anterior chamber depth	2.80 - 4.60 mm
Crystalline power	15.00 - 27.00 dpt.
Crystalline thickness	4.00 mm
Crystalline anterior radius	8.80 – 11.90 mm
Crystalline posterior radius	6.00 mm
Axial length	20.00 – 29.5 mm
Ocular power	54.00 – 65.00 dpt.
Iris abscissa	2.25 mm

Table 2

Refraction indexes for the eyeball optical system components

	Type	n_C (λ=656.2725 nm)	n_D (λ=589.2937nm)	n_F (λ=468.1327 nm)	n_g (λ=435.8343 nm)	ν_D (λ=587.2937 nm)
1	**Cornea**	1.3751	1.3771	1.3818	1.3857	56.2835
2	**Aqueous Humor**	1.3354	1.3374	1.3418	1.3454	52.7187
3	**Crystalline**	1.4175	1.4200	1.4254	1.4307	53.4645
4	**Vitrous Humor**	1.3341	1.3360	1.3404	1.3440	53.333

Table 2 presents the values of the refraction indexes for the human eye components [2].

Other bibliographic sources offer much less information related to refraction indexes, therefore their qualitative comparison is not possible. On the other hand, no work presents the method of establishing these values, that is uncomfortable due to the importance of the measurement method for further computations.

To these data one can add the relations that establish the age dependent change of the crystalline parameters and that simulate the accommodation of the human eye:

1. Relations that limit the age dependent changes of crystalline parameters:

Anterior chamber	-0.0215*age + 4.274
Crystalline thickness	0.0194*age + 3.088
Crystalline anterior radius	-0.0759*age +13.949
Posterior crystalline radius	0.0106*age - 6.436

2. Relations that govern the change of crystalline parameters in function of the accommodation A:

Crystalline anterior radius	12.0-0.4 A
Posterior crystalline radius	-5.224557+ 0.2 A
Crystalline thickness	2.7 – 0.04 A
Crystalline refraction index	1.42+ 0.00256 A – 0.00022 A2

3. The interpolation polynomials for the refraction indexes

Values taken from table 2 are enough for a first communication needed in current discussions, but are not at all satisfactory for a complete analysis of the optical aberrations of the eye.

In order to surpass this drawback, this minimal information will suffer a complex processing by constructing an interpolation polynomial using the least square method [3]. The used interpolation polynomial is in fact a Laurent series of shape:

$$n^2 = A_0 + A_1\lambda^2 + A_2\lambda^{-2} + A_3\lambda^{-4} + A_4\lambda^{-6} + A_5\lambda^{-8} \quad (1)$$

In order to solve the problem, it was supposed that the shape of the curve must be similar to optical glasses.

Figure 2 presents an exemplification of the results obtained for the cornea refraction index. It is emphasized that the eye can have visual sensations only for a specter range of 400nm…760nm, but the widening of the range was needed in order to obtain results.

Table 3 presents the results obtained for the interpolation polynomials of the refraction indexes of the components of the eyeball optical model.

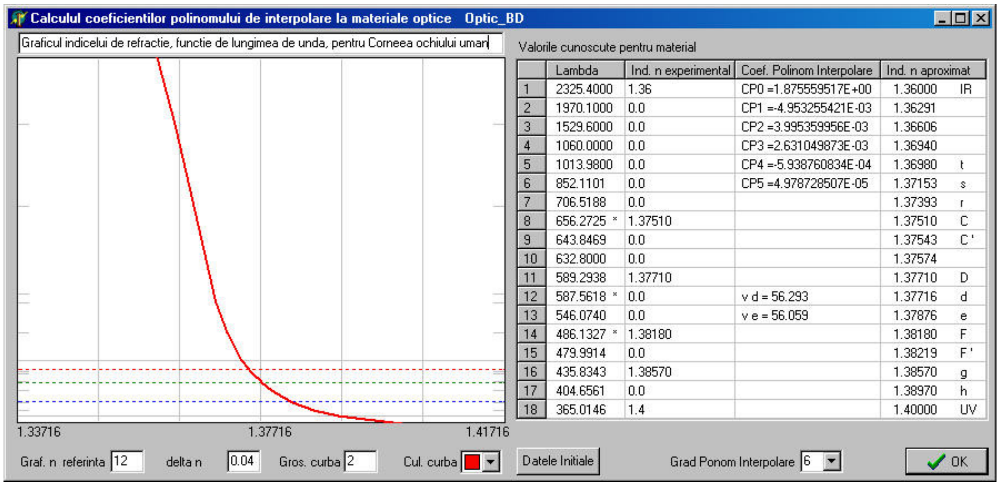

Fig. 2. Results obtained for the cornea refraction index.

Table 3

Coefficients of the interpolation polynomials for the components of the eyeball optical system

	Cornea	Aqueous humor	Crystalline	Vitrous humor
A_0	1.875559517 E+00	1.763641058 E+00	1.969028660 E+00	1.769441190 E+00
A_1	-4.953255421 E-03	-4.201908910 E-03	-1.036648516 E-02	-2.992355168 E-03
A_2	3.995359956 E-03	7.729400276 E-03	2.941570931 E-02	-2.500959603 E-03
A_3	2.631049873 E-03	1.595496417 E-03	-5.554141225 E-03	5.425751074 E-03
A_4	-5.938760834 E-04	-5.232898014 E-04	4.859624152 E-04	-1.143249299 E-03
A_5	4.978785507 E-05	5.038088834 E-05	1.043094134 E-05	8.730585366 E-05

4. The adopted model for the optical system of the eyeball

The adopted model for the optical system of the eyeball is a simplified model because it considers that the dioptre surfaces are spherical.

The parameters and the dimensions of this mathematical model were chosen accordingly to the statistical mean of the data obtained from indicated references, as to correspond also to the functioning outside the paraxial domain. These data are presented in figure 3, together with the main paraxial features.

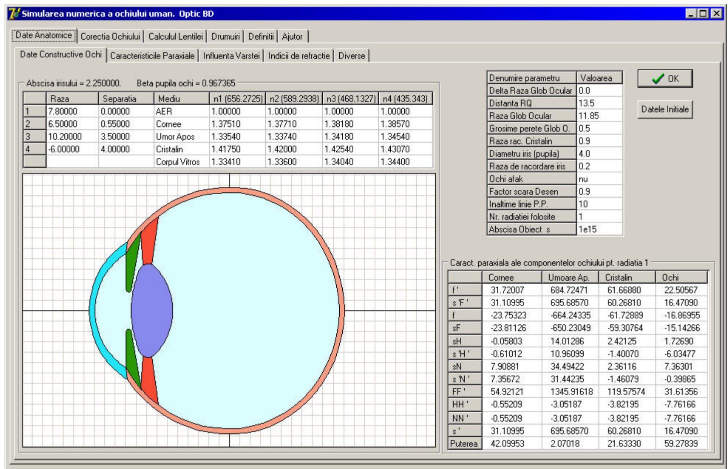

Fig. 3. Parameters and dimensions of the mathematical model.

The computation relations for the path of an optical ray through this biological optical system are the same used in the case of technical optical systems, fully presented in [3], and completed with the relations that give the coordinates of the point in which the optical ray picks the image sphere:

$$\begin{cases} \xi = X0' + s'\dfrac{L'}{N'} \\ \eta = Y0' + s'\dfrac{M'}{N'} \\ F = \sqrt{\xi^2 + \eta^2} \\ G = -L'\xi - M'\eta - N' R_{eyeball} \\ \Delta = G + \sqrt{G^2 - F} \\ x = L'\Delta + \xi \\ y = M'\Delta + \eta \\ z = N'\Delta + s' \end{cases} \qquad (2)$$

In this set of relations, (X0', Y0') denote the coordinates of the point in which the emergent ray picks the plane perpendicular to the optical axis and tangent to the posterior radius of the crystalline, (L',M',N') are the direction cosines of the emergent ray, (ξ,η) are the coordinates of the point where the emergent ray picks the plane perpendicular to the optical axis and tangent to the sphere that materializes the retina, and (x,y,z) are the coordinates of the point where the optical ray picks the sphere of the eyeball.

If these relations are attached to those that give the vector traversing, the paths of extraparaxial rays can be obtained, as shown in figure 4.

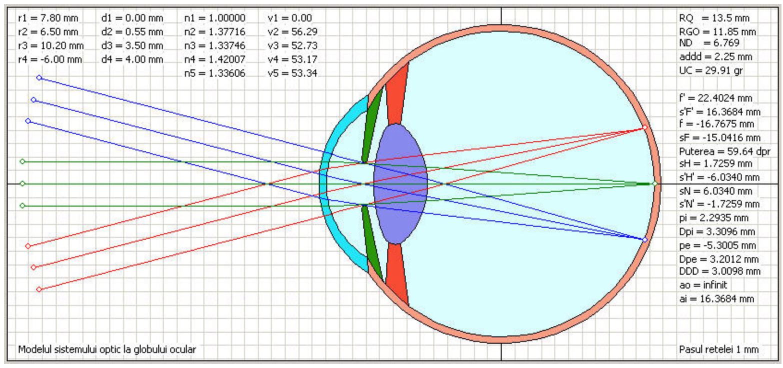

Fig. 4. Paths of extraparaxial rays.

The mechanism that simulates the pass of a bundle of optical rays through the model of the optical system of the eyeball allows the computation of all optical aberrations, monochromatic or chromatic. For instance, figure 5 presents the wavefront aberrations of the four base radiations, computed in the meridian plane and in the sagittal plane.

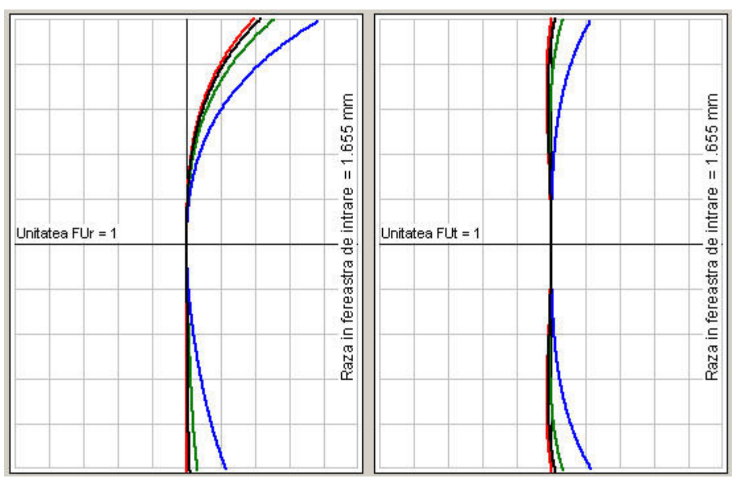

Fig. 5. Wavefront aberrations of the four base radiations, computed in the meridian plane and in the sagittal plane.

5. Computation method for the simulation of the variation of the crystalline refraction index

The crystalline plays an essential role in the resolution of the human eye and its accommodation to finite distances. It suffers a complex modification caused by the variation of the curvature radii, the narrowing of anterior chamber, the variation of crystalline thickness and the variation of the refraction index. The crystalline presents a poorly known gradual refraction index structure, with axial and radial variation.

From figure 5 it can be observed that the maximum wavefront aberration is of about 2.5 wavelengths for the C radiation. Considering that the diffusion spot on the retina must have at least the size of a cone cell, the value of the wavefront aberration is very high in this case. It is a foreseeable result, because the complex modification of the crystalline refraction index, the most important source of aberration correction imagined by nature, was not considered.

In order to correct this downside, it is presented onwards a method for computing an interpolation polynomial that simulates the variation of the crystalline refraction index in function of the incidence point in the plane perpendicular on the optical axis and tangent to the anterior radius of the crystalline $P(x_i,y_i)_{crystalline}$.

An uniform distribution in the eye input pupil, presented in figure 6, is generated for the computation. This distribution must be perpendicular on the main pupil ray. A bundle of optic rays will emerge from an object point and will pass through this distribution. For a certain optical ray from the bundle, referred in continuation, vector traversing will give the coordinates of the point $P(x_i,y_i)_{crystalline}$, as well as the wavefront aberration, denoted A_i. The refraction index of the crystalline attached to this point will be considered to be a parameter to be optimized. Accordingly, the function $f_i = W_i(A_i - T_i)$ is built, where: W_i represents a weight, A_i the considered aberration and T_i the target to be reached.

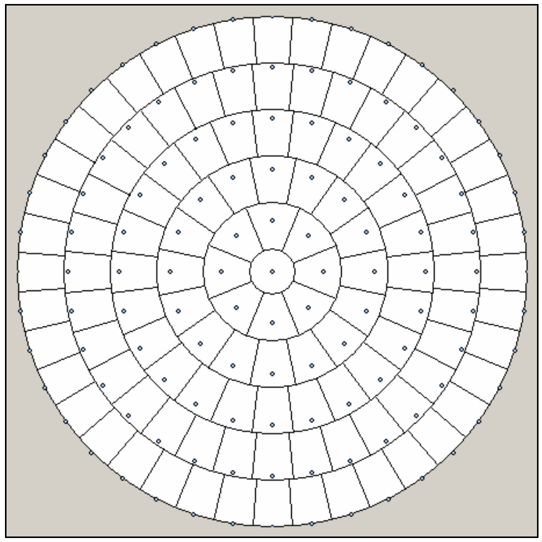

Fig. 6. Uniform distribution in the eye input pupil.

If an optimization algorithm is used in order to modify the crystalline refraction index, an optimum denoted with $(n_{optimum})_i$, where the function f_i will reach a minimum, will be obtained.

This value is attached to the coordinates of the point P. For the analyzed ray, the set $(x,y,n_{optimum})_i$ is obtained. If the described procedure is repeated for all the distribution points of the input pupil, data allowing the computation of the interpolation polynomial that would describe the complex modification of the crystalline refraction index will be obtained. The least squares method offers an easy way of doing it.

The computed interpolation polynomial will be used for a modified vector traversing that must change the crystalline refraction index in function of its incidence with the plane tangent to the anterior radius of the crystalline. If the analysis of results is not satisfactory, the method can be repeated changing the targets and the weights of the functions f_i.

6. Conclusions

As presented thus far, the modeling and therefore the simulation of the functioning of the human eye was based on simplifying hypotheses on the interpolation formula that gave the refraction indexes, on the fact that the dioptres were considered to be spherical and that the computation method proposed for the crystalline refraction index was considered correct. If these hypotheses are correct, the modeling of the optical system of the eyeball is correct and simulation will be close enough to the reality.

However, a number of remarks have to be done. The first remark refers to a better way of establishing the interpolation formulas for the refraction indexes of the optical components of the human eye by considering also the measured dispersion coefficients. The second remark aims the rigorous assignment of the weights and targets in conditions of assigning also the degree of the polynomial when the interpolation formula for the crystalline refraction index is determined.

If these remarks are respected and also aspheric dioptres are considered, the mathematical modeling of the functioning of the eyeball optical system will be very close to reality.

R E F E R E N C E S

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