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MHD MICROPOLAR FLUID FLOW OVER A MOVING PLATE UNDER SLIP CONDITIONS: AN APPLICATION OF LIE GROUP ANALYSIS

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In this paper, the boundary layer flow of an incompressible electrically conducting MHD micropolar fluid over a moving vertical plate with slip velocity and temperature jump at the boundary surface is theoretically studied in presence of first order chemical reaction. The governing equations are reduced to a set of coupled ordinary differential equations by applying the efficient method of Lie group analysis. The fourth order Runge-Kutta method with shooting technique is employed to solve the problem numerically. The impact of various pertinent parameters on flow characteristics are discussed in detail through graphs and tables. To see the validity of numerical code, the comparison of the present results is made with the existing literature.

Keywords: Lie group; Chemical reaction; Slip flow; Micropolar fluid

1. Introduction

Most of the fluids such as liquid crystals, polymers, body fluids, exotic lubricants, colloidal fluids, paints etc. are considered as non-Newtonian fluids in the engineering field. Micropolar fluid model is the most popular rheological model for non-Newtonian fluid. The theory of micropolar fluids which contain micro-constituents was first introduced by Eringen [1]. These fluids contain dilute suspension of rigid macro-molecules with individual motions that support stress body moments and are influenced by spin inertia, the presence of which can affect the hydrodynamics of the flow. The boundary layer theory for micropolar fluid was discussed by Peddison and Mcnitt [2]. The concept of thermo-micropolar fluids was developed by Eringen [3]. During the last several decades many research workers [4-12] have reported the results on the heat transfer problems of micropolar fluids. Recently, micropolar fluid flow through a porous channel was solved analytically by Mosayebidorcheh [13]. Turkyilmazoglu [14] examined micropolar fluid flow and heat transfer over a porous shrinking sheet.

In recent years, a large number of researchers [15-19] have been reported in the field of chemical reaction analysis to predict the reactor performance. The study of heat and mass transfer with chemical reaction is important in chemical

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and hydrometallurgical industries. The first-order chemical reaction where the rate of reaction is directly proportional to the species concentration is the simplest one. Formation of smog is an example of first-order homogeneous chemical reaction. Damseh et al. [20] discussed the combined effect of heat generation or absorption and first-order chemical reaction on micropolar fluid flow. Recently, Das [21] analyzed the effect of first-order chemical reaction and thermal radiation on micropolar fluid analytically.

The no-slip condition is not appropriate for the problem of fluid flow and heat transfer when the fluid is particulate such as suspensions, polymer solutions etc. It is also observed from the open literature [22-24] that the slip boundary condition is necessary when the flow pressure is very low or the characteristic size of the flow system is small. The fluid flow over a stretching sheet with constant suction and partial slip has been studied by Wang [25]. The model proposed by Wang [25] was extended by Van Gorder et al. [26] taking into account the various flow due to a stretching surface. Das [27] analyzed the micropolar fluid flow towards a shrinking sheet with slip boundary conditions. Zheng et al. [28] discussed micropolar fluid flow and heat transfer over a permeable plate under slip conditions.

Main objective of the present investigation is to study the combined effects of first order chemical reaction and internal heat source/sink on heat and mass transfer in a micropolar fluid flow over a moving flat plate under slip boundary conditions.

2. Mathematical Formulation of the Problem

Let us consider a steady laminar boundary layer flow of an incompressible electrically conducting micropolar fluid along a moving vertical plate (Fig.1). It is also assumed that the constant temperature of the wall is T_w and that of the surrounding fluid is T_∞ where $T_w > T_\infty$. A transverse magnetic field $B(\overline{x})$ is applied normal to the plate. It is assumed that the fluid flow is under slip boundary conditions. We consider temperature dependent heat source/sink in the flow region to get the effect of temperature difference between the plate and the ambient fluid. There exists a first order chemical reaction between the fluid and species concentration.

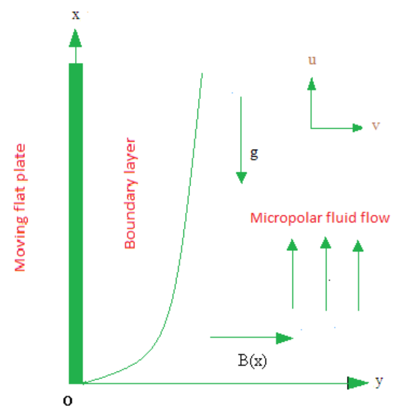

Fig.1. Physical model and coordinate system

Under these assumptions, the governing boundary layer equations are given by

$$\frac{\partial \bar{u}}{\partial \bar{x}}+\frac{\partial \bar{v}}{\partial \bar{y}}=0 \tag{1}$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}=\left(\frac{\mu+k}{\rho}\right)\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}+\frac{k}{\rho}\frac{\partial \bar{N}}{\partial \bar{y}}-\frac{\sigma B^2(\bar{x})}{\rho}\bar{u}+g\beta_1(T-T_\infty)+g\beta_2(\mathrm{C}-C_\infty) \tag{2}$$

$$\bar{u}\frac{\partial \bar{N}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{N}}{\partial \bar{y}}=\frac{\gamma}{\rho j}\frac{\partial^2 \bar{N}}{\partial \bar{y}^2}-\frac{k}{\rho j}\left(\frac{\partial \bar{u}}{\partial \bar{y}}+2\bar{N}\right) \tag{3}$$

$$\bar{u}\frac{\partial T}{\partial \bar{x}}+\bar{v}\frac{\partial T}{\partial \bar{y}}=\alpha\frac{\partial^2 T}{\partial \bar{y}^2}+\frac{Q_1}{\rho c_p}\left(T-T_\infty\right) \tag{4}$$

$$\bar{u}\frac{\partial C}{\partial \bar{x}}+\bar{v}\frac{\partial C}{\partial \bar{y}}=D\frac{\partial^2 C}{\partial \bar{y}^2}-k_r(\bar{x})\left(C-C_\infty\right) \tag{5}$$

where $\bar{u}$ and $\bar{v}$ are the velocity components along $\bar{x},\bar{y}$ axes, $\bar{N}$ is the angular velocity whose direction of rotation is in the $\bar{x}\,\bar{y}$-plane, σ is the electrical conductivity, c_p is the specific heat, ν is the kinematic viscosity, T is the temperature, k is the micropolar vortex viscosity, β_1 is the volumetric coefficient of thermal expansion, β_2 is the volumetric coefficient of species concentration, μ is the dynamic viscosity, j is the microinertia, α is the thermal diffusivity, n is a constant, γ is the micropolar spin gradient viscosity, Q_1 is the dimensional heat generation or absorption coefficient, C is the species concentration, D is the mass diffusivity and $k_r(x)$ is the chemical reaction rate constant.

The appropriate boundary conditions for the present model are given by

$$\left.\begin{array}{l} \bar{u}=U_w+N_1(\bar{x})\nu\dfrac{\partial\bar{u}}{\partial\bar{y}},\ \bar{v}=0,\ \bar{N}=-n\dfrac{\partial\bar{u}}{\partial\bar{y}},\ T=T_w+D_1(\bar{x})\dfrac{\partial T}{\partial\bar{y}},\ C=C_w \text{ at } \bar{y}=0 \\ \bar{u}=0,\ \bar{N}=0,\ T=T_\infty,\ C=C_\infty \text{ as } \bar{y}\to\infty \end{array}\right\} \quad (6)$$

where N_1 is the hydrodynamic slip factor and D_1 is the thermal slip factor.
To non-dimensionlize the above system of equations, let us introduce the stream function ψ and the following dimensionless variables:

$$\left.\begin{array}{l} x=\dfrac{\bar{x}}{L},\ y=\dfrac{\bar{y}}{L}\sqrt{\mathrm{Re}},\ u=\dfrac{\bar{u}}{U_w},\ v=\dfrac{\bar{v}}{U_w}\sqrt{\mathrm{Re}},\ N=\dfrac{\bar{N}L}{U_w\sqrt{\mathrm{Re}}},\theta=\dfrac{T-T_\infty}{T_w-T_\infty} \\ ,\ \mathrm{Re}=\dfrac{U_wL}{\nu},\ \phi=\dfrac{C-C_\infty}{C_w-C_\infty} \end{array}\right\} \quad (7)$$

Let us now launch the simplified form of Lie group transformations to convert the equations (2)-(6) into a system of ordinary differential equations as follows:

$$\left.\begin{array}{l} \Gamma:\ x=e^{-\varepsilon c_1}x^*, y=e^{-\varepsilon c_2}y^*, \psi=e^{-\varepsilon c_3}\psi^*, \theta=e^{-\varepsilon c_4}\theta^*, N=e^{-\varepsilon c_5}N^*, \phi=e^{-\varepsilon c_6}\phi^*, \\ \beta_1=e^{-\varepsilon c_7}\beta_1^*, \beta_2=e^{-\varepsilon c_8}\beta_2^*, B=e^{-\varepsilon c_9}B^*, D_1=e^{-\varepsilon c_{10}}D_1^*, j=e^{-\varepsilon c_{11}}j^*, \gamma=e^{-\varepsilon c_{12}}\gamma^*, \\ N_1=e^{-\varepsilon c_{13}}N_1^*, Q_1=e^{-\varepsilon c_{14}}Q_1^*, k_r=e^{-\varepsilon c_{15}}k_r^* \end{array}\right\} \quad (8)$$

After going proceeding through regular calculations, we achieve the following scaled transformations:

$$\left.\begin{array}{l} \eta=\dfrac{1}{\sqrt{2}}x^{-\frac{1}{2}}y, \psi=\sqrt{2}x^{\frac{1}{2}}f(\eta), \theta=\theta(\eta), \phi=\phi(\eta), \tilde{N}=\dfrac{1}{\sqrt{2}}x^{-\frac{1}{2}}h(\eta), \beta_1=\dfrac{(\beta_1)_0}{x}, \beta_2=\dfrac{(\beta_2)_0}{x}, \\ B=x^{-\frac{1}{2}}B_0, D_1=x^{\frac{1}{2}}(D_1)_0, j=x j_0, \gamma=x\gamma_0, N_1=x^{-\frac{1}{2}}(N_1)_0, Q_1=\dfrac{(Q_1)_0}{x}, k_r=\dfrac{(k_r)_0}{x} \end{array}\right\} \quad (9)$$

Consequently, we obtain the following self-similar equations as

$$(1+K)f'''+ff''+Kh'-Mf'+Gr\theta+Gm\phi=0 \quad (10)$$

$$\lambda h''+h'f+hf'-I(2h+f'')=0 \quad (11)$$

$$\theta''+\Pr f\theta'+\Pr Q\theta=0 \quad (12)$$

$$\phi''+Scf\phi'-Sck_r\phi=0 \quad (13)$$

The boundary conditions then turn into

$$\left.\begin{array}{l} f'=1+\xi f'',\ f=0,\ h=-nf'',\ \theta=1+\varsigma\theta',\ \phi=1 \text{ at } \eta=0 \\ f'\to 0,\ h\to 0,\ \theta\to 0,\ \phi\to 0 \text{ as } \eta\to\infty \end{array}\right\} \quad (14)$$

Here $K=k/\mu$ is the micropolar parameter, $Gr=2Lg\beta_1(T_w-T_\infty)/U_w^2$ is the Grashof number, $Gm=2Lg\beta_2(C_w-C_\infty)/U_w^2$ is the modified Grashof number, $M=2\sigma LB_0^2/\rho U_w$ is the magnetic field parameter, $\lambda=\gamma_0/\rho\nu j_0$ is the microrotational density parameter, $I=2Lk/\rho j_0U_w$ is the vortex viscosity parameter, $\Pr = \nu/\alpha$ is the Prandtl number, $\xi = \sqrt{\mathrm{Re}/2}\left(N_1\right)_0/L$ is the velocity slip parameter, $\varsigma = \sqrt{\mathrm{Re}/2}\left(D_1\right)_0/L$ is the thermal slip parameter, $Q = LQ_1/\mathrm{U}_w$ Is the heat generation or absorption parameter, $Sc = \nu/D$ is the Schmidt number, $K_r = k_r L/\mathrm{U}_w$ is the rate of chemical reaction parameter.

The reduced skin friction coefficient, reduced Nusselt number and the reduced Sherwood number are defined respectively as

$$C_{fr} = \left[1 + K(1-n)\right] f''(0) \text{ where } C_{fr} = C_f \sqrt{2\,\mathrm{Re}_{\bar{x}}} \tag{15}$$

$$Nur = -\theta'(0) \text{ where } \mathrm{Nur} = \frac{Nu}{\sqrt{\mathrm{Re}_{\bar{x}}/2}} \tag{16}$$

$$Shr = -\phi'(0) \text{ where } \mathrm{Shr} = \frac{Sh}{\sqrt{\mathrm{Re}_{\bar{x}}/2}} \tag{17}$$

where $\mathrm{Re}_{\bar{x}} = \dfrac{U_w \bar{x}}{\nu}$ is the local Reynolds number.

Table 1

Values of $-f''(0)$ for several values of K

K	$-f''(0)$	
	Ishak et al. [6]	Present results
0.0	0.6276	0.627516
0.5	0.5604	0.560433
1.0	0.5117	0.511688
2.0	0.4423	0.442331
4.0	0.3694	0.369393

3. Method of Solution

Since the transformed equations are highly non-linear, a numerical treatment would be more appropriate. Therefore, the above system of equations are converted to a system of first order differential equations as follows:

$$\left.\begin{aligned} & f' = f_1,\ f_1' = f_2,\ \mathrm{h}' = f_3,\ \theta' = f_4,\ \phi' = f_5, \\ & f_2' = -\left[ff_2 + Kf_3 + G_r\theta + G_m\phi - Mf_1\right]/(1+K), \\ & f_3' = -\left[ff_3 + hf_1 - I(2h + f_2)\right]/\lambda, \\ & f_4' = -\Pr\, ff_4 - Q\theta,\ f_5' = -Scff_5 + Kr\phi \end{aligned}\right\} \tag{18}$$

with the boundary conditions

$$\left.\begin{aligned} & f_1(0) = 1 + \xi f_2(0), f(0) = 0, \mathrm{h}(0) = -\mathrm{n}\ f_2(0), \\ & \theta(0) = 1 + \varsigma f_4(0), \phi(0) = 1 \end{aligned}\right\} \tag{19}$$

The above system of equations are solved numerically by employing a fourth-order Runge–Kutta method and shooting techniques. To check the validity of the present code, the values of $f''(0)$ are compared for different values of K with those of Ishak et al. [6] in Table 1. The values are found to be in excellent agreement, which justifies the use of the present numerical code for current model.

4. Results and Discussions

In order to analyze the effects of slip velocity parameters ξ, thermal slip parameter ζ and chemical reaction parameter Kr on the flow characteristics, the numerical results are computed and are displayed graphically in Figs. 2-9. Figure 2 depict the influence of slip velocity parameter ξ on the velocity distribution near the boundary layer region. It is seen that velocity increases with increasing the values of slip parameter ξ for η<1.7 (not precisely determined) but the effect is not significant for η>1.7 (not precisely determined). The reason behind this phenomenon is that the slip at the surface wall of the plate increases as slip parameter ξ increases. Figure 3 presents the variation of velocity distribution for various values of thermal slip parameter ζ in presence of slip velocity. One may note that the effect of ζ is prominent within the boundary layer region and the momentum boundary layer thickness decreases with the increase of ζ for both conducting and non-conducting fluid. It is observed from the table that the skin friction coefficient (absolute value) decreases with the surface slip parameter.

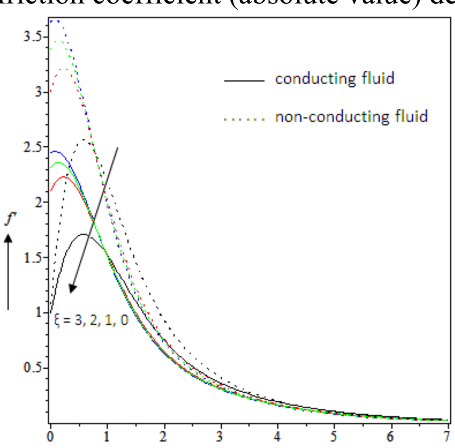

Fig. 2. Velocity profiles for various values of ξ

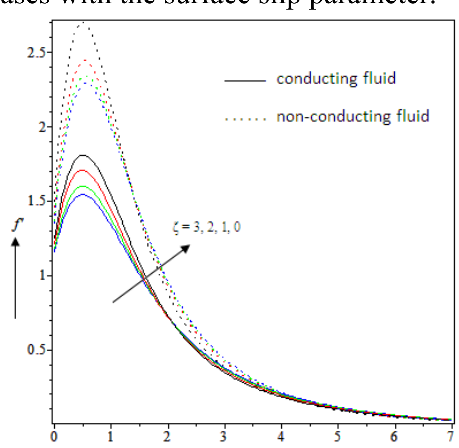

Fig. 3. Velocity profiles for various values of ζ

One may observe from the figure 4 that the angular velocity decreases drastically as slip velocity parameters ξ increases for both conducting and non-conducting fluid in the region near to the plate. Figure 5 illustrates the effect of thermal slip parameter ζ on angular velocity in presence of surface slip. It is revealed from the figure that the angular velocity decreases with increasing the values of ζ for both conducting and non-conducting fluid but the effect is prominent for conducting fluid.

From the figure 6 one may note that the effect of slip velocity parameter ξ on temperature distribution is not prominent for both conducting and non-conducting micropolar fluid. The effect of the thermal slip parameter ζ on the temperature distribution of micropolar fluid is shown in Fig. 7. It is seen that the thermal slip parameter decreases the fluid temperature near the surface of the plate for both type fluids. Thus, the thermal boundary layer thickness decreases with increasing the values of ζ and tends asymptotically to zero as the distance increase from the boundary. Finally, an important observation may be made about the temperature profiles as presented in Figures 6, 7 is that the fluid temperature is higher in case of conducting micropolar fluid than that in the case of non-conducting micropolar fluid. This phenomenon indicates that conducting micropolar fluid will be more effective in the cooling and heating processes. It is understood from tables 2 that the rate of heat transfers in absolute sense increase with increase of ξ and ζ.

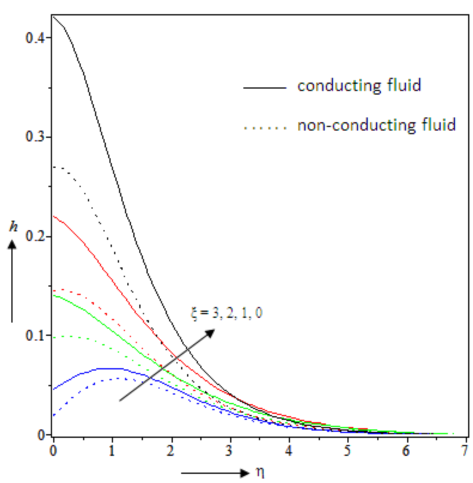

Fig. 4. Angular velocity profiles for various values of ξ

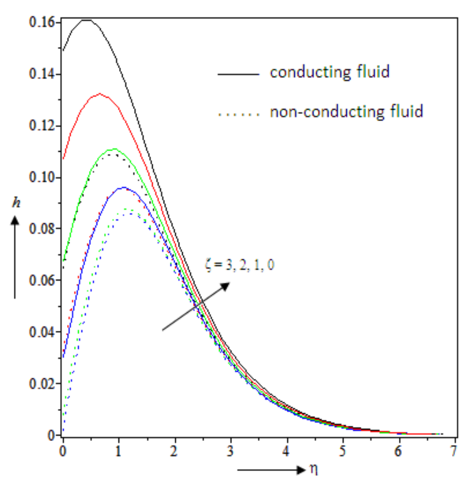

Fig. 5. Angular velocity profiles for various values of ζ

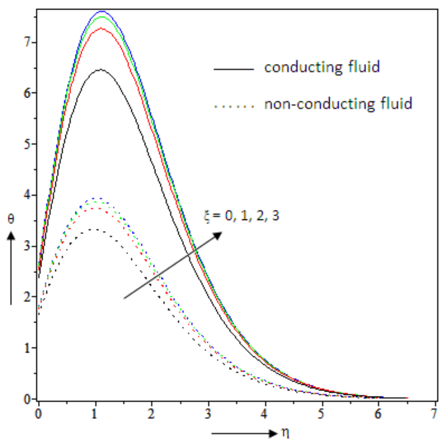

Fig. 6. Temperature profiles for various values of ξ

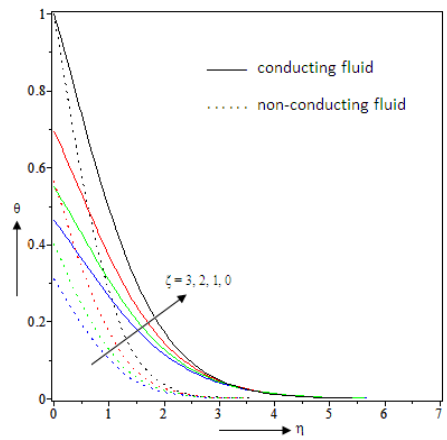

Fig. 7. Temperature profiles for various values of ζ

Fig. 8 depicts the concentration distribution for various values of chemical reaction parameter Kr (>0) for both conducting and non-conducting fluid. It is observed from the figure that the effect of increasing values of chemical reaction parameter Kr (>0) is to decrease the concentration of the conducting as well as non-conducting micropolar fluid. Figure 9 presents the variation of concentration distribution across the boundary layer for different values of chemical reaction parameter Kr (<0). It is clear from the figure that the concentration increases tremendously in presence of generative chemical reaction near the boundary surface of the plate. It is seen from the table 2 that the rate of mass transfer at the plate increases with increasing the values of chemical reaction parameter Kr.

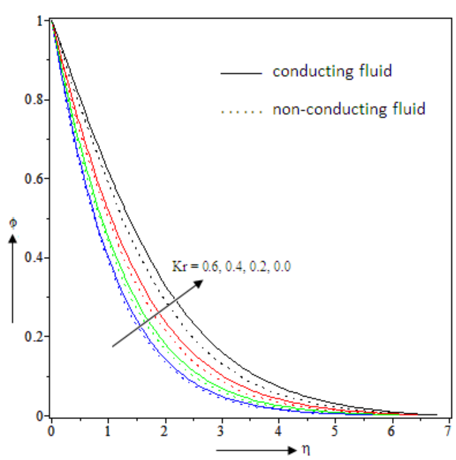

Fig. 8. Concentration profiles for various values of Kr (>0)

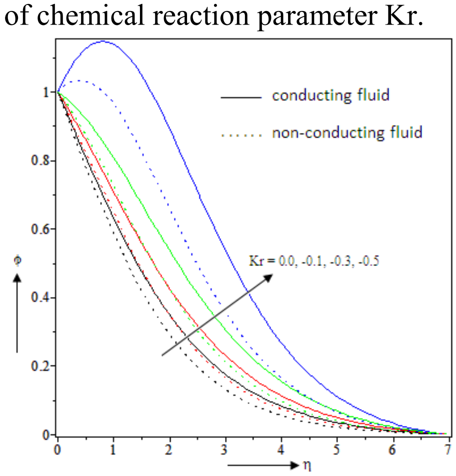

Fig. 9. Concentration profiles for various values of Kr (<0)

Table 2

Values of C_{fr}, Nu_r and Sh_r for various values of ξ, ζ and Kr

ξ	ζ	Kr	C_{fr}	Nu_r	Sh_r
0.0	1.0	0.1	-0.6304	0.1588	0.7322
1.0	-		-0.1707	-0.2658	0.7322
2.0	-		-0.0342	-0.5820	0.7322
1.0	0.0		-0.3526	-0.1873	0.7322
-	1.0		-0.3526	-0.2305	0.7322
-	2.0		-0.3526	-0.2995	0.7322
		-0.1	-0.5773	0.0946	0.1488
		0.0	-0.5989	0.0575	0.3268
		0.2	-0.6208	0.0144	0.5547

5. Conclusions

In this theoretical study, we have presented the combined effects of chemical reaction and internal heat source/sink on boundary layer flow of a MHD micropolar fluid past a moving flat plate with slip boundary conditions. The governing partial differential equations are solved numerically with the help of combined Lie group analysis. The main results of the present analysis are listed below:

1. The fluid velocity near the boundary layer region increases with increasing the values of surface slip parameter but effect is reverse for thermal slip parameter.
2. The increasing value of thermal slip parameter is to increase the absolute value of wall temperature gradient and so more heat is carried out of the sheet, resulting in a decrease of the thermal boundary layer thickness.
3. An increase in the chemical reaction parameter leads to decrease in the concentration boundary layer thickness for destructive chemical reaction but reverse effect occurs for constructive reaction.

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