MODELING WITH THE CHAOS GAME (I). SIMULATING SOME FEATURES OF REAL TIME SERIES

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Seriile numerice obținute prin derivarea seriilor care descriu cursurile de schimb a 10 monede naționale în raport cu dolarul american (USD) în perioada 1996-2006 sunt sintetizate prin superpoziția unor episoade de zgomot gaussian peste seria numită "the chaos game time series" (aproximată prin seria Yuan Chinez/USD). Prin integrare, se obțin serii care modelează dinamica seriilor originale. Panta spectrului Fourier şi complexitatea Lempel-Ziv pentru seriile reconstruite sunt în foarte bun acord cu aceleaşi mărimi ale seriilor reale, pentru caracteristici ale zgomotului gaussian alese în mod convenabil.

By superposition of episodes of Gaussian noise over the "chaos game time series" (approximated by the Chinese Yuan/USD series) we reconstruct changes of exchange rates of 10 national currencies with respect to the US Dollar during 1996-2006. By subsequent integration, a reconstruction of the real series is obtained. Their Lempel-Ziv complexity and the slope of the Fourier spectrum are in very good agreement with the corresponding characteristics of the original series, when convenient parameters of the Gaussian noise are used.

Keywords: Chaos game, Lempel-Ziv complexity, Gaussian Noise

1. Introduction

The powerful development of the theory of nonlinear phenomena stimulated by the significant increase of computing capabilities triggered a focusing of the interest of physics scientists onto the economic problems in the framework of the interdisciplinary field of econo-physics [1]. The treatment of financial time series could be performed in two main ways either interpreting the time evolution as exclusively random [2] or trying to understand their behaviour in terms of nonlinear theory, implying chaotic-deterministic dynamics [3]. Whatever the approach, the multifractal analysis is used as additional tool for explaining both the short and long range correlations [4].

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We identify the deterministic and stochastic contributions in exchange rate time series and demonstrate that the presence of both is essential for the understanding of the dynamics.

The following section presents the data, their source and selection arguments. Section 3 introduces the Iterated Function Systems (IFS) clumpiness test, particularly the chaos game and the Sierpinski gasket as prerequisites for identifying the deterministic and stochastic contributions to the dynamics of exchange rate time series. The IFS - Chaos Game is identified with the CHY-USD exchange rate [5] and is proved that the method of adding episodes of Gaussian white noise to the CHY-USD exchange rate is enough to replicate the considered characteristics of the investigated time series [6]. The computational results of the analyses and their interpretation make the object of the following section. Finally, the Conclusion section consists of a synthetic review of the important results.

2. Data

The analysed data represent daily exchange rates of the Chinese Yuan and other 9 national currencies with respect to the United States Dollar (USD) acquired from the electronic sites [http://www.oanda.com]. All of them (except Romania) are exhibiting a certain stability of their economies in the interval 1 Jan. 1996 - 31 Dec. 2006. These are: Brazil (Bra), Denmark (Den), Fiji (Fij), Israel (Isr), Japan (Jap), Kuwait (Kuw), Romania (RO), Singapore (Sin), and the United Kingdom (UK). The exchange rate time series consist of N=3957 data each.

If the individual data are denoted $x_n \ (n = 1, 2, \cdots, N)$ then the (price) changes series used in this study is defined as the derivative of the genuine series:

$$y_n = x_{n+1} - x_n \qquad (n = 1, 2, \cdots, N-1). \tag{1}$$

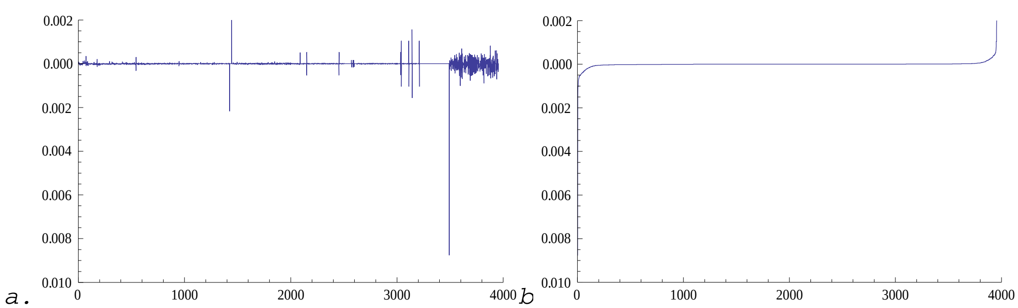

Fig.1. The changes of the Chinese Yuan (*a*) and its sorted values (*b*).

Due to its particular position in our study we present the CHY-USD exchange rate changes in Fig.1.

3. The chaos game

Here we are interested in a particular type of IFS system called the chaos game that is generated as follows [6]. Consider a number of points in a plane, particularly four points representing the vertices of a square.

Suppose only three of the four points are used which are numbered in clockwise direction, with the lower left vertex as number 1. Take an arbitrary point inside the triangle as the seed. At random, generate one of the three numbers, 1, 2 or 3. If the number 2 comes out then, halfway between the seed and the vertex number 2 a point is plotted. If the next number to come out is 1, then a new point is plotted at mid distance between the previously generated one and vertex number 1. The process is continued for a significant number of steps giving rise to the Sierpinski gasket shown on Fig.2*a.* If all the four vertices are used and the four numbers are generated at random, a uniform distribution of points on the square surface results (Fig.2*b*). When using all four vertices, but with two of them (say 3 and 4) in coincidence (situation that can be called as degeneracy of a vertex) in the sense that instead of "3" one would label again "4" and thus the vertex 3 would be completely missing, the pattern is in Fig.1*c*.

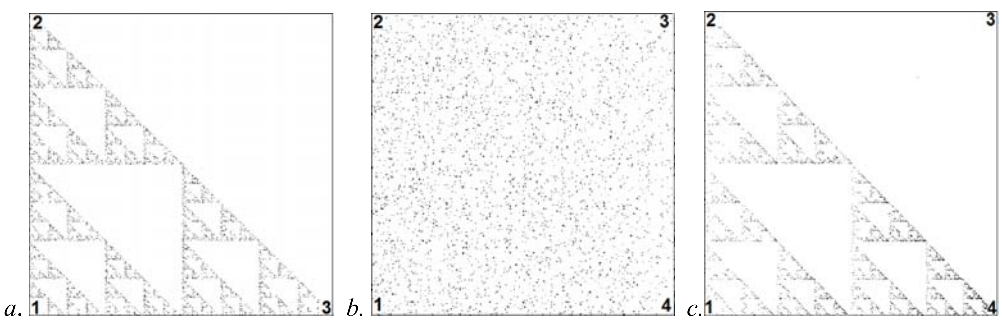

Fig.2. The chaos game with three vertices (*a*), four vertices (*b*), and four vertices with degeneracy (*c*) - the density of points increases from the upper left corner toward the lower right one

The resulting Sierpinski gasket is not symmetrical, the density of points is higher in the lower right vertex than in the upper left vertex.

4. Method

If the random generation is replaced by a correlated time series, the IFS technique can be used to create a pattern that helps to visualize the degree of correlation present in the respective series. The process begins with the sorting of the data y_n in increasing order from the minimum to the maximum value as shown in Fig.1 for the Chinese Yuan. Then, the string is divided by quartiles Q_j (j=1,2,3), such that each of the four groups (bins) contains the same number of entries (25%

of the total). The data in the original time series are classified (numbered) according to the bin where they belong. All the data below the lower quartile are marked with 1, the data below the median and above the lower quartile are marked with 2, the values above the median and below the upper quartile are denoted by 3 and those above the upper quartile, by 4. Then, the chaos game is played on a square with the four vertices labeled 1 to 4 in the clockwise direction for all the points in the series. The point associated with the first data in the series is plotted halfway between the center of the square (taken as the seed) and the corner whose number is the same as the number of the bin to which this data belongs. The second point is plotted at mid distance between this point and the corner with the same number as the bin to which the second data belongs.

The quartiles for the series of the Chinese Yuan/USD rate changes are

$$Q_1 = -1.1\times10^{-4}\ ;\ Q_2=0\ ;\ Q_3=0, \qquad (2)$$

therefore the labels of the vertices are 1, 2, and 4, and there is no vertex labeled “3”. Playing the chaos game with the CHY-USD exchange rate changes generates a pattern in excellent agreement with the Sierpinski gasket with the asymmetric density of the points due to the intrinsic degeneracy shown by (2). This result is explained as consequence of the severe control of the Chinese Government [6].

We noticed that the IFS pattern for the changes of all the considered exchange rates consists of a Sierpinski gasket over which an irregular spread of points is superimposed. Accordingly, the synthesis process consists of adding episodes of Gaussian noise of zero mean over the series of the CHY-USD exchange rate changes. The parameters of the Gaussian noise are the amplitude (standard deviation σ) and duration P (the noise length as percentage of the total length of the chaos game series).

5. Computational results

The comparison between the slope of the frequency spectrum (β) of the synthesized series and the genuine one in log-log coordinates is performed according to the algorithm shown in the Fig.3.

Unlike for the IFS clumpiness test, to compute the Lempel-Ziv (LZ) complexity, a numerical sequence of data is separated into two bins only. This is achieved by comparing the data with the median and whenever the value is larger than or equal to the threshold the particular data is replaced by 1, otherwise by 0. This operation attaches to the original series a digital one looking something like 1100011101...The next step is parsing the obtained symbolic sequence i.e. identifying the number of distinct words present in the sequence. The complexity counter is given by the number of distinct patterns contained in the sequence. Details on the parsing procedure can be found in [7]. The comparison between the

LZ complexity of the synthesized and the real series are performed by the same algorithm save for the last step. The results are presented in Table 1.

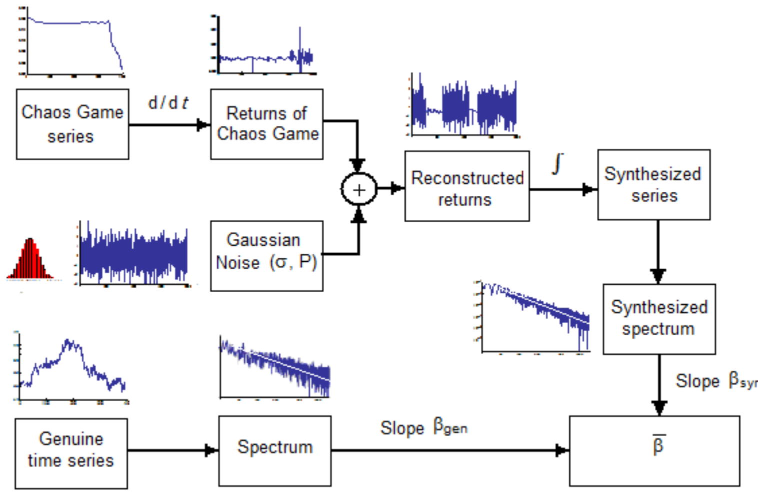

Fig.3 The synthesis process (the iterations are not shown)

The comparison between the genuine and the synthesized values is described by the parameter $\overline{Z} = 2\dfrac{|Z_{syn} - Z_{gen}|}{|Z_{syn} + Z_{gen}|}$ which stands for either the slope of the Fourier spectrum ($\overline{\beta}$) or the Lemepel-Ziv complexity ($\overline{LZ}$).

Table 1

Country	Bra	Den	Fij	Isr	Jap	Kuw	RO	Sin	UK
P (%)	78	92	20	88	92	56	75	85	92
$\sigma \times 10^{-2}$	0.061	1.7	0.032	1.0	1.7	2.8	1.8	2.8	1.8
$\overline{\beta}$ (%)	2.8	2.0	2.7	2.1	1.1	2.3	2.9	0.9	1.8
$\overline{LZ}$ (%)	10.14	9.09	10.48	9.10	8.22	9.71	11.57	8.19	9.32

As shown in [6], for the length of the noise episode, $P > 80\%$ the distribution of points in the clumpiness plots is practically uniform. For the series that can be reproduced by adding a Gaussian noise sample with $P \le 80\%$, the Sierpinsky gasket pattern of the chaos game is distinguishable. Episodes of white noise with $P \le 10\%$ do not observably change the chaos game distribution.

We applied these ideas to the case of the Romanian economic system that passed through structural changes consisting of the transition from command economy to a market economy [8]. However, the assumptions presented in Sect.3 should be applicable also in this case. The reconstruction obtained using a long time episode (*P*=75%) of Gaussian noise combined with the chaos game time series give results are in good agreement with the features of the real time series.

7. Conclusions

We demonstrate that some features of the changes of exchange rate time series of national currencies versus the USD can be reproduced as linear superposition between a mostly deterministic phenomenon (the chaos game) and episodes of Gaussian white noise. For some economic systems, the two components are intertwined almost permanently, while for others the chaos game time series is perturbed by Gaussian noise only on restricted time intervals. The reconstructed parameters are the L-Z complexity and the slope of the Fourier spectrum. The results of the analysis clearly show that a Gaussian component of too high amplitude can completely mask the chaos game dynamics and the result is similar to purely stochastic behavior. With too weak government corrections, the exchange rate time series can become completely random. This conclusion cannot be simply extrapolated to the whole financial system of an economy, however, the present financial crises suggests that this idea bears some truth.

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