AN APPLICATION OF SUBSTRUCTURE METHOD

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Lucrarea descrie unele aspecte matematice privind metoda substructurii aplicate unui sistem elastic cu mase concentrate, supus unei miscari armonice, in domeniul timp si frecventa.

Prin introducerea metodei substructurii devine posibila studierea unui sistem discretizat in doua subsisteme, pentru care sunt scrise ecuatiile dinamice de echilibru. Sistemul de ecuatii este rezolvat utilizand schema explicita Newmark de integrare in timp, obtinandu-se astfel contributiile celor doua subsisteme.

The paper deals with the mathematical description of the substructure method applied to an elastic lumped mass system subjected to an harmonic motion, in time and frequency domain.

By introducing the substructure method, it becomes possible to study a system separately in two subsystems (substructures), with an interface between them, and for which the dynamic equilibrium equations are written. The system of equations is solved using explicit Newmark time integration scheme, distinguishing between the quantities coming from the substructure 1 and substructure 2.

Keywords: substructure method, equilibrium equations, numerical integration, Fourier transform, impedances functions

Introduction

In case of omogen systems it is not necessary to make distinctions between some component parts, but in other cases, in which these components have different properties, the analyse on components it is useful and then the subsequent assembling of the results (kinematics or topological partition).

The substructures are disjunctive parts, with common boundary points but, for which internal or external (on boundaries) degrees-of-freedom are considered. Through substructure method, generally the reduction of dynamic problem dimensions is followed, by static or dynamic condensation, related to external degrees-of-freedom of substructures [1, 2].

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An elastic lumped mass system in time and frequency domain

The aim of this work is to identify the influence of each substructure in the modelling of interaction between those two substructures (subsystems) as matrices from the split system of equations matrix. The appropriate way to do this is to consider a *n*-degree-of-freedom liniar oscillator, supported on a rigid layer (substructure 1) and resting on another layer modelled with elastic isotropic homogenous halfspace (substructure 2), Fig.1.

The substructure 1 has a stiffness matrix [*K*], mass matrix [*M*] and damping matrix [*C*], satisfying the condition $[M]^{-1}[K][M]^{-1}[C]=[M]^{-1}[C][M]^{-1}[K]$, a necessary and sufficient condition for the substructure to admit decomposition into classical real modes [3].

The system has *n+2* significant degrees of freedom, namely, horizontal translation of each floor, horizontal translation of the base mass and rotation of the system in the plane of motion.

Table 1

Characteristics of the substructures

Substructure 1	Rigid layer	mass m_0, moment of inertia I_0,
	Structure	mass [*M*], (moment of inertia *I*), stiffness [*K*], damping [*C*], height *H*
	Displacement:	$v(t)+h\theta^I(t)+v_g^I(t)$
Substructure 2	Elastic halfspace	Poisson's ratio υ, mass density ρ, shear wave velocity c_s
	Rigid massless plate on the surface of the halfspace	its displacement compatibility with the lower surface of the rigid basemat
	Displacement:	$v_g(t)+v_g^I(t)$

The equilibrium equations of motions will be developed for the general case of *n* masses, in terms of the parameters of the overall system and the unknown displacements:

$v_g(t)$ - deformation at free-field surface,

$v(t)$ - deformation of the substructure 1 relative to the base,

$v_g^I(t)$ - base displacement caused by substructure 2-substructure 1 interaction, and

$\theta^I(t)$ - base rotation caused by substructure 2-substructure 1 interaction.

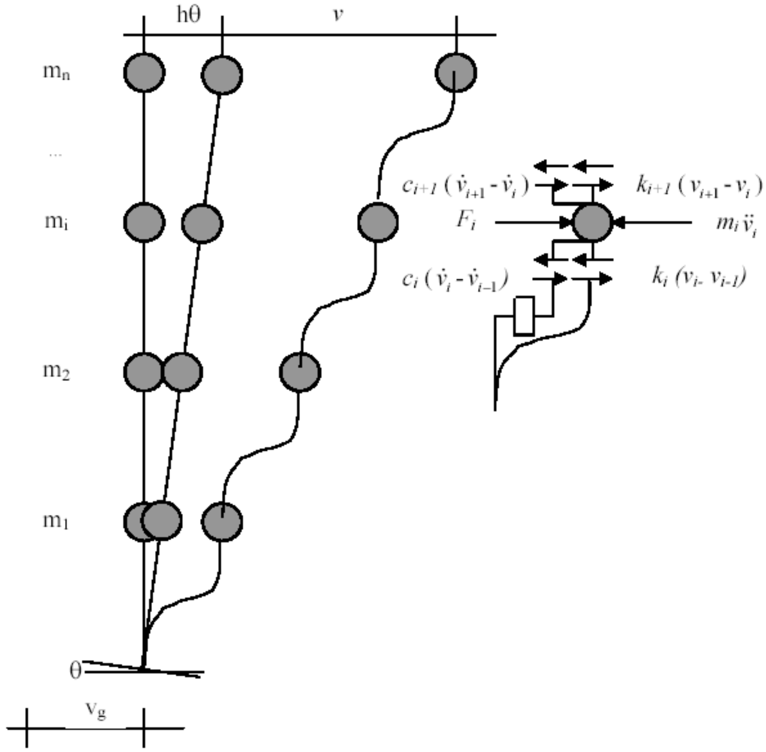

Fig. 1. – A lumped elastic system with *n*-degree-of-freedom (Substructure 1)

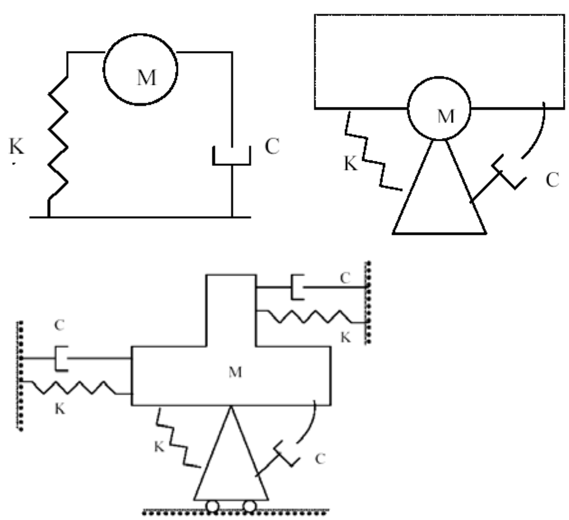

Fig. 2. - Physical models to represent dynamic stiffness for Substructure 2 (translation motion/rotational motion/coupling of horizontal and rocking motions)Substructure 1, in the time domain

Substructure 1, in the time domain

- all n masses are isolated in order to get the n horizontal force equilibrium equation:
-

$$m_1[\ddot{v}^1(t)+\ddot{\theta}^I\cdot h+\ddot{v}_g^I(t)]+c_1\dot{v}^1(t)+k_1v^1(t)-c_2[\dot{v}^2(t)-\dot{v}^1(t)]-k_2[v^2(t)-v^1(t)]=-m_1\ddot{v}_g$$

$$m_2\left[\ddot{v}^2(t)+\ddot{\theta}^I\cdot 2h+\ddot{v}_g^I(t)\right]+c_2\left[\dot{v}^2(t)+\dot{v}^1(t)\right]+k_2\left[v^2(t)-v^1(t)\right]-$$
$$-c_3\left[\dot{v}^3(t)+\dot{v}^2(t)\right]-k_3\left[v^3(t)-v^2(t)\right]=-m_2\ddot{v}_g\ldots$$

$$m_n[\ddot{v}^n(t)+\ddot{\theta}^I\cdot nh+\ddot{v}_g^I(t)]+c_n[\dot{v}^n(t)-\dot{v}^{n-1}(t)]+k_n[v^n(t)-v^{n-1}(t)]=-m_n\ddot{v}_g$$

- the entire structure is isolated from the elastic halfspace in order to get the horizontal force and moments about the centroidal x-axis of the basemat equilibrium equations:
-

$$m_1[\ddot{v}^1(t)+\ddot{\theta}^I\cdot h+\ddot{v}_g^I(t)]+m_2[\ddot{v}^2(t)+\ddot{\theta}^I\cdot 2h+\ddot{v}_g^I(t)]+\ldots+$$
$$m_n[\ddot{v}^n(t)+\ddot{\theta}^I\cdot nh+\ddot{v}_g^I(t)]+$$
$$m_1\ddot{v}_g(t)+m_2\ddot{v}_g(t)+\ldots+m_n\ddot{v}_g(t)+m_0[\ddot{v}_g(t)+\ddot{v}_g^I(t)]=V_0(t)$$

($V_0(t)$ is the base interaction shear force)

$$I_0\ddot{\theta}^I(t)+m_1\cdot h\cdot[\ddot{v}^1(t)+\ddot{\theta}^I\cdot h+\ddot{v}_g^I(t)]+m_2\cdot 2h\cdot[\ddot{v}^2(t)+\ddot{\theta}^I\cdot 2h+\ddot{v}_g^I(t)]+\ldots+$$

$$m_n\cdot nh\cdot[\ddot{v}^n(t)+\ddot{\theta}^I\cdot nh+\ddot{v}_g^I(t)]+m_1\cdot h\cdot\ddot{v}_g(t)+m_2\cdot 2h\cdot\ddot{v}_g(t)+\ldots+$$

$$m_n\cdot nh\cdot\ddot{v}_g(t)=M_0(t)$$

($M_0(t)$ is the base interaction moment)

In the matrix form one can obtain:

$$\begin{bmatrix} m_1 & 0 & 0 & \dots & 0 & m_1 & hm_1 \\ 0 & m_2 & 0 & \dots & 0 & m_2 & 2hm_2 \\ 0 & 0 & m_3 & \dots & 0 & m_3 & 3hm_3 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & m_n & m_n & nhm_n \\ m_1 & m_2 & m_3 & \dots & m_n & m_1+m_2+\dots+m_n+m_0 & hm_1+2hm_2+\dots+nhm_n \\ h & 2h & 3h & \dots & nh & hm_1+2hm_2+\dots+nhm_n & h^2m_1+4h^2m_2+\dots+n^2h^2m_n+J_0 \end{bmatrix}$$

$$\begin{Bmatrix} \ddot{v}^1(t) \\ \ddot{v}^2(t) \\ \ddot{v}^3(t) \\ \dots \\ \ddot{v}^n(t) \\ \ddot{v}_g^I(t) \\ \ddot{\theta}^I(t) \end{Bmatrix} + \begin{bmatrix} c_1+c_2 & -c_2 & 0 & \dots & 0 & 0 & 0 \\ -c_2 & c_2+c_3 & -c_3 & \dots & 0 & 0 & 0 \\ 0 & -c_3 & c_3+c_4 & \dots & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & c_n & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \dot{v}^1(t) \\ \dot{v}^2(t) \\ \dot{v}^3(t) \\ \dots \\ \dot{v}^n(t) \\ \dot{v}_g^I(t) \\ \dot{\theta}^I(t) \end{Bmatrix} +$$

$$+ \begin{bmatrix} k_1+k_2 & -k_2 & 0 & \dots & 0 & 0 & 0 \\ -k_2 & k_2+k_3 & -k_3 & \dots & 0 & 0 & 0 \\ 0 & -k_3 & k_3+k_4 & \dots & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & k_n & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} v^1(t) \\ v^2(t) \\ v^3(t) \\ \dots \\ v^n(t) \\ v_g^I(t) \\ \theta^I(t) \end{Bmatrix} =$$

$$= - \begin{Bmatrix} m_1 \\ m_2 \\ m_3 \\ \dots \\ m_n \\ m_1+m_2+\dots+m_n+m_0 \\ hm_1+2hm_2+\dots+nhm_n \end{Bmatrix} \ddot{v}_g + \begin{Bmatrix} 0 \\ 0 \\ 0 \\ \dots \\ 0 \\ V_0(t) \\ M_0(t) \end{Bmatrix}$$

The system of equations is integrated with respect to time using *explicit Newmark time integration scheme* with $\beta = 0$, $\gamma = \frac{1}{2}$ and

$$v_{i+1} = v_i + \Delta t \cdot \dot{v}_i + \frac{\Delta t^2}{2} \cdot \ddot{v}_i$$

$$\dot{v}_{i+1} = \dot{v}_i + \frac{\Delta t}{2} \cdot (\ddot{v}_i + \ddot{v}_{i+1})$$

Thus:

$$M\ddot{v}_{i+1} + C\dot{v}_{i+1} + Kv_{i+1} = P_{i+1}$$

$$\left(M + C \cdot \frac{\Delta t}{2} \right) \ddot{v}_{i+1} = P_{i+1} - v_i \cdot K - \dot{v}_i \cdot \left(C + K \cdot \Delta t \right) - \ddot{v}_i \cdot \left(C \cdot \frac{\Delta t}{2} + K \cdot \frac{\Delta t^2}{2} \right)$$

$\ddot{v}_{i+1} = f\left(v_i, \dot{v}_i, \ddot{v}_i \right)$ and the discrete system of linear equations at time t_{n+1} is $\bar{\bar{M}} \cdot \ddot{v}_{i+1} = \bar{\bar{P}}_{i+1}$.

Distinguishing between the quantities coming from the substructure 1 and substructure 2, the system $\bar{M} \cdot \ddot{v}_{i+1} = \bar{P}_{i+1}$ may be split into:

$$\begin{bmatrix} \bar{\bar{M}}_{substr1-substr1} & \bar{\bar{M}}_{substr1-substr2} \\ \bar{\bar{M}}_{substr2-substr1} & \bar{\bar{M}}_{substr2-substr2} \end{bmatrix} \begin{Bmatrix} \ddot{v}_{substr1} \\ \ddot{v}_{substr2} \end{Bmatrix} = \begin{Bmatrix} \bar{\bar{P}}_1 \\ \bar{\bar{P}}_2 \end{Bmatrix}, \textbf{ where } \bar{\bar{M}} = \bar{M} + C \cdot \frac{\Delta t}{2},$$

$$\bar{M}_{substr1-substr1} = \begin{bmatrix} m_1 & 0 & 0 & \dots & 0 \\ 0 & m_2 & 0 & \dots & 0 \\ 0 & 0 & m_3 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & m_n \end{bmatrix}, \ \bar{M}_{substr1-substr2} = \begin{bmatrix} m_1 & hm_1 \\ m_2 & 2hm_2 \\ m_3 & 3hm_3 \\ \dots & \dots \\ m_n & nhm_n \end{bmatrix},$$

$$\bar{M}_{substr2-substr1} = \bar{M}^T_{substr1-substr2},$$

$$\bar{M}_{substr2-substr2} = \begin{bmatrix} m_1 + m_2 + \dots + m_n + m_0 & hm_1 + 2hm_2 + \dots + nhm_n \\ hm_1 + 2hm_2 + \dots + nhm_n & h^2 m_1 + 4h^2 m_2 + \dots + n^2 h^2 m_n + I_0 \end{bmatrix}$$

In another simplified form, the mass matrix can be expressed as:

$$I = \begin{Bmatrix} 1 \\ 1 \\ 1 \\ \ldots \\ 1 \end{Bmatrix}; \qquad H = \begin{Bmatrix} h \\ 2h \\ 3h \\ \ldots \\ nh \end{Bmatrix}; \qquad \bar{\bar{M}} = \begin{bmatrix} \bar{\bar{M}}_{substr1-substr1} & \bar{\bar{M}}_{substr1-substr2} \\ \bar{\bar{M}}_{substr2-substr1} & \bar{\bar{M}}_{substr2-substr2} \end{bmatrix} \textbf{ or}$$

$$\bar{\bar{M}} = \begin{bmatrix} \bar{\bar{M}} & \bar{\bar{M}}I & \bar{\bar{M}}H \\ \left\{\bar{\bar{M}}I\right\}^T & m_0 + I^T\bar{\bar{M}}I & I^T\bar{\bar{M}}H \\ \left\{\bar{\bar{M}}H\right\}^T & \{I^T\bar{\bar{M}}H\}^T & J_0 + H^T\bar{\bar{M}}H \end{bmatrix}$$

Substructure 1, in the frequency domain, using the Fourier transforming of *n+2* equations and the impedances functions (because the stiffness and damping properties of the substructure 2 are frequency dependent)

The Fourier transform is a mathematical technique for converting time domain data ($v(t)$) to frequency domain data ($V(\omega)$), and reversely,

$$V(\omega) = \int_{-\infty}^{+\infty} v(t)e^{-i\omega t}dt$$

and is applied to each equilibrium equation for substructure 1.

More over, the equations of motion involve only the two substructure 2-substructure 1 interaction degrees-of-freedom, $v_g^I(t)$ and $\theta^I(t)$, and each impedances functions are expressed in terms of the halfspace:

$$\left(-m_1\bar{\omega}^2 + c_1 i\bar{\omega} + k_1 + c_2 i\bar{\omega} + k_2\right)V^1\left(i\bar{\omega}\right) - m_1\bar{\omega}^2 h\Theta^I(i\bar{\omega}) - m_1\bar{\omega}^2 V_g^I(i\bar{\omega}) - \left(c_2 i\bar{\omega} + k_2\right)V^2(i\bar{\omega}) +$$

$$+ m_1\ddot{V}_g\left(i\bar{\omega}\right) = 0$$

$$\left(-m_2\varpi^2 + c_2 i\varpi + k_2 + c_3 i\varpi + k_3\right)V^2\left(i\varpi\right) - \left(c_2 i\varpi + k_2\right)V^1\left(i\varpi\right) - m_2\varpi^2 2h\Theta^I(i\varpi) - m_2\varpi^2 V_g^I(i\varpi)$$
$$-\left(c_3 i\varpi + k_3\right)V^3(i\varpi) + m_2\ddot{V}_g(i\varpi) = 0$$
$$\ldots$$
$$\left(-m_n\varpi^2 + c_n i\varpi + k_n\right)V^n\left(i\varpi\right) - m_n\varpi^2 nh\Theta^I(i\varpi) - m_n\varpi^2 V_g^I(i\varpi) - \left(c_{n-1} i\varpi + k_{n-1}\right)V^{n-1}(i\varpi) +$$
$$+ m_n\ddot{V}_g\left(i\overline{\omega}\right) = 0$$

$$-m_1\varpi^2 V^1(i\varpi) - m_2\varpi^2 V^2(i\varpi) - \ldots - m_n\varpi^2 V^n(i\varpi) - \left(m_1\varpi^2 h + m_2\varpi^2 2h + \ldots + m_n\varpi^2 nh\right)\Theta^I(i\varpi)$$
$$-\left(m_1\varpi^2 + m_2\varpi^2 + \ldots + m_n\varpi^2 + m_0\varpi^2\right)V_g^I(i\varpi) + \left(m_1 + m_2 + \ldots + m_n + m_0\right)\ddot{V}_g(i\varpi) = V_0(i\varpi)$$

$$-m_1 h\overline{\omega}^2 V^1\left(i\overline{\omega}\right) - m_2 2h\overline{\omega}^2 V^2\left(i\overline{\omega}\right) - \ldots - m_n nh\overline{\omega}^2 V^n\left(i\overline{\omega}\right) -$$
$$-\left(m_1 h\overline{\omega}^2 + m_2\overline{\omega}^2 4h^2 + \ldots + m_n\overline{\omega}^2 n^2 h^2 I_0\right)\Theta^I\left(i\overline{\omega}\right) -$$
$$-\left(m_1 h\overline{\omega}^2 + m_2 2h\overline{\omega}^2 + \ldots + m_n nh\overline{\omega}^2\right)V_g^I\left(i\overline{\omega}\right) + \left(m_1 h + m_2 2h + \ldots + m_n nh\right)\ddot{V}_g\left(i\overline{\omega}\right) = M_0\left(i\overline{\omega}\right)$$

Using the complex impedance functions (frequency dependent, having the form $G(ia_0) = G^R(a_0) + iG^I(a_0)$), the interaction forces acting on *substructure 1* are given in the frequency domain by:

$$\begin{Bmatrix} -V_0(i\varpi) \\ -M_0(i\varpi) \end{Bmatrix} = \begin{bmatrix} G_{v_g^I v_g^I}(i\varpi) & G_{v_g^I \theta^I}(i\varpi) \\ G_{\theta^I v_g^I}(i\varpi) & G_{\theta^I \theta^I}(i\varpi) \end{bmatrix} \begin{Bmatrix} V_g^I(i\varpi) \\ \Theta^I(i\varpi) \end{Bmatrix} \text{, i.e.}$$

$$\begin{bmatrix} G_{11} & G_{12} & \ldots & G_{1n} & G_{1,n+1} & G_{1,n+2} \\ G_{21} & G_{22} & \ldots & G_{2n} & G_{2,n+1} & G_{2,n+2} \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ G_{n1} & G_{n2} & \ldots & G_{nn} & G_{n,n+1} & G_{n,n+2} \\ G_{n+1,1} & G_{n+1,2} & \ldots & G_{n+1,n} & G_{n+1,n+1} & G_{n+1,n+2} \\ G_{n+2,1} & G_{n+2,2} & \ldots & G_{n+2,n} & G_{n+2,n+1} & G_{n+2,n+2} \end{bmatrix} \begin{Bmatrix} V^1(i\varpi) \\ V^2(i\varpi) \\ \ldots \\ V^n(i\varpi) \\ V_g^I(i\varpi) \\ \Theta^I(i\varpi) \end{Bmatrix} =$$

$$= - \begin{Bmatrix} m_1 \\ m_2 \\ \ldots \\ m_n \\ m_1 + m_2 + \ldots + m_n + m_0 \\ hm_1 + 2hm_2 + \ldots + nhm_n \end{Bmatrix} \ddot{V}_g(i\varpi) + \begin{Bmatrix} 0 \\ 0 \\ \ldots \\ 0 \\ V_0(i\varpi) \\ M_0(i\varpi) \end{Bmatrix} \text{, where}$$

$$G_{11} = -m_1\overline{\varpi}^2 + c_1 i\overline{\varpi} + k_1 + c_2 i\overline{\varpi} + k_2$$
$$G_{12} = -(c_2 i\overline{\varpi} + k_2) = G_{21}$$
$$G_{13} = ... = G_{1n} = 0$$
$$G_{1,n+1} = -m_1\overline{\varpi}^2$$
$$G_{1,n+2} = -m_1\overline{\varpi}^2 h$$
$$G_{22} = -m_2\overline{\varpi}^2 + c_2 i\overline{\varpi} + k_2 + c_3 i\overline{\varpi} + k_3$$
$$G_{23} = -\left(c_3 i\overline{\varpi} + k_3\right)$$
$$G_{24} = ... = G_{2n} = 0$$
$$G_{2,n+1} = -m_2\overline{\varpi}^2$$
$$G_{2,n+2} = -m_2\overline{\varpi}^2 2h$$
$$\dots$$
$$G_{n1} = ... = G_{n-2} = 0$$
$$G_{n,n-1} = -\left(c_{n-1} i\overline{\varpi} + k_{n-1}\right)$$
$$G_{nn} = -m_n\overline{\varpi}^2 + c_n i\overline{\varpi} + k_n$$
$$G_{n,n+1} = -m_n\overline{\varpi}^2$$
$$G_{n,n+2} = -m_n\overline{\varpi}^2 nh$$
$$G_{n+1,1} = -m_1\overline{\varpi}^2$$
$$G_{n+1,2} = -m_2\overline{\varpi}^2$$
$$\dots$$
$$G_{n+1,n} = -m_n\overline{\varpi}^2$$
$$G_{n+1,n+1} = -\left(m_1 + ... + m_n + m_0\right)\overline{\varpi}^2 + G_{v_g^I v_g^I}(i\overline{\varpi})$$
$$G_{n+1,n+2} = -\left(m_1 h + m_2 2h + ... + m_n nh\right)\overline{\varpi}^2 + G_{v_g^I \theta^I}(i\overline{\varpi})$$
$$G_{n+2,1} = -m_1\overline{\varpi}^2 h$$
$$G_{n+2,2} = -m_2\overline{\varpi}^2 2h$$
$$\dots$$
$$G_{n+2,n} = -m_n\overline{\varpi}^2 nh$$
$$G_{n+2,n+1} = -\left(m_1 h + ... + m_n nh\right)\overline{\varpi}^2 + G_{\theta^I v_g^I}(i\overline{\varpi})$$
$$G_{n+2,n+2} = -\left(m_1 h^2 + m_2 2^2 h^2 + ... + m_n n^2 h^2 + I_0\right)\overline{\varpi}^2 + G_{\theta^I \theta^I}(i\overline{\varpi})$$

For substructure 2,

In dynamics, the dimensionless frequency a_0 is introduced, $a_0 = \frac{\omega r_0}{c_s}$, with r_0 representing a characteristic length and c_s the shear-wave velocity from the motion, $c_s = \sqrt{\frac{G}{\rho}}$, G shear modulus. Using the static-stiffness coefficient K, is formulated the dynamic stiffness coeficient, $S(a_0)$:

$$S(a_0) = K\left[k(a_0) + i a_0 c(a_0)\right]$$

The spring with the stiffness $k(a_0)$ governs the force, which is in phase with the displacement, and the damping coefficient $c(a_0)$ describes the force which is 90^0 out of phase. The dynamic-stiffness coefficient $S(a_0)$ can be interpreted as a spring with the frequency-dependent coefficient $Kk(a_0)$ and a dashpot in parallel with the frequency-dependent coefficient $\frac{r_0}{c_s} Kc(a_0)$, Fig. 3.

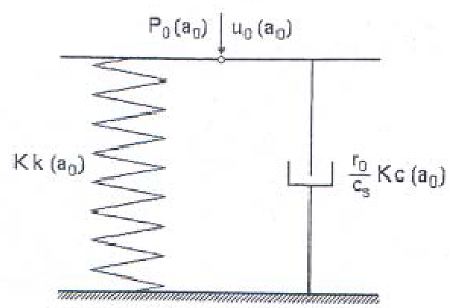

Fig. 3. - Interpretation of dynamic-stiffness coefficient for harmonic excitation as spring and as dashpot in parallel with frequency-dependent coefficients

Using cones to model the halfspace, there are the following expressions for stiffness and damping (Fig. 2) [1]:

$$k_{oriz}(a_0) = 1 - \frac{G}{\pi} \frac{z_0}{r_0} \frac{c_s^2}{c^2} a_0^2 ; \; c_{oriz}(a_0) = \frac{z_0}{r_0} \frac{c_s}{c} .$$

(for the horizontal motion, $c = c_s$, $G = 0$ for all υ, for the vertical motion, $c = c_p$, $G = 0$ for $\upsilon \le \frac{1}{3}$ and $c = 2\,c_s$ and $G \neq 0$ for $\frac{1}{3} \le \upsilon \le \frac{1}{2}$)

$$k_{rot}(a_0) = 1 - \frac{4}{3}\frac{G_\theta}{\pi}\frac{z_0}{r_0}\frac{c_s^2}{c^2}a_0^2 - \frac{1}{3}\frac{a_0^2}{\left(\frac{r_0 c}{z_0 c_s}\right)^2 + a_0^2};$$

$$c_{rot}(a_0) = \frac{1}{3}\frac{z_0}{r_0}\frac{c_s}{c}\frac{a_0^2}{\left(\frac{r_0 c}{z_0 c_s}\right)^2 + a_0^2}$$

(for the torsional motion, $c = c_s$, $G = 0$ for all υ, for the rocking motion, $c = c_p$, $G = 0$ for $\upsilon \le \frac{1}{3}$ and $c = 2\,c_s$ and $G \neq 0$ for $\frac{1}{3} \le \upsilon \le \frac{1}{2}$)

Also, for stiffness and damping there are the following formulae:

$$k_{oriz} = \frac{8}{2-\upsilon}Gr_0\,;\; c_{oriz} = \frac{4.6}{2-\upsilon}\rho c_s r_0^2\,;\; k_{rot} = \frac{8Gr_0^3}{3(1-\upsilon)}\,;\; c_{rot} = \frac{0.4}{1-\upsilon}\rho c_s r_0^4$$

Conclusions

The theoretical analysis of a lumped elastic system with *n* degree-of-freedom supported by a rigid layer resting on an elastic halfspace is presented. The quantities coming from the substructure 1 and the substructure 2 are put into evidence, being determined the matrices for and at the interface from the both structures. This aspect is important from mathematical point of view, but important too from practical point of view for the possibility to combine the numerical simulation of the analytical part of the system with the effective laboratory testing of the remaining part of the system.

The theoretical point of view is going to be continued with some numerical studies related to the substructure 2-substructure 1 interaction, using the computer modeling of different systems.

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