SOFT LOOPS

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The main objective of this paper is to introduce the notion of soft loops as a generalization of the soft groups which were defined by Aktas and Cagman in [1]. *We discuss structural properties of the soft loops and generalize some existing results of* [1] *and* [5] *for the theory of soft loops*

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1. Introduction

Foundation of soft sets was laid by Molodtsov [14] in 1999 in order to introduce a mathematical tool for modeling vagueness and uncertainty arising in computer sciences, natural sciences, social sciences, medical and many other fields. This is one of mathematical concepts like fuzzy sets [21], generalized fuzzy sets [6] and rough sets [16] given in order to overcome the difficulties in dealing with uncertainties arising in above mentioned fields and daily life, which were hard to handle with, in classical set theory.

A soft set is a parameterized family of subsets of a universal set. This can be considered as a neighborhood system, and is a generalized case of context-dependent fuzzy set. There does not occur the problem of setting the membership function, among other related problems in soft sets. This makes it very convenient and easy to apply in practice. Because of this friendly behavior, soft sets have various applications within and outside mathematics. For basic notions and the applications of soft sets, we incite to read [1, 4, 5, 10, 12, 13, 14, 15].

The study of non-associative structures is justified since two out of four basic binary operations are non-associative. A loop Q is a non-associative algebraic structure, which is defined as usual, a groupoid with binary operation '$\cdot$' having an element e such that $e \cdot x = x \cdot e = x$, for all $x \in Q$ and in the equation $x \cdot y = z$ any two of x, y, z uniquely determine the third. The study of loops started in 1920's and these were introduced for the first time in 1930's [19]. The theory of loops has its roots in geometry, algebra and combinatorics. This can be found in non-associative products in algebra, in combinatorics this is present in latin squares of particular form and in geometry it has connection with the analysis of web structures [18]. A detailed study of theory of the loops can be found in [2, 3, 7, 8, 9, 18].

In [1], authors defined soft groups and gave fundamental results in soft group theory as a generalization of fuzzy groups [20]. They showed that a fuzzy group is a special case of soft group. This gave a new dimension to soft set theory and involved algebra in it. This work was extended by Aslam and Qurashi in [5], discussing various structural properties of soft groups their substructures and structure preserving mappings.

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In this paper, we introduce soft loop as a generalization of the soft group. We discuss structural properties of soft loops, define its substructures. We show that the class of soft loops, soft subloops, soft normal loops, and soft normal subloops is closed under certain soft set theoretic operations.

Paper is organized as follows: Section 2 is devoted to the basic definitions and terminologies of soft sets and loops. In Section 3, we define soft loops, soft subloop and include related results. Section 4 contains the notions of soft normal loops, soft normal subloops and the results that the class of normal subloops is closed under certain soft set theoretic operations. At the end of section 4, we define soft loops homomorphism. In the last section we conclude our paper by mentioning the possible directions in which this work can be extended.

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2. **Preliminaries**

In this section, we give some fundamentals of the soft set theory and the theory of loops which will be required to develop the theory of soft loops in Section 3 and Section 4. We start with the notion of soft sets and operations defined on it. We consulted [4, 12, 13] for this purpose, readers are invited to go through these for details.

Throughout this section U denotes an initial universe, E is the set of parameters and $A \subseteq E$. We define a soft set (F, A) over U as:

Definition 2.1. *A soft set over U, is an ordered pair (F, A), where F is a function given by*

$$F : A \to P(U).$$

Definition 2.2. *A soft set (F, A) over U is a soft subset of another soft set (G, B) over U, denoted as $(F, A)\widetilde{\subset}(G, B)$, if $A \subseteq B$ and $F(a) \subseteq G(a)$, for all $a \in A$. Two soft sets over the same universe are equal if both are soft subsets of each other.*

Definition 2.3. *The union of two soft sets (F, A) and (G, B) over U is denoted as $(F, A)\widetilde{\cup}(G, B)$ and is a soft set (H, C) over U such that $C = A \cup B$ and*

$$H(x) = \begin{cases} F(x) & \text{if} \quad x \in A - B \\ G(x) & \text{if} \quad x \in B - A \\ F(x) \cup G(x) & \text{if} \quad x \in A \cap B \end{cases}$$

Definition 2.4. *Let (F, A) and (G, B) be two soft sets over U, then the intersection $(F, A)\widetilde{\cap}(G, B)$ of these soft sets is a soft set (H, C) over U such that $C = A \cap B$ and $H(c)$ is either $F(c)$ or $G(c)$ for all $c \in C$.*

Pei and Miao in [17] indicated that generally $F(c)$ or $G(c)$ may not be identical and thus revised the above definition as:

Definition 2.5. *Let (F, A) and (G, B) be two soft sets over U. The intersection $(F, A)\widetilde{\cap}(G, B)$ of these two soft sets is a soft set (H, C) over U such that $C = A \cap B$ and $H(c) = F(c) \cap G(c)$ for all $c \in C$.*

Kharal and Ahmad in [11] pointed out that in Definition 2.5, A and B must not be disjoint to avoid the degenerate case. So, the definition of intersection of two soft sets took the following form.

Definition 2.6. *Let (F, A) and (G, B) be two soft sets over U such that $A \cap B \neq \emptyset$, then their intersection $(F, A)\widetilde{\cap}(G, B)$ is a soft set (H, C) over U such that $C = A \cap B$ and $H(c) = F(c) \cap G(c)$, for all $c \in C$.*

Ali et al. in [4], defined two new operations on soft sets called *restricted intersection* and *extended intersection*, which are given as;

Definition 2.7. *Let (F, A) and (G, B) be two soft sets over a common universe U such that $A \cap B \neq \emptyset$. Then their restricted intersection is denoted and defined as $(F, A) \Cap (G, B) = (H, C)$ where $C = A \cap B$ and $H(c) = F(c) \cap G(c)$, for all $c \in C$.*

Definition 2.8. *Let (F, A) and (G, B) be two soft sets over a common universe U. Then their extended intersection, which is denoted by $(F, A) \sqcap (G, B)$ is a soft set (H, C), where $C = A \cup B$ and for all $c \in C$,*

$$H(c) = \begin{cases} F(c) & \text{if} \quad c \in A - B \\ G(c) & \text{if} \quad c \in B - A \\ F(c) \cap G(c) & \text{if} \quad c \in A \cap B \end{cases}$$

We remark that definitions of intersection of soft sets given by Kharal and Ahmad in [11] and that of restricted intersection of soft sets defined by Ali et al. in [4] are same. However, in our paper we use the definition of restricted intersection as given in [4].

Definition 2.9. *The restricted union of two soft sets (F, A) and (G, B) over U with $A \cap B \neq \emptyset$ is a soft set (H, C) such that $C = A \cap B$ and $H(c) = F(c) \cup G(c)$, for all $c \in C$. This notion is denoted as $(H, C) = (F, A) \cup_R (G, B)$.*

Definition 2.10. *The 'AND' operation between two soft sets (F, A) and (G, B) over a common universe U is denoted as $A \wedge B$ and is a soft set $(H, A \times B)$, where $H(a, b) = F(a) \cap G(b)$, for all $a \in A$ and $b \in B$.*

The rest of this section is devoted for the notions of loops, subloops, normal subloops and few important results about these notions which will be useful in setting the theory of soft loops.

Definition 2.11. *A loop $(Q, \cdot)$ is a groupoid with the following properties*
(L1) for each $a, b \in Q$ there exist unique elements $x, y \in Q$ such that

$$a \cdot x = b \text{ and } y \cdot a = b.$$

(L2) Q has an element e satisfying

$$e \cdot x = x \cdot e = x, for \ \ all \, x \in Q.$$

So, a loop is a quasigroup with an identity element. The binary operation table of a finite quasigroup is a square array with symbols arranged, so that each symbol appears once in each column and each row. This is called a latin square. Following is an example of finite loop with six symbols:

Example 2.1. *Let $Q_1 = \{e, a_1, a_2, a_3, a_4, a_5\}$ be a loop with identity e and latin square*

$(Q_1, \cdot)$						
$\cdot$	e	a_1	a_2	a_3	a_4	a_5
e	e	a_1	a_2	a_3	a_4	a_5
a_1	a_1	a_5	a_4	e	a_2	a_3
a_2	a_2	a_3	a_5	a_4	a_1	e
a_3	a_3	e	a_1	a_2	a_5	a_4
a_4	a_4	a_2	a_3	a_5	e	a_1
a_5	a_5	a_4	e	a_1	a_3	a_2

Definition 2.12. *Let $(Q, \cdot)$ be a loop, $H \subseteq Q$ is called a subloop of Q if $(H, \cdot)$ is a loop itself.*

Example 2.2. *The loop Q_1 given in Example 2.1 has 4 subloops, namely $H_1 = \{e\}$, $H_2 = \{e, a_4\}$, $H_3 = \{e, a_2, a_5\}$ and $H_4 = Q$. Out of these four subloops two subloops H_1 and H_4 are trivial subloops and other two subloops are non-trivial.*

Remark 2.1. *It is interesting to note that unlike the groups and subgroups, order of a subloop may not divide the order of loop.*

We support the above remark with an example of loop $Q_2 = \{e, a_1, a_2, a_3, a_4\}$ of order 5 with the following latin square

$(Q_2, *)$					
$*$	e	a_1	a_2	a_3	a_4
e	e	a_1	a_2	a_3	a_4
a_1	a_1	e	a_3	a_4	a_2
a_2	a_2	a_3	a_4	a_1	e
a_3	a_3	a_4	e	a_2	a_1
a_4	a_4	a_2	a_1	e	a_3

and has a subloop $\{e, a_1\}$ of order 2.
The following result from [9] can be easily verified.

Lemma 2.1. *Intersection of two subloops of a loop is a loop.*

Next, we define normal subloop, see [8].

Definition 2.13. *A subloop H of a loop $(Q, .)$ is called normal subloop of Q if*
(1) $aH = Ha$,
(2) $a(bH) = (ab)H$,
(3) $(Hb)a = H(ba)$, for every $a, b \in H$.
We denote this relation by $H \trianglelefteq Q$.

Example 2.3. *Let $Q_3 = \{e, a_1, a_2, a_3, a_4, a_5\}$ be a loop with identity e and latin square*

$(Q_3, *_3)$						
$*_3$	e	a_1	a_2	a_3	a_4	a_5
e	e	a_1	a_2	a_3	a_4	a_5
a_1	a_1	a_2	e	a_4	a_5	a_3
a_2	a_2	e	a_1	a_5	a_3	a_4
a_3	a_3	a_4	a_5	a_2	e	a_1
a_4	a_4	a_5	a_3	a_1	a_2	e
a_5	a_5	a_3	a_4	e	a_1	a_2

The loop Q_3 has a subloop $N = \{e, a_1, a_2\}$ which is normal in Q_3.

Here is an important result from [18] which has a key role in the development of the theory of normal soft loops and normal soft subloops.

Lemma 2.2. *The intersection of any non-empty family of normal subloops of a loop is a normal subloop.*

Definition 2.14. *Let* $(Q, *)$ *and* $(Q^{'}, *^{'})$ *be two loops, a map*

$$f : Q \longrightarrow Q^{'}$$

is said to be a loop homomorphism if

$$f(x * y) = f(x) *^{'} f(y),\ \forall\, x, y \in Q.$$

Unlike the associative algebraic structures, the homomorphic image of a loop is not always a loop. However if Q is a loop with identity e, and its homomorphic image $f(Q) = Q^{'}$ is a quasigroup, then $Q^{'}$ is also a loop with identity $e^{'} = f(e)$.

A bijective homomorphism between two loops is called an isomorphism. Two loops are said to be isomorphic if there exists at least one isomorphism between them and this relation is denoted by the symbol $' \cong '$. Isomorphy is an equivalence relation and its equivalence classes are said to be isomorphy classes.

As in group theory an isomorphism of a loop to itself is called an automorphism and all the automorphisms on a loop form a group [18]. We end this section with the following lemma which will be useful in defining the notion of soft loop homomorphism.

Lemma 2.3. *Let* $(Q, *_1)$ *and* $(Q^{'}, *_2)$ *be two isomorphic loops under isomorphism*

$$f : Q_1 \longrightarrow Q_2.$$

If H *is a subloop of* Q_1*, then* $H^{'} = f(H)$ *is a subloop of* Q_2*.*

3. **Soft Loops And Soft Subloops**

It is straightforward fact to verify that any relation R of a loop Q and a set Λ, that is $R \subset \Lambda \times Q$ induces a map

$$\Phi : \Lambda \longrightarrow P(Q)$$

which is defined, for all $a \in \Lambda$ as:

$$\Phi(a) = \{q \in Q \mid (a, q) \in R\},$$

thus a soft set (Φ, Λ) over Q.
Conversely, every map

$$\Phi : \Lambda \longrightarrow P(Q)$$

gives rise to a binary relation $R \subset \Lambda \times Q$ which is defined as $R = \{(x, q) \mid q \in \Phi(x)\}$. Next, we define the notion of soft loops which is the main object of this paper.

Definition 3.1. *A soft set* (Φ, Λ) *over a loop* Q *is a soft loop if* $\Phi(x)$ *is a subloop of* Q*, for all* $x \in Q$*.*

Following are two trivial examples of soft loops which can be defined over any loop.

Example 3.1. *Let* Q *be a loop with an identity* e*,* Λ *be any set of parameters . Then a soft loop* (I, Λ) *over* Q*, where* I *is defined as:* $I(x) = \{e\}$*, for all* $x \in \Lambda$*, is called an identity soft loop over* Q*.*

Example 3.2. *A soft loop* (Θ, Λ) *over a loop* Q *is called a full soft loop over* Q*, if* $\Theta(x) = Q$*, for all* $x \in \Lambda$*.*

We give the following non-trivial example by defining a soft loop over a loop of order 5.

Example 3.3. *Let $Q = \{e, a_1, a_2, a_3, a_4\}$ be a loop with identity e as in Example 2.1. It has four subloops H_1, H_2, H_3 and H_4. Let G be a symmetric group with the following finite presentation*

$$\langle x, y \mid x^2 = y^3 = (xy)^2 = 1\rangle$$

The group G has six elements e, x, y, y^2, xy, xy^2. We define a relation from G to the set of subloops of Q as $g \sim H_i$ if and only if $o(g) = |H_i|$, where $o(g)$ is the order of the element g and $|H_i|$ denotes the cardinality of H_i, $i = 1, 2, 3, 4$. The function $\Phi : G \longrightarrow P(Q)$ defined as

$$\Phi(g) = \{H_i \mid g \sim H_i\},$$

is induced by the relation just defined above. That is $\Phi(e) = H_1$, $\Phi(x) = \Phi(xy) = \Phi(xy^2) = H_2$, $\Phi(y) = \Phi(y^2) = H_3$. Then (G, Φ) is a soft loop over Q.

Now we give a series of results about the soft loops, which show that the class of soft loops is closed under the operations of intersection, extended intersection, 'AND' operation, union and restricted union.

Theorem 3.1. *Let (Φ_1, Λ_1) and (Φ_2, Λ_2) be two soft loops over a loop Q such that $\Lambda_1 \cap \Lambda_2 \neq \emptyset$ then their restricted intersection $(\Phi_1, \Lambda_1) \Cap (\Phi_2, \Lambda_2)$ is a soft loop over Q.*

Proof. Let $(\Phi_1, \Lambda_1) \Cap (\Phi_2, \Lambda_2) = (\Phi, \Lambda)$. Then (Φ, Λ) is a soft set over Q, where $\Lambda = \Lambda_1 \cap \Lambda_2$ and

$$\Phi : \Lambda \longrightarrow P(Q)$$

defined as $\Phi(x) = \Phi_1(x) \cap \Phi_1(x)$. Then by Lemma 2.1, $\Phi(x)$ is a subloop of Q being the intersection of two subloops, for all $x \in \Lambda$. This completes the proof.■

Corollary 3.1. *Let I be an indexing set, $(\phi_i, \Lambda_i)_{i \in I}$ be a family of soft loops over Q. Then the restricted intersection $\Cap_{i \in I}(\phi_i, \Lambda_i)$ is a soft loop over Q.*

Theorem 3.2. *extended intersection $(\Phi_1, \Lambda_1) \sqcap (\Phi_2, \Lambda_2)$ of two soft loops (Φ_1, Λ_1) and (Φ_2, Λ_2) over Q is a soft loop over Q.*

Proof. Let $(\Phi_1, \Lambda_1) \sqcap (\Phi_2, \Lambda_2) = (\Phi, \Lambda)$. Then by definition (Φ, Λ) is a soft set over Q, where $\Lambda = \Lambda_1 \cup \Lambda_2$ and

$$\Phi : \Lambda \longrightarrow P(Q)$$

is a map defined as

$$\Phi(x) = \begin{cases} \Phi_1(x) & \text{if} \quad x \in \Lambda_1 - \Lambda_2 \\ \Phi_2(x) & \text{if} \quad x \in \Lambda_2 - \Lambda_1 \\ \Phi_1(x) \cap \Phi_2(x) & \text{if} \quad x \in \Lambda_1 \cap \Lambda_2 \end{cases}$$

The image $\Phi(x)$ is a subloop of Q in all above situations, consequently $(\Phi_1, \Lambda_1) \sqcap (\Phi_2, \Lambda_2)$ is a soft loop over Q.■

Corollary 3.2. *The extended intersection $\sqcap_{i \in I}(\phi_i, \Lambda_i)$ of a family $(\phi_i, \Lambda_i)_{i \in I}$ of soft loops over a loop Q indexed by a set I is also a soft loop over Q.*

Theorem 3.3. *Let (Φ_1, Λ_1) and (Φ_2, Λ_2) be soft loops over Q such that $\Lambda_1 \cap \Lambda_2 = \emptyset$, then their union $(\Phi_1, \Lambda_1) \widetilde{\cup} (\Phi_2, \Lambda_2)$ is also a soft loop over Q.*

Proof. Let $(\Phi_1, \Lambda_1)\widetilde{\cup}(\Phi_2, \Lambda_2)$. Then by definition (Φ, Λ) is a soft set over Q, where $\Lambda = \Lambda_1 \cup \Lambda_2$ and

$$\Phi : \Lambda \longrightarrow P(Q)$$

is a map defined as

$$\Phi(x) = \begin{cases} \Phi_1(x) & \text{if} \quad x \in \Lambda_1 - \Lambda_2 \\ \Phi_2(x) & \text{if} \quad x \in \Lambda_2 - \Lambda_1 \\ \Phi_1(x) \cup \Phi_2(x) & \text{if} \quad x \in \Lambda_1 \cap \Lambda_2 \end{cases}$$

Since $\Lambda_1 \cap \Lambda_2 = \emptyset$, $\Phi(x) = \Phi_i(x)$ for $i = 1, 2$, is a subloop of Q for all $x \in \Lambda$. We have the required result.■

Corollary 3.3. *Let Q be a loop, $(\phi_i, \Lambda_i)_{i\in I}$ be an indexed family of soft loops over Q, such that Λ_i are pairwise disjoint for all $i \in I$. Then the union $\widetilde{\cup}_{i\in I}(\phi_i, \Lambda_i)$ is also a soft loop over Q.*

Theorem 3.4. *The 'AND' operation $(\Phi_1, \Lambda_1) \wedge (\Phi_2, \Lambda_2)$ of two soft loops over Q gives another soft loop over Q.*

Proof. Assume that $(\Phi_1, \Lambda_1) \wedge (\Phi_2, \Lambda_2) = (\Phi, \Lambda)$. Then as we know (Φ, Λ) is a soft loop over Q where $\Lambda = \Lambda_1 \times \Lambda_2$ and

$$\Phi : \Lambda \longrightarrow P(Q)$$

defined as $\Phi(x_1, x_2) = \Phi_1(x_1) \cap \Phi_2(x_2)$, where $x_1 \in \Lambda_1$ and $x_2 \in \Lambda_2$. Then $\Phi(x_1, x_2)$ being an intersection of subloops is a subloop of Q. Hence the assertion follows.■

Corollary 3.4. *The 'AND' operation $\wedge_{i\in I}(\phi_i, \Lambda_i)$ of an indexed family $(\phi_i, \Lambda_i)_{i\in I}$ of soft loops over a loop Q is a soft loop over Q.*

Definition 3.2. *Let (Φ, Λ) and (Ψ, Γ) be the soft loops over Q, (Ψ, Λ) is said to be a soft subloop of (Φ, Γ) if*
(i) $\Lambda \subset \Gamma$,
(ii) $\Psi(a)$ is a subloop of $\Phi(a)$, for all $a \in A$.
We shall denote this relation as $(\Psi, \Lambda)\widetilde{<}(\Phi, \Gamma)$.

Example 3.4. *Let (G, Φ) be a soft loop over the loop Q as in example 3.3. Let $H = \{1, x\} \subseteq G$. Then (H, Ψ) is a soft subloop of (G, Φ), where Ψ is a restriction of Φ on H.*

Theorem 3.5. *Let $(\Phi_i, \Lambda_i)_{i\in I}$ be an indexed family of soft subloops of soft loop (Φ, Λ) over Q. Then*
(1) The restricted intersection $\Cap_{i\in I}(\Phi_i, \Lambda_i)$ is a soft subloop.
(2) The extended intersection $\sqcap_{i\in I}(\Phi_i, \Lambda_i)$ is a soft subloop.
(3) $\widetilde{\cup}_{i\in I}(\Phi_i, \Lambda_i)$ is a soft subloop if Λ_i are pairwise disjoint for all $i \in I$.
(4) $\wedge_{i\in I}(\Phi_i, \Lambda_i)$ is a soft subloop.

Proof. Proof of the theorem can be obtained on the same lines as we did for soft loops.■

4. Soft normal loop and Soft Normal Subloops

Definition 4.1. *A soft loop (Φ, Λ) over Q is called normal soft loop if $\Phi(a)$ is a normal subloop of Q, for all $a \in \Lambda$.*

There are always two normal soft loops over any loop, namely the identity loop (I, Λ) and full soft loop (Θ, Λ). These are trivial normal soft loops, following is an example of a non-trivial normal soft loop.

Example 4.1. *Let*

$(Q_4, *_4)$								
$*_4$	e	a_1	a_2	a_3	a_4	a_5	a_6	a_7
e	e	a_1	a_2	a_3	a_4	a_5	a_6	a_7
a_1	a_1	a_3	a_4	a_5	a_6	e	a_7	a_2
a_2	a_2	a_7	a_3	a_6	a_1	a_4	e	a_5
a_3	a_3	a_5	a_6	e	a_7	a_1	a_2	a_4
a_4	a_4	a_2	a_5	a_7	a_3	a_6	a_1	e
a_5	a_5	e	a_7	a_1	a_2	a_3	a_4	a_6
a_6	a_6	a_4	e	a_2	a_5	a_7	a_3	a_1
a_7	a_7	a_6	a_1	a_4	e	a_2	a_5	a_3

be a latin square of a loop $Q_4 = \{e, a_1, a_2, a_3, a_4, a_5, a_6, a_7\}$ *on eight symbols. Let* $\Lambda = \{\kappa_1, \kappa_2, \kappa_3, \kappa_4, \kappa_5, \kappa_6\}$ *be any set of parameters. We define a function*

$$\Phi : \Lambda \longrightarrow P(Q_4)$$

by

$$\Phi(\kappa_1) = \{e\},\ \Phi(\kappa_2) = \{e, a_3\},\ \Phi(\kappa_3) = \{e, a_1, a_5\},\ \Phi(\kappa_4) = \{e, a_2, a_6\},$$
$$\Phi(\kappa_5) = \{e, a_4, a_7\},\ \Phi(\kappa_6) = \{e, a_1, a_2, a_3, a_4, a_5, a_6, a_7\}.$$

Then (Φ, Λ) *is a normal soft loop over* Q.

Theorem 4.1. *Let* $\Omega = \{\Phi_i, \Lambda_i\}$ *be a family of normal soft loops defined over* Q *which is indexed by the set* I. *Then the set* Ω *is closed under the soft set operations restricted intersection, extended intersection and 'AND' operation.*

Proof. Statement follows by using the Theorem 3.1 , Theorem 3.2, Theorem 3.4 and Lemma 2.2.■

Definition 4.2. *A soft subloop* (Ψ, Γ) *of a soft loop* (Φ, Λ) *over* Q *is called a normal soft subloop if* $\Psi(a)$ *is a normal subloop of* $\Phi(a)$, *for all* $a \in \Gamma$. *We denote this relation of* (Ψ, Γ) *and* (Φ, Λ) *by* $(\Psi, \Gamma)\ddot{\triangleleft}(\Phi, \Lambda)$.

Theorem 4.2. *Let* (Φ, Λ) *be a soft loop over* Q *and* (Φ_i, Λ_i) *be a family of normal soft subloops of* (Φ, Λ) *which is indexed by the set* I. *Then,*
(1) The restricted intersection $\Cap_{i\in I}(\Phi_i, \Lambda_i)$ *is a soft normal subloop.*
(2) The extended intersection $\sqcap_{i\in I}(\Phi_i, \Lambda_i)$ *is a soft normal subloop.*
(3) $\tilde{\cup}_{i\in I}(\Phi_i, \Lambda_i)$ *is a soft normal subloop if* Λ_i *are pairwise disjoint for all* $i \in I$.
(4) $\wedge_{i\in I}(\Phi_i, \Lambda_i)$ *is a soft normal subloop.*

Proof. Assertions follow by using Theorem 3.5 and Lemma 2.2.■

We end this section by giving the definition of soft loops homomorphism.

Definition 4.3. *Let* (Ψ, Γ) *and* (Φ, Λ) *be two soft loops over a loop* Q. *A pair of mappings* (f, g) *is said to be soft loops homomorphism from* (Ψ, Γ) *to* (Φ, Λ), *if*
(i) f *is an automorphism on* Q,
(ii) g *is a surjective mapping from* Γ *to* Λ,
(iii) $f(\Psi(x)) = \Phi(g(x))$, *for all* $x \in \Gamma$.

If g is a bijective mapping from Γ to Λ then the pair (f, g) is said to be a soft loop isomorphism, and the soft loops are said to be isomorphic. We denote this relation between the two loops (Ψ, Γ) and (Φ, Λ) by $(\Psi, \Gamma)\ddot{\cong}(\Phi, \Lambda)$.

5. **Conclusion**

In this paper we introduce the notions of soft loops, their substructures and structure preserving mappings. We include various results about the soft loops, soft subloops, soft normal loops and soft normal subloops in order to extend a theoretical study of soft loops. In the end we define soft loops homomorphism which is an analogue of similar notion in soft group theory.

Molodtsov in his seminal paper [14] demonstrated various applications of soft sets in probability theory, game theory, operations research, measurement theory, smoothness of functions, Riemann integration, Perron integration. Later in [13], authors exhibited application of soft set theory in decision making problem.

The theory of soft algebraic structures is in its initial phase. The present paper is an effort to lay the foundation of theory of soft loops. One can extend this work in both application and theoretical directions. Still the quotients of soft loops and soft loops homomorphism theorems are open but hard problems.

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