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NEW FIXED POINT RESULTS AND THE PROPERTY (P) ON ORDERED INTUITIONISTIC FUZZY METRIC SPACES

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We provide some fixed point results for some contractive mappings on complete ordered triangular intuitionistic fuzzy metric spaces. Also, we give some results about the property (P).

Keywords: Contractive mapping, fixed point, complete ordered triangular intuitionistic metric space, the property (P).

1. Introduction

In 1922, Banach proved the principle contraction result [9]. As we know, fixed points results for different kinds of contractions are of great interest for fixed point theorists on some spaces such quasi-metric spaces [11, 28], cone metric spaces [3], convex metric spaces [35], partially ordered metric spaces [1, 4, 6, 7, 10, 12, 14, 39, 43, 51], G-metric spaces [8, 13, 41, 45, 46, 47], (quasi-)partial metric spaces [42, 44], Menger spaces [33], and fuzzy metric spaces [25, 27, 32]. The concept of fuzzy sets introduced by Zadeh in 1965 [50]. In 1975, Kramosil and Michalek introduced the notion of fuzzy metric spaces [32] and George and Veeramani modified the concept in 1994 [26]. They also defined the notion of Hausdorff topology in fuzzy metric spaces [26]. This notion has very important applications in quantum particle physics particularly in connection with both string and E-infinity theory which introduced by El Naschie and Sigalotte [18, 19, 20, 21, 22, 23, 49]. Motivated by the potential applicability of fuzzy topology to quantum particle physics, Park introduced the notion of intuitionistic fuzzy metric space [36]. He showed that for each intuitionistic fuzzy metric space $(X, M, N, *, \diamondsuit)$, the topology generated by the intuitionistic fuzzy metric (M, N) coincides with the topology generated by the fuzzy metric M. We shall use this fact in our results throughout this manuscript. Actually, Park's notion is useful in modeling some phenomena where it is necessary to study the relationship between two probability functions. Some authors have introduced and discussed several notions of intuitionistic fuzzy metric spaces in different ways (see for example [2, 5, 15]). Grabiec obtained a fuzzy version of the Banach contraction principle in fuzzy metric spaces in Kramosil and Michalek's sense [25], and since then many authors have proved fixed point theorems in fuzzy metric spaces [16, 31, 34, 37]. In 2007, Rhoades defined the property (P) on metric spaces [29] and [30]. Denote as usual, by $F(T)$ the set of fixed points of the mapping $T\colon X \to X$. We say that a selfmap T has the property (P) whenever $F(T) = F(T^n)$ for all $n \geq 1$. Note that, $F(T) \subseteq F(T^n)$ for all

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$n \geq 1$. Recently, Ghorbanian, Rezapour and Shahzad generalized some old results about the property (P) [24]. In 2012, Samet et. al. introduced the concept of α-ψ-contractive type mappings [40]. In this paper, we combine all the idea of these papers and provide some fixed point results for some contractive mappings on complete ordered triangular intuitionistic fuzzy metric spaces. Also, we give some results about the property (P).

2. **Preliminaries**

Here, we recall some basic notions. For basic notions such continuous t-norm, intuitionistic fuzzy metric space and the induced topology which is denoted by $\tau_{(M,N)}$, one can study [36] and [48]. A sequence $\{x_n\}$ in an intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$ is said to be Cauchy whenever for each $\varepsilon > 0$ and $t > 0$, there exists a natural number n_0 such that $M(x_n, x_m, t) > 1 - \varepsilon$ and $N(x_n, x_m, t) < \varepsilon$ for all $n, m \geq n_0$. Also, $(X, M, N, *, \Diamond)$ is called complete whenever every Cauchy sequence is convergent with respect $\tau_{(M,N)}$.

Definition 2.1 ([16])**.** Let $(X, M, N, *, \Diamond)$ be an intuitionistic fuzzy metric space. The fuzzy metric (M, N) is called *triangular* whenever

$$\frac{1}{M(x,y,t)} - 1 \leq \frac{1}{M(x,z,t)} - 1 + \frac{1}{M(z,y,t)} - 1$$

and $N(x, y, t) \leq N(x, z, t) + N(z, y, t)$ for all $x, y, z \in X$ and $t > 0$.

Definition 2.2 ([27])**.** A sequence $\{x_n\}$ in intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$ is called *intuitionistic fuzzy contractive sequence* whenever there exists $0 < k < 1$ such that $\frac{1}{M(x_{n+1}, x_{n+2}, t)} - 1 \leq k\Big(\frac{1}{M(x_n, x_{n+1}, t)} - 1\Big)$ and $N(x_{n+1}, x_{n+2}, t) \leq kN(x_n, x_{n+1}, t)$, for all n and $t > 0$.

Lemma 2.1 ([31])**.** *Let $(X, M, N, *, \Diamond)$ be a triangular intuitionistic fuzzy metric space and $\{x_n\}$ an intuitionistic fuzzy contractive sequence in X. Then $\{x_n\}$ is a Cauchy sequence.*

Denote with Ψ the family of non-decreasing functions $\psi\colon [0, +\infty) \to [0, +\infty)$ such that $\sum_{n=1}^{\infty} \psi^n(t) < \infty$ for all $t > 0$. It is known that $\psi(t) < t$ for all $t > 0$.

Definition 2.3 ([40])**.** Let (X, d) be a metric space, $\alpha\colon X \times X \to [0, +\infty)$ a mapping and T a selfmap on X. We say that T is α-*admissible* whenever $\alpha(x, y) \geq 1$ implies $\alpha(Tx, Ty) \geq 1$ for all $x, y \in X$.

Example 2.4. Let $X = [0, +\infty)$ and $d(x, y) = |x - y|$. Define the selfmap T on X and $\alpha\colon X \times X \to [0, +\infty)$, respectively by the formulas $Tx = \sqrt{x}$, and $\alpha(x, y) = \exp(x - y)$, whenever $x \geq y$ and $\alpha(x, y) = 0$ whenever $x < y$ for all $x, y \in X$. Then T is α-admissible.

Definition 2.5 ([38])**.** Let $(X, M, N, *, \Diamond)$ be an intuitionistic fuzzy metric space. A selfmap f on X is called *intuitionistic fuzzy contractive* whenever there exists $k \in (0, 1)$ such that $N(f(x), f(y), t) \leq kN(x, y, t)$ and $\frac{1}{M(f(x), f(y), t)} - 1 \leq k\Big(\frac{1}{M(x, y, t)} - 1\Big)$, for all $x, y \in X$ and $t > 0$.

Definition 2.6. Let $(X, M, N, *, \Diamond)$ be an intuitionistic fuzzy metric space, T a selfmap on X, $\psi \in \Psi$ and $\alpha\colon X \times X \to [0, +\infty)$ a mapping. We say that T is a α-ψ-*contraction* whenever $\alpha(x, y)N(x, y, t) \leq \psi(N(x, y, t))$ and $\alpha(x, y)\Big(\frac{1}{M(Tx, Ty, t)} - 1\Big) \leq \psi\Big(\frac{1}{M(x, y, t)} - 1\Big)$, for all $x, y \in X$.

If $(X, \leq)$ is a partially ordered set, then we define

$$X_{\leq} = \{(x, y) \in X \times X : x \leq y \,\text{or}\, y \leq x\}.$$

Also, we say that a selfmap $T\colon X \to X$ is orbitally continuous at x whenever for each sequence $\{n(i)\}_{i\geq 1}$ with $T^{n(i)}x \to a$ for some $a \in X$, we have $T^{n(i)+1} \to Ta$. By [24], here $T^{m+1} = T(T^m)$. Finally, we define the orbit of T at x by

$$O(x, \infty) := \{x, Tx, T^2x, \ldots, T^nx, \ldots\}.$$

We say that T has the strongly comparable property whenever $(T^{n-1}y, T^ny) \in X_{\leq}$ for all $n \geq 1$ and $m \geq 2$, where $y \in F(T^m)$, see [24].

3. Main Results

First, we give the following result which includes some special mappings that could be discontinuous.

Theorem 3.1. *Let $(X, M, N, *, \Diamond)$ be a complete ordered triangular intuitionistic fuzzy metric space, $\lambda \in (0, 1)$ and T a selfmap on X satisfy the condition*

$$\min\left\{\frac{[1-M(Tx,Ty,t)]^2}{M^2(Tx,Ty,t)}, \frac{[1-M(x,y,t)][1-M(Tx,Ty,t)]}{M(x,y,t)M(Tx,Ty,t)}, \frac{[1-M(y,Ty,t)]^2}{M^2(y,Ty,t)}\right\}$$
$$-\min\left\{\frac{[1-M(x,Tx,t)]^2}{M^2(x,Tx,t)}, \frac{[1-M(y,Ty,t)][1-M(x,Ty,t)]}{M(y,Ty,t)M(x,Ty,t)}, \frac{[1-M(y,Tx,t)]^2}{M^2(y,Tx,t)}\right\}$$
$$\leq \lambda\frac{[1-M(x,Tx,t)][1-M(y,Ty,t)]}{M(x,Tx,t)M(y,Ty,t)}$$

for all $x, y \in X_{\leq}$. If there exists $x_0 \in X$ such that $(T^{n-1}x_0, T^nx_0) \in X_{\leq}$ for all $n \geq 1$ and T is orbitally continuous at x_0, then T has a fixed point. Moreover, if T has the strongly comparable property, then T has the property (P).

Proof. Define $x_{n+1} = Tx_n = T^{n+1}x_0$ for all $n \geq 0$. If $x_{n_0} = x_{n_0-1}$ for some natural number n_0, then $x_n = x_{n_0}$ for all $n \geq n_0$ and x_{n_0} is a fixed point of T. Suppose that $x_n \neq x_{n-1}$ for all $n \geq 1$. Now for each $n \geq 1$, by using the assumption, we can put $x = x_{n-1}$ and $y = x_n$ in the condition. Thus we obtain

$$\min\left\{\frac{[1-M(x_n,x_{n+1},t)]^2}{M^2(x_n,x_{n+1},t)}, \frac{[1-M(x_{n-1},x_n,t)][1-M(x_n,x_{n+1},t)]}{M(x_{n-1},x_n,t)M(x_n,x_{n+1},t)}\right\}$$
$$\leq \lambda\frac{[1-M(x_{n-1},x_n,t)][1-M(x_n,x_{n+1},t)]}{M(x_{n-1},x_n,t)M(x_n,x_{n+1},t)}.$$

Since $\lambda < 1$,

$$\min\left\{\frac{[1-M(x_n,x_{n+1},t)]^2}{M^2(x_n,x_{n+1},t)}, \frac{[1-M(x_{n-1},x_n,t)][1-M(x_n,x_{n+1},t)]}{M(x_{n-1},x_n,t)M(x_n,x_{n+1},t)}\right\}$$
$$= \frac{[1-M(x_n,x_{n+1},t)]^2}{M^2(x_n,x_{n+1},t)}$$

Hence,

$$\frac{1}{M(x_n,x_{n+1},t)} - 1 \leq \lambda\Big(\frac{1}{M(x_{n-1},x_n,t)} - 1\Big).$$

By continuing this process we obtain

$$\frac{1}{M(x_n,x_{n+1},t)} - 1 \leq \lambda^n\Big(\frac{1}{M(x_0,x_1,t)} - 1\Big)$$

for all $n \geq 1$. Thus for each natural number k we have

$$\frac{1}{M(x_n, x_{n+k}, t)} - 1 \leq \sum_{i=n}^{n+k-1} \Big(\frac{1}{M(x_i, x_{i+1}, t)} - 1\Big) \leq \sum_{i=n}^{n+k-1} \lambda^i \Big(\frac{1}{M(x_0, x_1, t)} - 1\Big)$$

$$\leq \frac{\lambda^n}{1-\lambda}\Big(\frac{1}{M(x_0, x_1, t)} - 1\Big).$$

Therefore, $\{x_n\}$ is a Cauchy sequence. Since $(X, M, N, *, \diamondsuit)$ is a complete intuitionistic fuzzy metric space, there exists $u \in X$ such that $x_n \to u$. Since T is orbitally continuous, $x_{n+1} = Tx_n \to Tu$. This implies that $Tu = u$.

Now, we prove that T has the property (P). Let $n \geq 2$ be given and $v \in F(T^n)$. Since T has the strongly comparable property, we can put $x = T^{n-1}v$ and $v = T^n v$ in the condition. Thus, we obtain

$$\min\Big\{\frac{[1 - M(T^n v, T^{n+1}v, t)]^2}{M^2(T^n v, T^{n+1}v, t)}, \frac{[1 - M(T^{n-1}v, T^n v, t)][1 - M(T^n v, T^{n+1}v, t)]}{M(T^{n-1}v, T^n v, t)M(T^n v, T^{n+1}v, t)}\Big\}$$

$$\leq \lambda \frac{[1 - M(T^{n-1}v, T^n v, t)][1 - M(T^n v, T^{n+1}v, t)]}{M(T^{n-1}v, T^n v, t)M(T^n v, T^{n+1}v, t)}.$$

Thus,

$$\min\Big\{\frac{[1 - M(v, Tv, t)]^2}{M^2(v, Tv, t)}, \frac{[1 - M(T^{n-1}v, v, t)][1 - M(v, Tv, t)]}{M(T^{n-1}v, v, t)M(v, Tv, t)}\Big\}$$

$$\leq \lambda \frac{[1 - M(T^{n-1}v, v, t)][1 - M(v, Tv, t)]}{M(T^{n-1}v, v, t)M(v, Tv, t)}$$

and so we get two cases.

Case I. This is $\dfrac{[1 - M(v, Tv, t)]^2}{M^2(v, Tv, t)} \leq \lambda \dfrac{[1 - M(T^{n-1}v, v, t)][1 - M(v, Tv, t)]}{M(T^{n-1}v, v, t)M(v, Tv, t)}$

We claim that $\frac{1}{M(v,Tv,t)} - 1 = 0$.

If $\frac{1}{M(v,Tv,t)} - 1 > 0$, then $\frac{1}{M(v,Tv,t)} - 1 = \frac{1}{M(T^n v, T^{n+1}v, t)} - 1 \leq \lambda \frac{1}{M(T^{n-1}v, T^n v, t)} - 1$. Again, by putting $x = T^{n-2}v$ and $y = T^{n-1}v$ in condition, we obtain

$$\min\Big\{\frac{[1 - M(T^{n-1}v, T^n v, t)]^2}{M^2(T^{n-1}v, T^n v, t)}, \frac{[1 - M(T^{n-2}v, T^{n-1}v, t)][1 - M(T^{n-1}v, T^n v, t)]}{M(T^{n-2}v, T^{n-1}v, t)M(T^{n-1}v, T^n v, t)}\Big\}$$

$$\leq \lambda \frac{[1 - M(T^{n-2}v, T^{n-1}v, t)][1 - M(T^{n-1}v, T^n v, t)]}{M(T^{n-2}v, T^{n-1}v, t)M(T^{n-1}v, T^n v, t)}.$$

Again, we get two cases. Let

$$\frac{[1 - M(T^{n-1}v, T^n v, t)]^2}{M^2(T^{n-1}v, T^n v, t)} \leq \lambda \frac{[1 - M(T^{n-2}v, T^{n-1}v, t)][1 - M(T^{n-1}v, T^n v, t)]}{M(T^{n-2}v, T^{n-1}v, t)M(T^{n-1}v, T^n v, t)}.$$

If $\frac{1}{M(T^{n-1}v, T^n v, t)} - 1 = 0$, then $T^{n-1}v = v$ and so $v = T^n v = Tv$. If $\frac{1}{M(T^{n-1}v, T^n v, t)} - 1 > 0$, then $\frac{1}{M(T^{n-1}v, T^n v, t)} - 1 \leq \lambda[\frac{1}{M(T^{n-2}v, T^{n-1}v, t)} - 1]$. Now, let

$$\frac{[1 - M(T^{n-2}v, T^{n-1}v, t)][1 - M(T^{n-1}v, T^n v, t)]}{M(T^{n-2}v, T^{n-1}v, t)M(T^{n-1}v, T^n v, t)}$$

$$\leq \lambda \frac{[1 - M(T^{n-2}v, T^{n-1}v, t)][1 - M(T^{n-1}v, T^n v, t)]}{M(T^{n-2}v, T^{n-1}v, t)M(T^{n-1}v, T^n v, t)}.$$

In this case we should have $\frac{1}{M(T^{n-2}v,T^{n-1}v,t)}-1=0$ or $\frac{1}{M(T^{n-1}v,T^{n}v,t)}-1=0$ (and so $v=Tv$), because if $\frac{1}{M(T^{n-2}v,T^{n-1}v,t)}-1>0$ and $\frac{1}{M(T^{n-1}v,T^{n}v,t)}-1>0$, then we get $\lambda\geq 1$ which is a contradiction. By continuing this process, we obtain

$$\frac{1}{M(v,Tv,t)}-1=\frac{1}{M(T^{n}v,T^{n+1}v,t)}-1\leq\lambda\Big(\frac{1}{M(T^{n-1}v,T^{n}v,t)}-1\Big)$$
$$\leq\lambda^{2}\Big(\frac{1}{M(T^{n-2}v,T^{n-1}v,t)}-1\Big)\leq\cdots\leq\lambda^{n}\Big(\frac{1}{M(Tv,Tv,t)}-1\Big)$$

which leads us to $\lambda\geq 1$ which is a contradiction.

Therefore, in this case we have $\frac{1}{M(v,Tv,t)}-1=0$ and so $Tv=v$.

Case II. $\dfrac{[1-M(T^{n-1}v,v,t)][1-M(v,Tv,t)]}{M(T^{n-1}v,v,t)M(v,Tv,t)}\leq\lambda\dfrac{[1-M(T^{n-1}v,v,t)][1-M(v,Tv,t)]}{M(T^{n-1}v,v,t)M(v,Tv,t)}.$

In this case, we should have $\frac{1}{M(T^{n-1}v,Tv,t)}-1=0$ or $\frac{1}{M(v,Tv,t)}-1=0$ (and so $v=Tv$). In fact, if $\frac{1}{M(T^{n-1}v,v,t)}-1>0$ and $\frac{1}{M(v,Tv,t)}-1>0$, then $\lambda\geq 1$ which is a contradiction. Thus we consequence that $F(T^{n})\subseteq F(T)$. Therefore, T has the property (P). □

The following example shows that there are nonlinear and discontinuous mappings which satisfy the condition of Theorem 3.1.

Example 3.1. Let $X=[0,\infty)$, be endowed with $d(x,y)=|x-y|$, $M(x,y,t)=\dfrac{t}{t+d(x,y)}$ and $N(x,y,t)=\dfrac{d(x,y)}{t+d(x,y)}$ for all $x,y\in X$ and $t\geq 0$. Define the selfmap T on X by $Tx=0$ whenever $0\leq x\leq 10$, $Tx=x-10$ whenever $10\leq x\leq 11$ and $Tx=1.1$ whenever $x\geq 11$. Then by putting $\lambda=\frac{1}{2}$. T satisfies the condition of Theorem 3.1.

Theorem 3.2. *Let $(X,M,N,*,\diamondsuit)$ be a complete ordered triangular intuitionistic fuzzy metric space, $b\in[0,1)$, $c\geq 0$, m a nonnegative integer and T a selfmap on X satisfy the condition*

$$\frac{[1-M(T^{m+1}x,T^{m+2}y,t)]^{2}}{M^{2}(T^{m+1}x,T^{m+2}y,t)}\leq b\frac{[1-M(T^{m}x,T^{m+1}x,t)][1-M(T^{m+1}y,T^{m+2}y,t)]}{M(T^{m}x,T^{m+1}x,t)M(T^{m+1}y,T^{m+2}y,t)}$$
$$+c\frac{[1-M(T^{m}x,T^{m+2}y,t)][1-M(T^{m+1}y,T^{m+1}x,t)]}{M(T^{m}x,T^{m+2}y,t)M(T^{m+1}y,T^{m+1}x,t)},$$

for all $x,y\in X_{\leq}$. Suppose that there exists $x_0\in X$ such that $(T^{n-1}x_0,T^{n}X_0)\in X_{\leq}$ for all $n\geq 1$. If T is orbitally continuous at x_0 or $m=0$, then T has a fixed point. Moreover, T has a unique fixed point whenever $c<1$. If T has the strongly comparable property, then T has the property (P).

Proof. Define $x_1=T^{m+1}x_0$ and $x_{n+1}=Tx_n$ for all $n\geq 1$. Then

$$\frac{[1-M(x_1,x_2,t)]^{2}}{M^{2}(x_1,x_2,t)}=\frac{[1-M(T^{m+1}x_0,T^{m+2}x_0,t)]^{2}}{M^{2}(T^{m+1}x_0,T^{m+2}x_0,t)}$$
$$\leq b\frac{[1-M(T^{m}x_0,T^{m+1}x_0,t)][1-M(T^{m+1}x_0,T^{m+2}x_0,t)]}{M(T^{m}x_0,T^{m+1}x_0,t)M(T^{m+1}x_0,T^{m+2}x_0,t)}$$
$$+c\frac{[1-M(T^{m}x_0,T^{m+2}x_0,t)][1-M(T^{m+1}x_0,T^{m+1}x_0,t)]}{M(T^{m}x,T^{m+2}x_0,t)M(T^{m+1}x_0,T^{m+1}x_0,t)}$$
$$=b\frac{[1-M(T^{m}x_0,T^{m+1}x_0,t)][1-M(T^{m+1}x_0,T^{m+2}x_0,t)]}{M(T^{m}x_0,T^{m+1}x_0,t)M(T^{m+1}x_0,T^{m+2}x_0,t)}$$

$$= b\frac{[1-M(T^m x_0, T^{m+1}x_0,t)][1-M(x_1,x_2,t)]}{M(T^m x_0, T^{m+1}x_0,t)M(x_1,x_2,t)}.$$

If $\frac{1}{M(x_1,x_2,t)} - 1 = 0$, then $Tx_1 = x_2 = x_1$ and so T has a fixed point. If $\frac{1}{M(x_1,x_2,t)} - 1 > 0$, then $\frac{1}{M(x_1,x_2,t)} - 1 \leq b[\frac{1}{M(T^m x_0,x_1,t)} - 1]$. Similarly, we have

$$\frac{[1-M(x_2,x_3,t)]^2}{M^2(x_2,x_3,t)} = \frac{[1-M(T^{m+2}x_0,T^{m+3}x_0,t)]^2}{M^2(T^{m+2}x_0,T^{m+3}x_0,t)}$$

$$\leq b\frac{[1-M(T^{m+1}x_0,T^{m+2}x_0,t)][1-M(T^{m+2}x_0,T^{m+3}x_0,t)]}{M(T^{m+1}x_0,T^{m+2}x_0,t)M(T^{m+2}x_0,T^{m+3}x_0,t)}$$

$$+c\frac{[1-M(T^{m+1}x_0,T^{m+3}x_0,t)][1-M(T^{m+2}x_0,T^{m+2}x_0,t)]}{M(T^{m+1}x,T^{m+3}x_0,t)M(T^{m+2}x_0,T^{m+2}x_0,t)}$$

$$= b\frac{[1-M(T^{m+1}x_0,T^{m+2}x_0,t)][1-M(T^{m+2}x_0,T^{m+3}x_0,t)]}{M(T^{m+1}x_0,T^{m+2}x_0,t)M(T^{m+2}x_0,T^{m+3}x_0,t)}$$

$$= b\frac{[1-M(x_1,x_2,t)][1-M(x_2,x_3,t)]}{M(x_1,x_2,t)M(x_2,x_3,t)}.$$

If $\frac{1}{M(x_2,x_3,t)} - 1 = 0$, then $Tx_2 = x_3 = x_2$ and so T has a fixed point. If $\frac{1}{M(x_2,x_3,t)} - 1 > 0$, then $\frac{1}{M(x_2,x_3,t)} - 1 \leq b[\frac{1}{M(x_1,x_2,t)} - 1]$ and so $\frac{1}{M(x_2,x_3,t)} - 1 \leq b^2[\frac{1}{M(T^m x_0,x_1,t)} - 1]$. By continuing this process we get that $\frac{1}{M(x_n,x_{n+1},t)} - 1 \leq b^n[\frac{1}{M(T^m x_0,x_1,t)} - 1]$ for all $n \geq 1$. This implies that $\{x_n\}$ is a Cauchy sequence. Since $(X, M, N, *, \diamondsuit)$ is a complete intuitionistic fuzzy metric space, there exists $u \in X$ such that $x_n \to u$. If T is orbitally continuous, then $Tx_n \to Tu$. Hence, $Tu = u$.

If $m = 0$, then for each $n \geq 2$ we have

$$\frac{[1-M(Tu,T^n x_0,t)]^2}{M^2(Tu,T^n x_0,t)} \leq b\frac{[1-M(u,Tu,t)][1-M(Tx_{n-2},T^2x_{n-2},t)]}{M(u,Tu,t)M(Tx_{n-2},T^2x_{n-2},t)}$$

$$+c\frac{[1-M(u,T^2x_{n-2},t)][1-M(Tx_{n-2},Tu,t)]}{M(u,T^2x_{n-2},t)M(Tx_{n-2},Tu,t)}.$$

Since $x_n \to u$, we have

$$\frac{1}{M(Tu,u,t)} - 1 \leq c\frac{[1-M(u,u,t)][1-M(u,Tu,t)]}{M(u,u,t)M(u,Tu,t)} = 0$$

and so $Tu = u$. Now, we show that T has a unique fixed point whenever $c < 1$. Let u and v be fixed points of T. Then, we have

$$\Big(\frac{1}{M(u,v,t)} - 1\Big)^2 = \Big(\frac{1}{M(T^{m+1}u,T^{m+2}v,t)} - 1\Big)^2$$

$$\leq b\frac{[1-M(T^m u,T^{m+1}u,t)][1-M(T^{m+1}v,T^{m+2}v,t)]}{M(T^m u,T^{m+1}u,t)M(T^{m+1}v,T^{m+2}v,t)}$$

$$+c\frac{[1-M(T^m u,T^{m+2}v,t)][1-M(T^{m+1}v,T^{m+1}u,t)]}{M(T^m u,T^{m+2}v,t)M(T^{m+1}v,T^{m+1}u,t)} = c\Big(\frac{1}{M(u,v,t)} - 1\Big)^2.$$

Hence, $\frac{1}{M(u,v,t)} - 1 = 0$ because $c < 1$. Thus, $u = v$ and so T has a unique fixed point.

Finally, we prove that T has the property (P) whenever T has the strongly comparable property. Let $n \geq 2$ be given and $v \in F(T^n)$. We consider the following cases.

Case I. $m = 0$. In this case, we have

$$\Big(\frac{1}{M(v,Tv,t)} - 1\Big)^2 = \Big(\frac{1}{M(T(T^{n-1}v),T^2(T^{n-1}v),t)} - 1\Big)^2$$

$$\leq b\frac{[1-M(T^{n-1}v,T^nv,t)][1-M(T^nv,T^{n+1}v,t)]}{M(T^{n-1}v,T^nv,t)M(T^nv,T^{m+1}v,t)}$$

$$+c\frac{[1-M(T^{n-1}v,T^{n+1}v,t)][1-M(T^nv,T^nv,t)]}{M(T^{n-1}v,T^{n+1}v,t)M(T^nv,T^nv,t)}=b\frac{[1-M(T^{n-1}v,v,t)][1-M(v,Tv,t)]}{M(T^{n-1}v,v,t)M(v,Tv,t)}.$$

If $\frac{1}{M(v,Tv,t)}-1=0$ then $Tv=v$. If $\frac{1}{M(u,v,t)}-1>0$, then $\frac{1}{M(T^nv,T^{n+1}v,t)}-1\leq b[\frac{1}{M(T^{n-1}v,T^nv,t)}-1]$. By continuing the process and using a similar argument as in Theorem 3.1, we obtain

$$\frac{1}{M(v,Tv,t)}-1=\frac{1}{M(T^nv,T^{n+1}v,t)}-1\leq b\Big(\frac{1}{M(T^{n-1}v,T^nv,t)}-1\Big)$$

$$\leq b^2\Big(\frac{1}{M(T^{n-2}v,T^{n-1}v,t)}-1\Big)\leq\cdots\leq b^n\Big(\frac{1}{M(v,Tv,t)}-1\Big).$$

Since $b<1$, $Tv=v$.

Case II. $m\geq 1$ and $n\leq m$. In this case, choose a natural number r and an integer number $0\leq s<n$ such that $m+1=rn+s$. Then, we have $T^n(T^{n-s}v)=T^{m+1}(T^{n-s}v)=v$, and so

$$\Big(\frac{1}{M(v,Tv,t)}-1\Big)^2=\Big(\frac{1}{M(T^{m+1}(T^{n-s}v),Tm+2(T^{n-s}v),t)}-1\Big)^2$$

$$\leq b\frac{[1-M(T^m(T^{n-s}v),T^{m+1}(T^{n-s}v),t)][1-M(T^{m+1}(T^{n-s}v),T^{m+2}(T^{n-s}v),t)]}{M(T^m(T^{n-s}v),T^{m+1}(T^{n-s}v),t)M(T^{m+1}(T^{n-s}v),T^{m+2}(T^{n-s}v),t)}$$

$$+c\frac{[1-M(T^m(T^{n-s}v),T^{m+2}(T^{n-s}v),t)][1-M(T^{m+1}(T^{n-s}v),T^{m+1}(T^{n-s}v),t)]}{M(T^m(T^{n-s}v),T^{m+2}(T^{n-s}v),t)M(T^{m+1}(T^{n-s}v),T^{m+1}(T^{n-s}v),t)}$$

$$=b\frac{[1-M(T^{n-1}v,v,t)][1-M(v,Tv,t)]}{M(T^{n-1}v,v,t)M(v,Tv,t)}.$$

If $\frac{1}{M(v,Tv,t)}-1=0$, then $Tv=v$. If $\frac{1}{M(v,Tv,t)}-1>0$, then $\frac{1}{M(T^nv,T^{n+1}v,t)}-1\leq b[\frac{1}{M(T^{n-1}v,T^nv,t)}-1]$. By continuing the process and using a similar argument as in Theorem 3.1, We obtain

$$\frac{1}{M(v,Tv,t)}-1=\frac{1}{M(T^nv,T^{n+1}v,t)}-1\leq b\Big(\frac{1}{M(T^{n-1}v,T^nv,t)}-1\Big)$$

$$\leq b^2\Big(\frac{1}{M(T^{n-2}v,T^{n-1}v,t)}-1\Big)\leq\cdots\leq b^n\Big(\frac{1}{M(v,Tv,t)}-1\Big).$$

Since $b<1$, $Tv=v$. Thus, $F(T^n)\subseteq F(T)$. Therefore, T has the property (P). □

The following example shows that there are nonlinear and discontinuous mappings satisfy the condition of Theorem 3.2.

Example 3.2. Let $X=[0,\infty)$, $d(x,y)=|x-y|$, $M(x,y,t)=\frac{t}{t+d(x,y)}$ and $N(x,y,t)=\frac{d(x,y)}{t+d(x,y)}$ for all $x,y\in X$ and $t\geq 0$. Define the selfmap T on X by $Tx=0$ whenever $0\leq x\leq 100$, $Tx=x-100$ whenever $100\leq x\leq 100.1$ and $Tx=0.15$ whenever $x\geq 100.1$. Then, by putting $m=0$, $b=\frac{1}{2}$. T satisfies the condition of Theorem 3.2.

Definition 3.3 ([24]). Let $(X,M,N,*,\Diamond)$ be a intuitionistic fuzzy metric space and T a selfmap on X. Then, T is said to be a *convex contraction of order* 2 if there exist $a,b\in(0,1)$ with $a+b<1$ such that

$$\frac{1}{M(T^2x,T^2y,t)}-1\leq a\Big(\frac{1}{M(Tx,Ty,t)}-1\Big)+b\Big(\frac{1}{M(x,y,t)}-1\Big),$$

for all $x, y \in X$ and $t > 0$. Also, T is said to be a convex contraction of order 2 if there exist $a_1, a_2, b_1, b_2 \in (0, 1)$ with $a_1 + a_2 + b_1 + b_2 < 1$ such that

$$\frac{1}{M(T^2x, T^2y, t)} - 1 \leq a_1\Big(\frac{1}{M(x, Tx, t)} - 1\Big) + a_2\Big(\frac{1}{M(Tx, T^2x, t)} - 1\Big)$$

$$+b_1\Big(\frac{1}{M(y, Ty, t)} - 1\Big) + b_2\Big(\frac{1}{M(Ty, T^2y, t)} - 1\Big),$$

for all $x, y \in X$.

Theorem 3.3. *Let $(X, M, N, *, \Diamond)$ be a complete order triangular intuitionistic fuzzy metric space, $a, b \in (0, 1)$ with $a + b < 1$ and T an orbitally continuous selfmap on X satisfy the condition*

$$\frac{1}{M(T^2x, T^2y, t)} - 1 \leq a\Big(\frac{1}{M(Tx, Ty, t)} - 1\Big) + b\Big(\frac{1}{M(x, y, t)} - 1\Big)$$

for all $x, y \in X_{\leq}$, then T has a unique fixed point. Also, $F(T) = F(T^2)$.

Proof. Define $x_n = T^nx_0$ for all $n \geq 1$, $v = \frac{1}{M(Tx_0, T^2x_0, t)} - 1 + \frac{1}{M(x_0, T_0^x, t)} - 1$, and $\lambda = a + b$. Thus $\frac{1}{M(T^2x_0, Tx_0, t)} - 1 \leq v$. Now, by using the assumption, we can put $x = Tx_0$ and $y = x_0$ in the condition. Thus, we obtain

$$\frac{1}{M(T^3x_0, T^2x_0, t)} - 1 \leq a\Big(\frac{1}{M(T^2x_0, Tx_0, t)} - 1\Big) + b\Big(\frac{1}{M(x_0, Tx_0, t)} - 1\Big) \leq \lambda v.$$

Now, by putting $x = T^2x_0$ and $y = Tx_0$ in the condition, we get

$$\frac{1}{M(T^4x_0, T^3x_0, t)} - 1 \leq a\Big(\frac{1}{M(T^3x_0, T^2x_0, t)} - 1\Big) + b\Big(\frac{1}{M(T^2x_0, x_0, t)} - 1\Big)$$

$$\leq a^2\Big(\frac{1}{M(T^2x_0, Tx_0, t)} - 1\Big) + ab\Big(\frac{1}{M(x_0, Tx_0, t)} - 1\Big) + b\Big(\frac{1}{M(T^2x_0, Tx_0, t)} - 1\Big) \leq \lambda^2 v.$$

Again, by putting $x = T^3x_0$ and $y = T^2x_0$ in the condition, we obtain

$$\frac{1}{M(T^5x_0, T^4x_0, t)} - 1 \leq a\Big(\frac{1}{M(T^4x_0, T^3x_0, t)} - 1\Big) + b\Big(\frac{1}{M(T^3x_0, T^2x_0, t)} - 1\Big)$$

$$\leq (a^3 + ab)\Big(\frac{1}{M(T^2x_0, Tx_0, t)} - 1\Big) + a^2b\Big(\frac{1}{M(x_0, Tx_0, t)} - 1\Big)$$

$$+ab\Big(\frac{1}{M(T^2x_0, Tx_0, t)} - 1\Big) + b\Big(\frac{1}{M(x_0, Tx_0, t)} - 1\Big)$$

$$= (a^3 + 2ab)\Big(\frac{1}{M(T^2x_0, Tx_0, t)} - 1\Big) + (a^2b + b^2)\Big(\frac{1}{M(x_0, Tx_0, t)} - 1\Big) \leq \lambda^3 v.$$

By continuing this process, we get $\frac{1}{M(T^{n+1}x_0, T^nx_0, t)} - 1 \leq \lambda^{n-1}v$ for all $n \geq 3$. This implies that

$$\frac{1}{M(T^mx_0, T^nx_0, t)} - 1 \leq \sum_{i=m}^{n-1}\Big(\frac{1}{M(T^ix_0, T^{i+1}x_0, t)} - 1\Big) \leq \sum_{i=n}^{n-1}\lambda^{i-2}v \leq \frac{\lambda^{m-2}}{1-\lambda}v$$

for all $n > m \geq 3$. Hence, $\{x_n\}$ is a Cauchy sequence. Since $(X, M, N, *, \Diamond)$ is a complete intuitionistic fuzzy metric space, there exists $u \in X$ such that $x_n \to u$. Since T is orbitally continuous, $Tx_n \to Tu$ and so $Tu = u$. Now, we show that T has a unique fixed point. Let y and z be fixed points of T. Then

$$\frac{1}{M(y,z,t)} - 1 = \frac{1}{M(T^2y, T^2z, t)} - 1 \le a\Big(\frac{1}{M(Ty,Tz,t)} - 1\Big) + b\Big(\frac{1}{M(y,z,t)} - 1\Big)$$
$$= (a+b)\Big(\frac{1}{M(y,z,t)} - 1\Big).$$

Since $a + b < 1$, we get $Ty = y$. □

Example 3.4. Let $X = \{1, 3, 5\}$, $d(x, y) = |x - y|$, $M(x, y, t) = \frac{t}{t+d(x,y)}$ and $N(x, y, t) = \frac{d(x,y)}{t+d(x,y)}$ for all $x, y \in X$ and $t \ge 0$. Define $\le = \{(1, 1), (3, 3), (5, 5)\}$ and T be a selfmap on X by $T1 = 3$, $T3 = 1$ and $T5 = 5$. Then, by putting $x_0 = 5$, $a = \frac{1}{2}$ and $b = \frac{1}{4}$, it is easy to see that T satisfies the condition of Theorem 3.3.

Theorem 3.4. *Let $(X, M, N, *, \Diamond)$ be a complete order triangular intuitionistic fuzzy metric space, $a_1, a_2, b_1, b_2 \in (0, 1)$ with $a_1 + a_2 + b_1 + b_2 < 1$ and T an orbitally continuous selfmap on X satisfy the condition*

$$\frac{1}{M(T^2x, T^2y, t)} - 1 \le a_1\Big(\frac{1}{M(x,Tx,t)} - 1\Big) + a_2\Big(\frac{1}{M(Tx,T^2x,t)} - 1\Big)$$
$$+ b_1\Big(\frac{1}{M(y,Ty,t)} - 1\Big) + b_2\Big(\frac{1}{M(Ty,T^2y,t)} - 1\Big)$$

for all $x, y \in X_\le$. If there exists $x_0 \in X$ such that $T^{n-1}x_0, T^nx_0) \in X_\le$ for all $n \ge 1$, then T has a unique fixed point. Also $F(T) = F(T^2)$.

Proof. Define $x_n = T^nx_0$, for all $n \ge 1$, and set $v = \frac{1}{M(Tx_0, T^2x_0, t)} - 1 + \frac{1}{M(x_0, T_0^x, t)} - 1$. Also, put $\lambda = a_1 + a_2 + b_1$ and $\beta = 1 - b_2$. We prove that

$$\frac{1}{M(T^{n+1}x_0, T^nx_0, t)} - 1 \le \Big(\frac{\lambda}{\beta}\Big)^{n-2} v$$

for all $n \ge 3$. Note that

$$\frac{1}{M(T^3x_0, T^2x_0, t)} - 1 \le a_1\Big(\frac{1}{M(x_0,Tx_0,t)} - 1\Big) + a_2\Big(\frac{1}{M(Tx_0,T^2x_0,t)} - 1\Big)$$
$$+ b_1\Big(\frac{1}{M(Tx_0,T^2x_0,t)} - 1\Big) + b_2\Big(\frac{1}{M(T^3x_0,T^2x_0,t)} - 1\Big)$$
$$\le a_1v + (a_2 + b_1)v + b_2\Big(\frac{1}{M(T^3x_0,T^2x_0,t)} - 1\Big).$$

Hence, $\frac{1}{M(T^3x_0,T^2x_0,t)} - 1 \le (\frac{\lambda}{\beta})v$. Now, by using the assumption, we can put $x = Tx_0$ and $y = T^2x_0$ in the condition. Thus, we obtain

$$\frac{1}{M(T^3x_0, T^4x_0, t)} - 1 \le a_1\Big(\frac{1}{M(Tx_0,T^2x_0,t)} - 1\Big) + a_2\Big(\frac{1}{M(T^2x_0,T^3x_0,t)} - 1\Big)$$
$$+ b_1\Big(\frac{1}{M(T^2x_0,T^3x_0,t)} - 1\Big) + b_2\Big(\frac{1}{M(T^3x_0,T^4x_0,t)} - 1\Big)$$
$$\le a_1v + (a_2 + b_1)\frac{a_1 + a_2 + b_1}{1 - b_2}v + b_2\Big(\frac{1}{M(T^3x_0,T^4x_0,t)} - 1\Big).$$

Hence, $\frac{1}{M(T^3x_0,T^4x_0,t)} - 1 \le (\frac{\lambda}{\beta})v$. Similarly, we have

$$\frac{1}{M(T^5x_0, T^4x_0, t)} - 1 \le a_1\Big(\frac{1}{M(T^3x_0,T^2x_0,t)} - 1\Big) + a_2\Big(\frac{1}{M(T^4x_0,T^3x_0,t)} - 1\Big)$$

$$+b_1\Big(\frac{1}{M(T^4x_0,T^3x_0,t)}-1\Big)+b_2\Big(\frac{1}{M(T^5x_0,T^4x_0,t)}-1\Big)$$
$$\le a_1\frac{a_1+a_2+b_1}{1-b_2}v+(a_2+b_1)\frac{a_1+a_2+b_1}{1-b_2}v+b_2\Big(\frac{1}{M(T^5x_0,T^4x_0,t)}-1\Big).$$

Hence, $\frac{1}{M(T^5x_0,T^4x_0,t)}-1\le(\frac{\lambda}{\beta})^2v$. Also, by using the assumption and putting $x=T^3x_0$ and $y=T^4x_0$ in the condition, we obtain

$$\frac{1}{M(T^5x_0,T^6x_0,t)}-1\le a_1\Big(\frac{1}{M(T^3x_0,T^4x_0,t)}-1\Big)+a_2\Big(\frac{1}{M(T^4x_0,T^5x_0,t)}-1\Big)$$
$$+b_1\Big(\frac{1}{M(T^4x_0,T^5x_0,t)}-1\Big)+b_2\Big(\frac{1}{M(T^5x_0,T^6x_0,t)}-1\Big)$$
$$\le a_1\frac{\lambda}{\beta}v+(a_2+b_1)(\frac{\lambda}{\beta})^2v+b_2\Big(\frac{1}{M(T^5x_0,T^6x_0,t)}-1\Big)$$
$$=(\frac{\lambda}{\beta})^2\Big(a_1(\frac{\beta}{\lambda})v+(a_2+b_1)v+(\frac{\beta}{\lambda})^2b_2\Big(\frac{1}{M(T^5x_0,T^6x_0,t)}-1\Big)\Big)$$
$$\le(\frac{\lambda}{\beta})^2\Big(a_1(\frac{\beta}{\lambda})v+(\frac{\beta}{\lambda})(a_2+b_1)v+(\frac{\beta}{\lambda})^2b_2\Big(\frac{1}{M(T^5x_0,T^6x_0,t)}-1\Big)\Big)$$
$$=(\frac{\lambda}{\beta})^2\Big((\frac{\beta}{\lambda})(a_1+a_2+b_1)v+(\frac{\beta}{\lambda})^2b_2\Big(\frac{1}{M(T^5x_0,T^6x_0,t)}-1\Big)\Big)$$
$$\le(\frac{1}{\beta})^2\Big((\frac{\beta}{\lambda})(a_1+a_2+b_1)^3v+(\beta)^2b_2\Big(\frac{1}{M(T^5x_0,T^6x_0,t)}-1\Big)\Big)$$

which implies

$$(\frac{\lambda}{\beta})\beta^3\Big(\frac{1}{M(T^5x_0,T^6x_0,t)}-1\Big)\le(1-b_2)^3\Big(\frac{1}{M(T^5x_0,T^6x_0,t)}-1\Big)\le(\lambda)^3v.$$

Hence

$$\Big(\frac{1}{M(T^5x_0,T^6x_0,t)}-1\Big)\le(\frac{\lambda}{\beta})^3v.$$

By continuing the process, we get $(\frac{1}{M(T^5x_0,T^6x_0,t)}-1)\le(\frac{\lambda}{\beta})^{n-2}$ for all $n\ge3$. This implies

$$\frac{1}{M(T^mx_0,T^nx_0,t)}-1\le\sum_{i=m}^{n-1}\Big(\frac{1}{M(T^ix_0,T^{i+1}x_0,t)}-1\Big)\le\sum_{i=m}^{n-1}(\frac{\lambda}{\beta})^{i-2}v\le\frac{(\frac{\lambda}{\beta})^{m-2}}{1-(\frac{\lambda}{\beta})}v,$$

for all $n>m\ge3$. Hence $\{x_n\}$ is a Cauchy sequence. Since $(X,M,N,*,\Diamond)$ is a complete intuitionistic fuzzy metric space, there exists $u\in X$ such that $x_n\to u$. Since T is orbitally continuous, $Tx_n\to Tu$ and so $Tu=u$. Now, we show that T has a unique fixed point. Let y and z be fixed point of T. Then,

$$\frac{1}{M(y,z,t)}-1=\frac{1}{M(T^2y,T^2z,t)}-1\le a_1\Big(\frac{1}{M(Ty,y,t)}-1\Big)+a_2\Big(\frac{1}{M(Ty,T^2y,t)}-1\Big)$$
$$+b_1\Big(\frac{1}{M(z,Tz,t)}-1\Big)+b_2\Big(\frac{1}{M(Tz,T^2z,t)}-1\Big)$$

and so $y=z$. Now, we prove that $F(T)=F(T^2)$. Let $y\in F(T^2)$. Then, we have

$$\frac{1}{M(y,Ty,t)}-1=\frac{1}{M(T^2y,T^2Ty,t)}-1\le a_1\Big(\frac{1}{M(Ty,T^2y,t)}-1\Big)+a_2\Big(\frac{1}{M(T^2y,T^3y,t)}-1\Big)$$
$$+b_1\Big(\frac{1}{M(y,Ty,t)}-1\Big)+b_2\Big(\frac{1}{M(Ty,T^2y,t)}-1\Big)=(a_1+a_2+b_1+b_2)\Big(\frac{1}{M(y,Ty,t)}-1\Big).$$

Since $a+b<1$, we get $Ty=y$. □

Example 3.5. Let $X = \{1, 3, 5\}$, $d(x, y) = |x - y|$, $M(x, y, t) = \frac{t}{t+d(x,y)}$ and $N(x, y, t) = \frac{d(x,y)}{t+d(x,y)}$ for all $x, y \in X$ and $t \geq 0$. Define $\leq = \{(1, 1), (3, 3), (5, 5)\}$ and T be a selfmap on X by $T1 = 3$, $T3 = 1$ and $T5 = 5$. Then, by putting $x_0 = 5$, $a_1 = a_2 = b_1 = b_2 = \frac{1}{4}$, it is easy to see that T satisfies the condition of Theorem 3.4.

4. Conclusion

In this article, we provide fixed point results for some contractive mappings on complete ordered triangular intuitionistic fuzzy metric spaces. Also, we gave some results about the property (P). Our results are extensions of several results as in relevant items from the reference section of this paper, as well as in the literature in general. For stability results related to our fixed point research, please see [17], [35].

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