U.P.B. Sci. Bull., Series D, Vol. 71, Iss. 1, 2009 ISSN 1454-2358

KINEMATICS OF A 3-*PRP* PLANAR PARALLEL ROBOT

Damien CHABLAT[1], Stefan STAICU[2]

Articolul prezintă o modelare recurentă pentru cinematica unui robot paralel plan 3-PRP. Trei lanţuri cinematice plane ce conectează platforma mobilă a manipulatorului sunt situate în plan vertical. Cunoscând mişcarea platformei, se dezvoltă cinematica inversă şi se determină poziţiile, vitezele şi acceleraţiile robotului. Unele ecuaţii matriceale oferă expresii iterative şi grafice pentru deplasările, vitezele şi acceleraţiile celor trei acţionori de translaţie.

Recursive modelling for the kinematics of a 3-PRP planar parallel robot is presented in this paper. Three planar chains connecting the moving platform of the manipulator are located in a vertical plane. Knowing the motion of the platform, we develop the inverse kinematics and determine the positions, velocities and accelerations of the robot. Several matrix equations offer iterative expressions and graphs for the displacements, velocities and accelerations of the three prismatic actuators.

Keywords: kinematics, planar parallel robot, singularity

1. Introduction

Compared with serial manipulators, the potential advantages of the parallel architectures are a higher kinematical precision, lighter weight, better stiffness, greater loading capability, stable capacity and suitable positional actuator arrangements. However, they present limited workspace and complicated singularities [1].

Considerable efforts have been devoted to the kinematics and dynamic analysis of fully parallel manipulators. The class of manipulators known as Stewart-Gough platform focused particular attention (Stewart [2]; Merlet [3]; Parenti-Castelli and Di Gregorio [4]) and is used in flight simulators and, more recently, for Parallel Kinematics Machines. The Delta parallel robot (Clavel [5]; Tsai and Stamper [6]; Staicu and Carp-Ciocardia [7]) as well as the Star parallel manipulator (Hervé and Sparacino [8]) are equipped with three motors, which train on the mobile platform in a three-degree-of-freedom general translation

[1] Dr., Institut de Recherche en Communications et Cybernétique de Nantes, UMR CNRS 6597, France
[2] Prof., Department of Mechanics, University POLITEHNICA of Bucharest, Romania, e-mail: stefanstaicu@yahoo.com

motion. Angeles, Gosselin, Gagné and Wang [9, 10, 11] analysed the kinematics, dynamics and singularity loci of a spherical robot with three actuators.

A mechanism is considered as a *planar robot* if all the moving links in the mechanism perform planar motions; the loci of all points in all links can be drawn conveniently on a plane and the axes of the revolute joints remains normal to the plane of motion, while the direction of translation of a prismatic joint preserves parallel to the plane of motion.

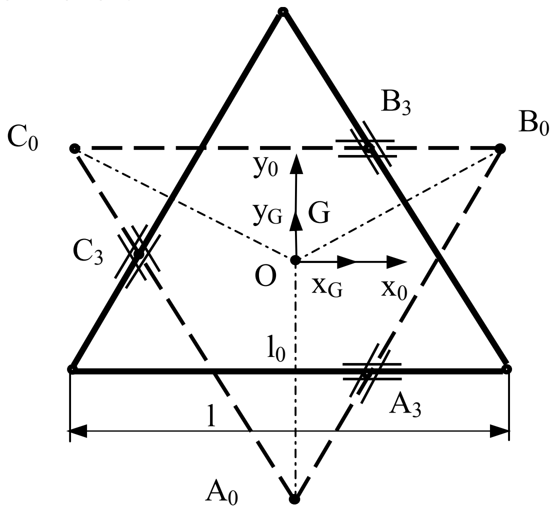

Fig. 1 The 3-*PRP* planar parallel robot

Aradyfio and Qiao [12] examine in their paper the inverse kinematics solution for the three different 3-DOF planar parallel robots. Gosselin and Angeles [13] and Pennock and Kassner [14] each present a kinematical study of a planar parallel robot, where a moving platform is connected to a fixed base by three links or legs, each leg consisting of two binary links and three parallel revolute joints. Sefrioui and Gosselin [15] give a numerical solution in the inverse and direct kinematics of this kind of robot. Mohammadi-Daniali et al. [16] present a study of velocity relationships and singular conditions for general planar parallel robots.

Merlet [17] solved the forward posed kinematics problem for a broad class of planar parallel manipulators. Williams and Reinholtz [18] analysed the dynamics and the control of a planar three-degree-of-freedom parallel manipulator at Ohio University, while Yang et al. [19] concentrate on the singularity analysis of a class of 3-*RRR* planar parallel robots developed in their laboratory. Bonev, Zlatanov and Gosselin [20] describe several types of singular configurations by studying the direct kinematics model of a 3-*RPR* planar parallel robot with actuated base joints. Mohammadi-Daniali et al. [21] analysed the kinematics of a planar 3-DOF parallel manipulator using the three *PRP* legs, where the three revolute joint axes are perpendicular to the plane of motion, while the prismatic joint axes lie in the same plane.

A recursive method is developed in the present paper for deriving the inverse kinematics of the 3-*PRP* planar parallel robot in a numerically efficient way.

2. Kinematics modelling

The planar 3-*PRP* parallel robot is a special symmetrical closed-loop mechanism composed of three planar kinematical chains with identical topology, all connecting the fixed base to the moving platform. Three points A_0, B_0, C_0 represent the summits of a fixed triangular base and other three points define the geometry of the moving platform. Each leg consists of two links, with one revolute and two prismatic joints. The parallel mechanism with seven links $(T_k, k = 1,2,...,7)$ consists of three revolute and six prismatic joints (Fig.1). Grübler mobility equation predicts that the device has certainly three degrees of freedom.

In the actuation scheme *PRP* each prismatic joint is an actively controlled prismatic cylinder. Thus, all prismatic actuators can be located on the fixed base. We attach a Cartesian frame $x_0 y_0 z_0 (T_0)$ to the fixed base with its origin located at triangle centre O, the z_0 axis perpendicular to the base and the x_0 axis pointing along the $C_0 B_0$ direction. Another, mobile reference frame $x_G y_G z_G$ is attached to the moving platform. The origin of the central reference frame $x_G y_G z_G$ is located at the centre G of the moving triangle (Fig. 2).

In the following we shall represent the intermediate reference systems by two axes only, like in many robotics papers [1, 3, 9]. We note that the relative translation $\lambda_{k,k-1}$ and the rotation angle $\varphi_{k,k-1}$ point along or about the direction of z_k axis.

We consider that the moving platform is initially located at a *central configuration*, where the platform is not rotated with respect to the fixed base and the mass centre G coincides with the origin O of the fixed frame. One of the three active legs (for example leg A) consists of a prismatic joint, which is a linear drive **1** as well, linked at the $x_1^A y_1^A z_1^A$ frame, having a rectilinear motion with displacement λ_{10}^A, velocity $v_{10}^A = \dot{\lambda}_{10}^A$ and acceleration $\gamma_{10}^A = \ddot{\lambda}_{10}^A$. Next to the link of the leg a rigid body **2** is bound to the $x_2^A y_2^A z_2^A$ frame, having a relative rotation about the z_2^A axis with the angle φ_{21}^A, velocity $\omega_{21}^A = \dot{\varphi}_{21}^A$ and acceleration $\varepsilon_{21}^A = \ddot{\varphi}_{21}^A$. A prismatic passive joint is introduced at the planar moving platform as an equilateral triangle with the edge $l = l_0\sqrt{3}$, which translates relatively with the displacement λ_{32}^A and the velocity $v_{32}^A = \dot{\lambda}_{32}^A$ along the z_3^A axis.

Also, we consider that at the central configuration all legs are symmetrically extended and that the angles of orientation of the three edges of the fixed platform are given by

$$\alpha_A = \frac{\pi}{3}, \alpha_B = \pi, \alpha_C = -\frac{\pi}{3}. \tag{1}$$

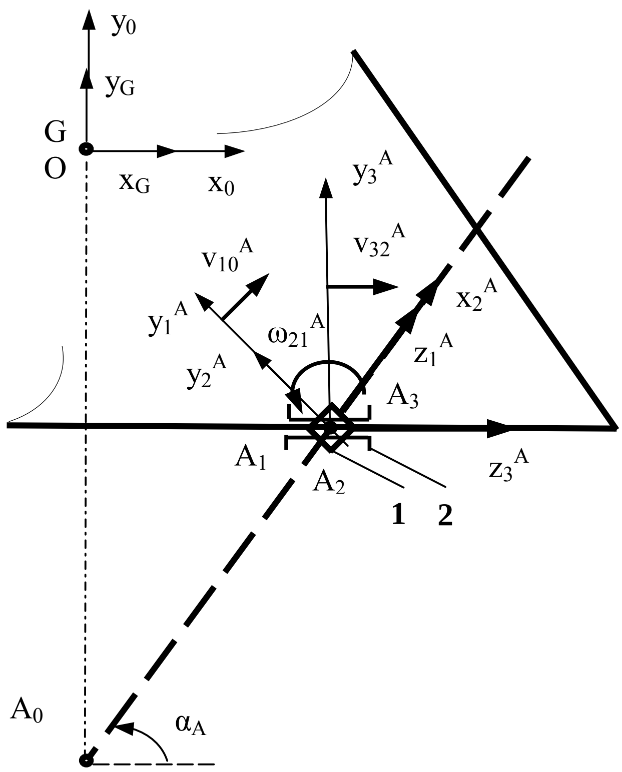

Fig. 2 Kinematical scheme of first leg A of the mechanism

In the following, we apply the method of successive displacements during the geometric analysis of the closed-loop chains and we note that a joint variable is the displacement required to move a link from the initial location to the actual position. If every link is connected to at least two other links, the chain forms one or more independent closed-loops.

The variable angle $\varphi_{k,k-1}^i$ of rotation about the joint axis z_k^i is the parameter needed to bring the next link from a reference configuration to the next configuration. We call the matrix $q_{k,k-1}^{\varphi}$, for example, the 3×3 orthogonal transformation matrix of the relative rotation with the angle $\varphi^i_{k,k-1}$ of the link T^i_k around z^i_k.

In the study of the kinematics of robotic manipulators, we are interested in deriving a matrix equation relating the location of an arbitrary body T^i_k to the joint variables. When the change of coordinates is successively considered, the corresponding matrices are multiplied. So, starting from the reference origin O and pursuing the three legs $OA_0A_1A_2A_3$, $OB_0B_1B_2B_3$, $OC_0C_1C_2C_3$, we obtain the following transformation matrices [22]:

$$q_{10} = \theta_1\theta^i_\alpha,\; q_{21} = q^\varphi_{21}\theta_1^T,\; q_{32} = \theta_1\theta_2 \quad \text{with } (q = a,\ b,\ c),\ (i = A,\ B,\ C) \tag{2}$$

where
$$q^\varphi_{k,k-1} = \begin{bmatrix} \cos\varphi^i_{k,k-1} & \sin\varphi^i_{k,k-1} & 0 \\ -\sin\varphi^i_{k,k-1} & \cos\varphi^i_{k,k-1} & 0 \\ 0 & 0 & 1 \end{bmatrix},\; \theta^i_\alpha = \begin{bmatrix} \cos\alpha_i & \sin\alpha_i & 0 \\ -\sin\alpha_i & \cos\alpha_i & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\theta_1 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix},\; \theta_2 = \frac{1}{2}\begin{bmatrix} 1 & -\sqrt{3} & 0 \\ \sqrt{3} & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \tag{3}$$

$$q_{k0} = \prod_{s=1}^{k} q_{k-s+1,k-s} \quad (k = 1, 2, 3).$$

The displacements λ^A_{10}, λ^B_{10}, λ^C_{10} of the active links are the joint variables that give the input vector $\vec{\lambda}_{10} = [\lambda^A_{10} \quad \lambda^B_{10} \quad \lambda^C_{10}]^T$ for the position of the mechanism. In the inverse geometric problem however, we can consider that the position of the mechanism is completely given by the coordinates x^G_0, y^G_0 of the mass centre G of the moving platform and the orientation angle ϕ of the movable frame $x_Gy_Gz_G$. The orthogonal rotation matrix of the moving platform from $x_0y_0z_0$ to $x_Gy_Gz_G$ reference system is

$$R = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{4}$$

Further, we suppose that the position vector $\vec{r}_0^{\,G} = [x_0^G \;\; y_0^G \;\; 0]^T$ of the centre G and the orientation angle Φ, which are expressed by following analytical functions

$$\frac{x_0^G}{x_0^{G*}} = \frac{y_0^G}{y_0^{G*}} = \frac{\phi}{\phi^*} = 1 - \cos\frac{\pi}{3}t\,, \tag{5}$$

can describe the general absolute motion of the moving platform in its *vertical plane*. The values $2x_0^{G*}, 2y_0^{G*}, 2\phi^*$ denote the final position of the moving platform.

The conditions concerning the absolute orientation of the moving platform are expressed by three identities

$$\overset{\circ}{q}{}_{30}^{T} q_{30} = R, \quad (q = a, b, c)\,, \tag{6}$$

where the resulting matrix q_{30} is obtained by multiplying the three basic matrices

$$q_{30} = q_{32} q_{21} q_{10}\,,\; \overset{\circ}{q}_{30} = q_{30}(t=0) = \theta_1 \theta_2 \theta_\alpha^i \quad (i = A, B, C)\,. \tag{7}$$

From these conditions one obtains the first relations between the angles of rotation

$$\varphi_{21}^A = \varphi_{21}^B = \varphi_{21}^C = \phi\,. \tag{8}$$

Six independent variables $\lambda_{10}^A, \lambda_{32}^A, \lambda_{10}^B, \lambda_{32}^B, \lambda_{10}^C, \lambda_{32}^C$ will be determined by several vector-loop equations as follows

$$\vec{r}_{10}^{\,i} + \sum_{k=1}^{2} q_{k0}^T \, \vec{r}_{k+1,k}^{\,i} + q_{30}^T \, \vec{r}_3^{\,Gi} = \vec{r}_0^{\,G}\,, \;\; (q = a, b, c) \;\; (i = A, B, C) \tag{9}$$

where

$$\vec{r}_{10}^{\,i} = \vec{r}_{00}^{\,i} + (l_0/\sqrt{3} + \lambda_{10}^i) q_{10}^T \vec{u}_3\,,\; \vec{r}_{00}^{\,A} = l_0 [0 \quad -1 \quad 0]^T$$

$$\vec{r}_{00}^{\,B} = \frac{1}{2} l_0 [\sqrt{3} \;\; 1 \;\; 0]^T,\; \vec{r}_{00}^{\,C} = \frac{1}{2} l_0 [-\sqrt{3} \;\; 1 \;\; 0]^T$$

$$\vec{r}_{21}^{\,i} = \vec{0},\; \vec{r}_{32}^{\,i} = \lambda_{32}^i q_{32}^T \vec{u}_3\,,\; \vec{r}_3^{\,Gi} = \frac{1}{2} l_0 [0 \;\; 1 \;\; -\frac{\sqrt{3}}{3}]^T$$

$$\vec{u}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \vec{u}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \vec{u}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \tilde{u}_3 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{10}$$

Actually, these vector equations mean that

$$\left(\frac{l_0}{\sqrt{3}}+\lambda_{10}^i\right)\cos\alpha_i+\lambda_{32}^i\cos\left(\phi-\frac{\pi}{3}+\alpha_i\right)=x_0^G-x_{00}^i-\frac{l_0}{2\sqrt{3}}\cos(\phi+\alpha_i)+\frac{l_0}{2}\sin(\phi+\alpha_i)$$

$$\left(\frac{l_0}{\sqrt{3}}+\lambda_{10}^i\right)\sin\alpha_i+\lambda_{32}^i\sin\left(\phi-\frac{\pi}{3}+\alpha_i\right)=y_0^G-y_{00}^i-\frac{l_0}{2\sqrt{3}}\sin(\phi+\alpha_i)-\frac{l_0}{2}\cos(\phi+\alpha_i)\quad(11)$$

with $(i=A,\ B,\ C)$.

Developing the inverse kinematics problem, we determine the velocities and accelerations of the manipulator, supposing that the planar motion of the moving platform is known. So, we compute the linear and angular velocities of each leg in terms of the angular velocity $\vec{\omega}_0^G=\dot{\phi}\vec{u}_3$ and the centre's velocity $\vec{v}_0^G=\dot{\vec{r}}_0^G$ of the moving platform.

The rotational motion of the elements of each leg (leg *A*, for example) are characterized by recursive relations using the following skew-symmetric matrices

$$\tilde{\omega}_{k0}^A=a_{k,k-1}\tilde{\omega}_{k-1,0}^A a_{k,k-1}^T+\omega_{k,k-1}^A\tilde{u}_3,\ \omega_{k,k-1}^A=\dot{\varphi}_{k,k-1}^A\quad(k=1,2,3)\,,\qquad(12)$$

which are *associated* to the absolute angular velocities

$$\vec{\omega}_{10}^A=\vec{0},\vec{\omega}_{20}^A=a_{21}\vec{\omega}_{10}^A+\vec{\omega}_{21}^A=\dot{\phi}\vec{u}_3,\ \vec{\omega}_{30}^A=a_{32}\vec{\omega}_{20}^A+\vec{\omega}_{32}^A=\dot{\phi}\vec{u}_3.\qquad(13)$$

The following relations give the velocities $\vec{v}_{k0}^A$ of joints A_k

$$\vec{v}_{10}^A=\dot{\lambda}_{10}^A\vec{u}_3,\ \vec{v}_{21}^A=\vec{0},\ \vec{v}_{32}^A=\dot{\lambda}_{32}^A\vec{u}_3$$

$$\vec{v}_{k0}^A=a_{k,k-1}\vec{v}_{k-1,0}^A+a_{k,k-1}\tilde{\omega}_{k-1,0}^A\vec{r}_{k,k-1}^A+v_{k,k-1}^A\vec{u}_3.\qquad(14)$$

The geometrical equations of constraints (8) and (9) when differentiated with respect to the time lead to the following *matrix conditions of connectivity* [23]

$$v_{10}^A\vec{u}_j^T a_{10}^T\vec{u}_3+v_{32}^A\vec{u}_j^T a_{30}^T\vec{u}_3=\vec{u}_j^T\dot{\vec{r}}_0^G-\omega_{21}^A\vec{u}_j^T\{\lambda_{32}^A a_{20}^T\tilde{u}_3 a_{32}^T\vec{u}_3+a_{20}^T\tilde{u}_3 a_{32}^T\vec{r}_3^{GA}\}\ \ (j=1,2)$$

$$\omega_{21}^A=\dot{\phi}\,,\qquad(15)$$

where $\tilde{u}_3$ is a skew-symmetric matrix associated to the unit vector $\vec{u}_3$, pointing in the positive direction of the z_k axis. From these equations, we obtain the relative velocities $v_{10}^A,\omega_{21}^A,v_{32}^A$ as functions of the angular velocity of the platform and the velocity of the mass centre G and the *complete* Jacobian matrix of the manipulator. This matrix is a fundamental element for the analysis of the robot workspace and the particular configuration of singularities, where the manipulator becomes uncontrollable.

By rearranging, the derivatives with respect to time for the six constraint equations (11) lead to the matrix equation

$$J_1 \dot{\lambda}_{10} = J_2 [\dot{x}_0^G \quad \dot{y}_0^G \quad \dot{\phi}]^T \tag{16}$$

for the planar robot with prismatic actuators.

The matrices J_1 and J_2 are the inverse and the forward Jacobian of the manipulator and can be expressed as

$$J_1 = diag\{\delta_A \quad \delta_B \quad \delta_C\}$$

$$J_2 = \begin{bmatrix} \beta_1^A & \beta_2^A & \beta_3^A \\ \beta_1^B & \beta_2^B & \beta_3^B \\ \beta_1^C & \beta_2^C & \beta_3^C \end{bmatrix}, \tag{17}$$

with

$$\delta_i = \sin(\phi - \frac{\pi}{3}) \quad (i = A, B, C)$$

$$\beta_1^i = \sin(\phi - \frac{\pi}{3} + \alpha_i), \beta_2^i = -\cos(\phi - \frac{\pi}{3} + \alpha_i) \tag{18}$$

$$\beta_3^i = (x_0^G - x_{00}^i)\cos(\phi - \frac{\pi}{3} + \alpha_i) + \\ + (y_0^G - y_{00}^i)\sin(\phi - \frac{\pi}{3} + \alpha_i) - \\ - (\frac{l_0}{\sqrt{3}} + \lambda_{10}^i)\cos(\phi - \frac{\pi}{3}).$$

The singular configurations of the three closed-loop kinematical chains can easily be determined through the analysis of two Jacobian matrices J_1 and J_2 [24, 25]. For the matrix J_1, the determinant vanishes in $\phi = \pi/3$, which leads also to a singular configuration of J_2.

Concerning the relative accelerations $\gamma_{10}^A, \varepsilon_{21}^A, \gamma_{32}^A$ of the robot, new connectivity conditions are obtained by the time derivative of equations in (15), which are [26]

$$\gamma_{10}^A \vec{u}_j^T a_{10}^T \vec{u}_3 + \gamma_{32}^A \vec{u}_j^T a_{30}^T \vec{u}_3 = \vec{u}_j^T \ddot{\vec{r}}_0^G -$$

$$-\omega_{21}^{A}\omega_{21}^{A}\vec{u}_j^{\,T}\,\{a_{20}^{T}\tilde{u}_3\tilde{u}_3a_{32}^{T}\vec{u}_3+a_{20}^{T}\vec{u}_3\tilde{u}_3a_{32}^{T}\vec{r}_3^{\,GA}\}-$$

$$-\varepsilon_{21}^{A}\vec{u}_j^{\,T}\,\{\lambda_{32}^{A}a_{20}^{T}\tilde{u}_3a_{32}^{T}\vec{u}_3+a_{20}^{T}\tilde{u}_3a_{32}^{T}\vec{r}_3^{\,GA}\}-2\omega_{21}^{A}v_{32}^{A}\vec{u}_j^{\,T}a_{20}^{T}\tilde{u}_3a_{32}^{T}\vec{u}_3\quad(j=1,2)$$

$$\varepsilon_{21}^{A}=\dot{\phi}\,. \tag{19}$$

The formulations in (15) and (19) are for A only and they also apply to the legs B and C when the superscript A is replaced by either B or C.

The following recursive relations give the angular accelerations $\vec{\varepsilon}_{k0}^{\,A}$ and the accelerations $\vec{\gamma}_{k0}^{\,A}$ of joints A_k:

$$\vec{\gamma}_{10}^{\,A}=\ddot{\lambda}_{10}^{A}\vec{u}_3,\ \vec{\gamma}_{21}^{\,A}=\vec{0},\ \vec{\gamma}_{32}^{\,A}=\ddot{\lambda}_{32}^{A}\vec{u}_3$$

$$\vec{\varepsilon}_{10}^{\,A}=\vec{0},\ \vec{\varepsilon}_{21}^{\,A}=\ddot{\phi}\,\vec{u}_3,\ \vec{\varepsilon}_{32}^{\,A}=\vec{0}$$

$$\vec{\varepsilon}_{k0}^{\,A}=a_{k,k-1}\vec{\varepsilon}_{k-1,0}^{\,A}+\varepsilon_{k,k-1}^{A}\vec{u}_3+\omega_{k,k-1}^{A}a_{k,k-1}\tilde{\omega}_{k-1,0}^{A}a_{k,k-1}^{T}\vec{u}_3$$

$$\begin{aligned}\tilde{\omega}_{k0}^{A}\tilde{\omega}_{k0}^{A}+\tilde{\varepsilon}_{k0}^{A}&=a_{k,k-1}\left(\tilde{\omega}_{k-1,0}^{A}\tilde{\omega}_{k-1,0}^{A}+\tilde{\varepsilon}_{k-1,0}^{A}\right)a_{k,k-1}^{T}+\omega_{k,k-1}^{A}\omega_{k,k-1}^{A}\tilde{u}_3\tilde{u}_3+\varepsilon_{k,k-1}^{A}\tilde{u}_3+\\&+2\omega_{k,k-1}^{A}a_{k,k-1}\tilde{\omega}_{k-1,0}^{A}a_{k,k-1}^{T}\tilde{u}_3\end{aligned} \tag{20}$$

$$\begin{aligned}\vec{\gamma}_{k0}^{\,A}&=a_{k,k-1}\vec{\gamma}_{k-1,0}^{\,A}+a_{k,k-1}(\tilde{\omega}_{k-1,0}^{A}\tilde{\omega}_{k-1,0}^{A}+\tilde{\varepsilon}_{k-1,0}^{A})\vec{r}_{k,k-1}^{\,A}+\\&+2v_{k,k-1}^{A}a_{k,k-1}\tilde{\omega}_{k-1,0}^{A}a_{k,k-1}^{T}\vec{u}_3+\vec{\gamma}_{k,k-1}^{\,A}\vec{u}_3,\ (k=1,2,3)\end{aligned}$$

We can notice that, for this robot, the displacement of the leg is very simple. The displacement of body 2 is along a fixed line only, and its velocity and acceleration is equal to those of the associated, actuated joint.

For simulation purposes let us consider a planar robot, which has the following characteristics:

$$x_0^{G*}=0.025\,m\,,\ y_0^{G*}=0.025\,m\,,\ \phi^{*}=\frac{\pi}{12},\ \Delta t=3\ s$$

$$l_0=OA_0=OB_0=OC_0=0.3m,\ l=l_0\sqrt{3}$$

A program, which implements the suggested algorithm, is developed in MATLAB to solve the inverse kinematics of the planar *PRP* parallel robot. For illustration, it is assumed that for a period of three seconds the platform starts at rest from a central configuration and rotates or moves along two orthogonal directions. A numerical study of the robot kinematics is carried out by computation of the displacements $\lambda_{10}^{A},\ \lambda_{10}^{B},\ \lambda_{10}^{C}$, the velocities $v_{10}^{A},\ v_{10}^{B},\ v_{10}^{C}$, and the accelerations $\gamma_{10}^{A},\ \gamma_{10}^{B},\ \gamma_{10}^{C}$ of three prismatic actuators.

The following examples are given to illustrate the simulation. As a first example, we consider the *rotation motion* of the moving platform about the axis z_0 with a variable angular acceleration, while all the other positional parameters are held equal to zero.

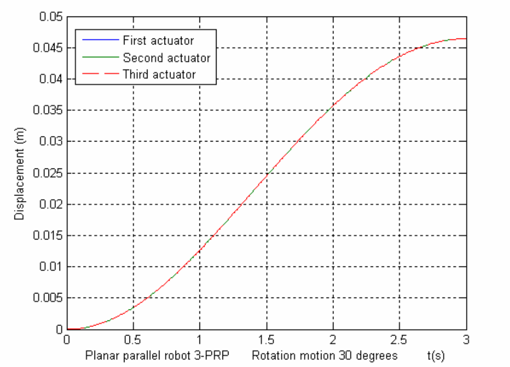

Fig. 3 Displacements λ_{10}^A, λ_{10}^B, λ_{10}^C

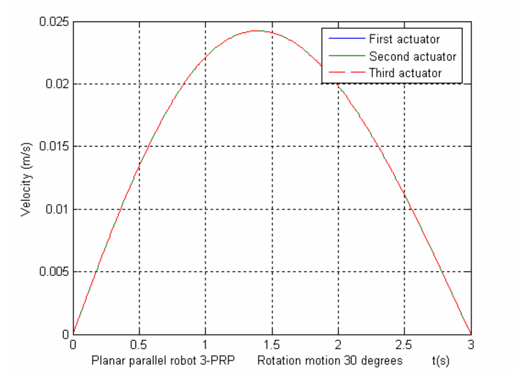

Fig. 4 Velocities v_{10}^A, v_{10}^B, v_{10}^C

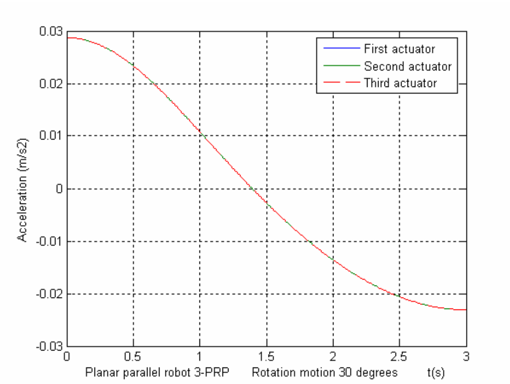

Fig. 5 Accelerations γ_{10}^A, γ_{10}^B, γ_{10}^C

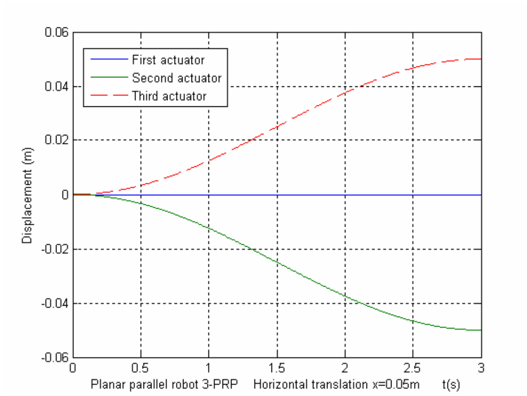

Fig. 6 Displacements λ_{10}^A, λ_{10}^B, λ_{10}^C

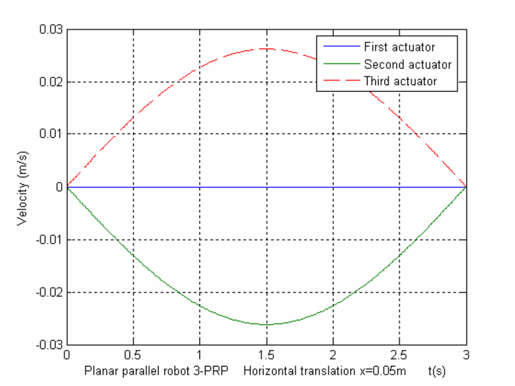

Fig. 7 Velocities v_{10}^A, v_{10}^B, v_{10}^C

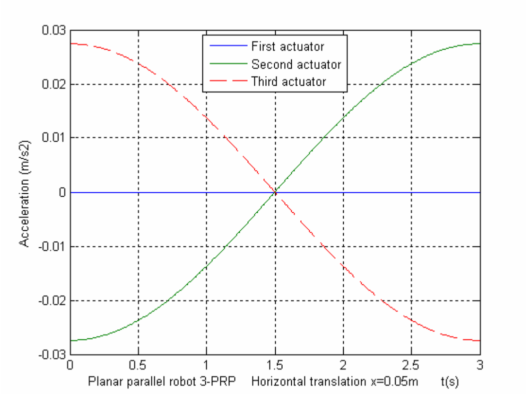

Fig. 8 Accelerations γ_{10}^A, γ_{10}^B, γ_{10}^C

As can be seen from Figs. 3, 4, 5, during the rotational motion of the platform all displacements, velocities and accelerations of all three actuators are identically distributed.

In a second example, the presumed motion of the platform is a *translation* along the horizontal axis x_0 (Figs. 6, 7, 8).

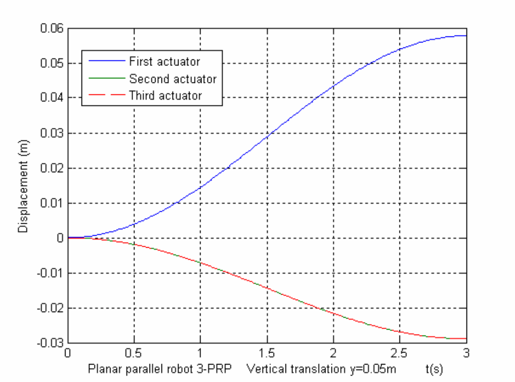

Fig. 9 Displacements λ_{10}^A, λ_{10}^B, λ_{10}^C

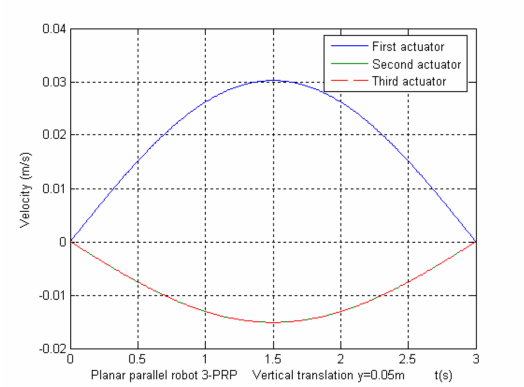

Fig. 10 Velocities v_{10}^A, v_{10}^B, v_{10}^C

Concerning the comparison in the case when the centre G moves along a *rectilinear trajectory* along the axis y_0, without any rotation of the platform, we remark that the distribution of the displacement, velocity and acceleration, as calculated by the program and depicted in Figs. 9, 10, 11 is the same, at any instant, for two of the three actuators.

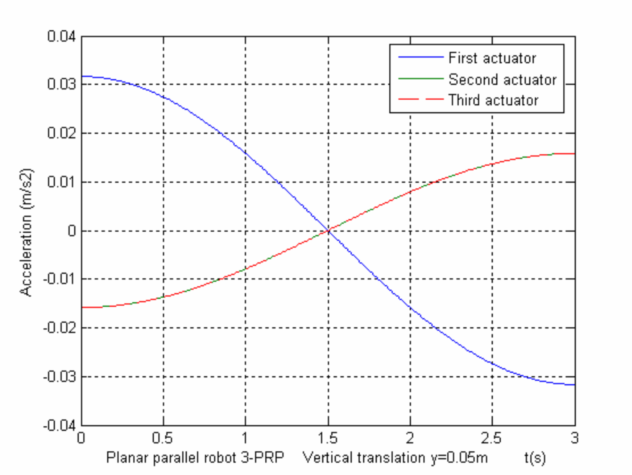

Fig. 11 Accelerations γ_{10}^{A}, γ_{10}^{B}, γ_{10}^{C}

The simulation through the MATLAB program certify that a major advantage of the current matrix recursive approach is the well structured way to formulate a kinematical model, which leads to computational efficiency. The proposed method can be applied to various types of complex robots, when the number of components of the mechanism is increased.

3. Conclusions

Within the inverse kinematics analysis some exact relations that give the position, velocity and acceleration of each element of the parallel robot in real-time have been established. The method described is quite available in forward and inverse mechanics of all serial or planar parallel mechanisms, the platform of which behaves in translation, rotation or general 3-DOF evolution .

R E F E R E N C E S

[1]. *L-W Tsai*, Robot analysis: the mechanics of serial and parallel manipulator, John Wiley & Sons, Inc., 1999
[2]. *D. Stewart*, A Platform with Six Degrees of Freedom, Proc. Inst. Mech. Eng., 1, 15, 180, pp. 371-378, 1965
[3]. *J-P. Merlet,* Parallel robots, Kluwer Academic Publishers, 2000
[4]. *V. Parenti-Castelli, R. Di Gregorio,* A new algorithm based on two extra-sensors for real-time computation of the actual configuration of generalized Stewart-Gough manipulator, Journal of Mechanical Design, 122, 2000
[5]. *R. Clavel,* Delta: a fast robot with parallel geometry, Proceedings of 18th International Symposium on Industrial Robots, Lausanne, pp. 91-100, 1988
[6]. *L-W. Tsai, R. Stamper,* A parallel manipulator with only translational degrees of freedom, ASME Design Engineering Technical Conferences, Irvine, CA, 1996

[7]. *S. Staicu, D.C. Carp-Ciocardia,* Dynamic analysis of Clavel's Delta parallel robot, Proceedings of the IEEE International Conference on Robotics & Automation ICRA'2003, Taipei, Taiwan, pp. 4116-4121, 2003
[8]. *J-M. Hervé, F. Sparacino,* Star. A New Concept in Robotics, Proceedings of the Third International Workshop on Advances in Robot Kinematics, Ferrara, pp.176-183, 1992
[9]. *J. Angeles,* Fundamentals of Robotic Mechanical Systems: Theory, Methods and Algorithms, Springer-Verlag, 2002
[10]. *C. Gosselin, M. Gagné,* Dynamic models for spherical parallel manipulators", Proceedings of the IEEE International Conference on Robotics & Automation, Milan, Italy, 1995
[11]. *J, Wang, C. Gosselin,* A new approach for the dynamic analysis of parallel manipulators", Multibody System Dynamics, 2, 3, 317-334, 1998
[12]. *D.D. Aradyfio, D. Qiao,* Kinematic Simulation of Novel Robotic Mechanisms Having Closed Chains, ASME Mechanisms Conference, Paper 85-DET-81, 1985
[13]. *C. Gosselin, J. Angeles,* The optimum kinematic design of a planar three-degree-of-freedom parallel manipulator, ASME journal of Mechanisms, Trans. and Automation in Design., 110, 1, pp. 35-41, 1988
[14]. *G.R. Pennock, D.J. Kassner,* Kinematic Analysis of a Planar Eight-Bar Linkage: Application to a Platfoem-type Robot", ASME Mechanisms Conference, Paper DE - 25, pp. 37-43, 1990
[15]. *J. Sefrioui, C. Gosselin,* On the quadratic nature of the singularity curves of planar three-degrees-of-freedom parallel manipulators, Mechanism and Machine Theory, 30, 4, pp. 533-551, 1995
[16]. *H. Mohammadi-Daniali, H.P. Zsombor-Murray, J. Angeles,* Singularity Analysis of Planar Parallel Manipulators, Mechanism and Machine Theory, 30, 5, pp. 665-678, 1995
[17]. *J-P. Merlet,* Direct kinematics of planar parallel manipulators, Proceedings of the IEEE International Conference on Robotics & Automation, Minneapolis, Minnesota, pp. 3744-3749, 1996
[18]. *R.L. Williams II, C.F. Reinholtz,* Closed-Form Workspace Determination and Optimization for Parallel Mechanisms, The 20th Biennial ASME, Mechanisms Conference, Kissimmee, Florida, DE, Vol. 5-3, pp. 341-351, 1988
[19]. *G. Yang, W. Chen, I-M. Chen*, A Geometrical Method for the Singularity Analysis of 3-RRR Planar Parallel Robots with Different Actuation Schemes, Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems, Lausanne, Switzerland, pp. 2055-2060, 2002
[20]. *I. Bonev, D. Zlatanov, C. Gosselin, ,* Singularity analysis of 3-DOF planar parallel mechanisms via screw theory, Journal of Mechanical, 25, 3, pp. 573-581, 2003
[21]. *H. Mohammadi-Daniali,P. Zsombor-Murray, J. Angeles,* The kinematics of 3-DOF planar and spherical double-triangular parallel manipulators, Computational Kinematics, edited by J. Angeles et al., Kluwer Academic Publishers, Dordrecht, Netherlands, pp. 153-164, 1993
[22]. *S. Staicu, X-J. Liu, J. Wang,* Inverse dynamics of the HALF parallel manipulator with revolute actuators, Nonlinear Dynamics, Springer Press, 50, 1-2, pp. 1-12, 2007
[23]. *S. Staicu, D. Zhang, R. Rugescu,* Dynamic modelling of a 3-DOF parallel manipulator using recursive matrix relations, Robotica, Cambridge University Press, 24, 1, 2006
[24]. *A. Pashkevich, D. Chablat, P., Wenger,* Kinematics and workspace analysys of a three-axis parallel manipulator: the Orthoglide, Robotica, Cambridge University Press, 24, 1, 2006
[25]. *X-J. Liu, X. Tang, J. Wang,* A Kind of Three Translational-DOF Parallel Cube-Manipulator, Proceedings of the 11th World Congress in Mechanism and Machine Science, Tianjin, China, 2004
[26] *S. Staicu,* Inverse dynamics of a planetary gear train for robotics, Mechanism and Machine Theory, 43, 7, pp. 918-927, 2008

[27]. *Y-W. Li, J. Wang, L-P. Wang, X-.J Liu,* Inverse dynamics and simulation of a 3-DOF spatial parallel manipulator, Proceedings of the IEEE International Conference on Robotics & Automation ICRA'2003, Taipei, Taiwan, pp. 4092-4097, 2003
[28]. *B. Dasgupta, T.S. Mruthyunjaya,* A Newton-Euler formulation for the inverse dynamics of the Stewart platform manipulator, *Mechanism and Machine Theory*, 34, 1998
[29]. *S. Guegan, W. Khalil, D. Chablat, P. Wenger,* Modélisation dynamique d'un robot parallèle à 3-DDL: l'Orthoglide, Conférence Internationale Francophone d'Automatique, Nantes, France, 8-10 Juillet 2002
[30]. *Z. Geng, L.S. Haynes, J.D. Lee, R.L. Carroll,* On the dynamic model and kinematic analysis of a class of Stewart platforms, Robotics and Autonomous Systems, 9, 1992
[31]. *K. Miller, R. Clavel,* The Lagrange-Based Model of Delta-4 Robot Dynamics, Robotersysteme, 8, pp. 49-54, 1992
[32]. *C-D. Zhang, S-M. Song,* An Efficient Method for Inverse Dynamics of Manipulators Based on Virtual Work Principle, Journal of Robotic Systems, 10, 5, pp. 605-627, 1993
[33]. *S. Staicu, D. Zhang,* A novel dynamic modelling approach for parallel mechanisms analysis" Robotics and Computer-Integrated Manufacturing, 24, 1, pp. 167-172, 2008
[34]. *S. Staicu,* Relations matricielles de récurrence en dynamique des mécanismes, Revue Roumaine des Sciences Techniques - Série de Mécanique Appliquée, 50, 1-3, pp. 15-28, 2005