STUDY OF THE SOLITONS PROPAGATION THROUGH OPTICAL FIBERS

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Caracteristicile specifice: a) tipul neliniarității, b) proprietățile dispersive ale mediului de propagare, c) caracteristicile de coerență ale pulsurilor, d) ecuațiile de propagare, e) modalitățile de întreținere, f) aplicațiile tehnice, etc. au fost analizate atât pentru principalele tipuri de solitoni optici [spre ex. solitonii Schrödinger neliniar: (i) pătratici (QNLS), (ii) cubici (Kerr, CNLS), (iii) cu neliniaritate saturabilă (SNLS), (iv) cu neliniaritate discretizată (DNLS)], precum şi pentru unii solitoni acustici [în particular, cei de tipurile: (i) Korteweg-de Vries, (ii) Boussinesq, (iii) Burgers, (iv) sine-Gordon]. Un studiu detaliat al discretizărilor folosite în cadrul unor diferite tipuri de simulări numerice, precum şi compararea pulsurilor solitare de tipurile "clopot" (breather), respectiv "kink" a fost realizată de asemenea.

The specific features: a) non-linearity type, b) dispersive properties of the propagation medium, c) pulses coherence features, d) propagation equations, e) maintenance procedures, f) technical applications, etc. were studied both for some typical optical [e.g. the: (i) quadratic (QNLS), (ii) Kerr (cubic, CNLS), (iii) saturable (SNLS), (iv) discretized (DNLS), Schrödinger non-linear kind solitons] and for some acoustic [e.g.: (i) Korteweg-de Vries, (ii) Boussinesq, (iii) Burgers, (iv) sine-Gordon] solitary pulses. A detailed study of the discretizations used by the different numerical simulations and a comparison of the solitary pulses of the "bell" (breather) and "kink" types was also achieved.

Keywords: Non-linearity, dispersive properties, pulses coherence, optical (NLS) solitons, acoustic solitary waves, solitons propagation simulations

1. Introduction

As it is well-known, the first studies of the solitons transmission through optical fibers were achieved by the works [1], [2], while the demonstration of the compensated losses by means of: a) optical amplifiers doped with Er^{3+} ions, b) Raman gain, were accomplished by works [3] and [4], respectively.

Inside the paper introduction it will be specified the present stage of the researches in the studied field (being cited the corresponding references), indicating also the paper goals.

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2. Main types of physical properties of solitons propagation media

2.1. The non-linearity type

Figure 1 presents the basic types of non-linearity.

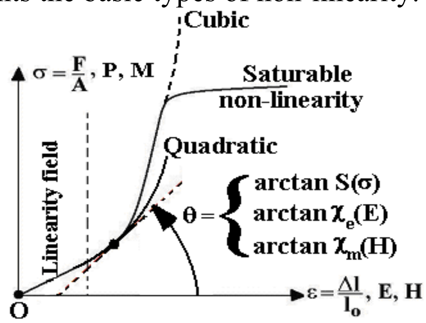

Fig. 1. Plots of stress / polarisation / magnetisation parameters vs. the strain / field intensities.

2.2. Basic properties of dispersive media

The basic physical properties of the optical fibers and their use in the optical communications were studied beginning from 1954 and with - many improved technological elements - in [5].

2.1. Balance of the non-linear and dispersive properties

We have to underline that - for the solitary waves - it is achieved a perfect balance between the non-linear and the dispersive properties. E.g. for the elementary case of the KdV solitons, described by the equations:

$$w = A\cdot\cosh^{-2}\Phi, \text{ where: } \Phi = \xi\left[x-\left(V_o+4d_1\cdot\xi^2\right)\right], \tag{1}$$

one obtains:

$$nw\cdot w' = -n\cdot 2A^2\xi\sinh\Phi\cdot\cosh^{-5}\Phi,\ d_1\cdot w''' = 8d_1A\cdot\xi^3\left[3\sinh\Phi\cdot\cosh^{-5}\Phi-\sinh\Phi\cdot\cosh^{-3}\Phi\right].$$

One finds that the non-linear term is neutralized by the dispersion term, in conditions when the pseudo-vector: $\xi = \sqrt{nA/12d_1}$, (2)

the (nonlinear) KdV equation being so rigorously fulfilled:

$$\dot{w} = -V_o w' - nw\cdot w' - d_1 w'''. \tag{3}$$

3. Coherence properties, generation and maintenance procedures

3.1. Coherence properties

According to the coherence theory, the spectral components of a pulse are coherent if $\frac{\Delta\omega}{\omega}\le\frac{T}{\tau}$, where T is the corresponding oscillations period, while τ is the observation time. The acoustic solitary waves correspond usually to a unique pulse, the (laser) optical solitons represent in fact (the envelope of) a wave group (see [5c], and figure 2).

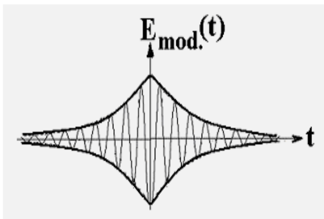

Fig. 2. Structure of an optical soliton.

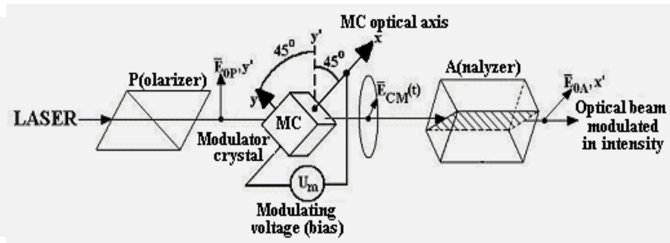

Fig. 3. Basic procedure of the optical solitons generation by means of the amplitude modulation of a light beam.

3.2. Generation procedures

While the acoustic solitons could result frequently as a consequence of some natural processes in water channels, oceans, Earth atmosphere, etc. [7], in order to be used (e.g. in the optical communications systems), the optical solitons represent some artifacts (they have to be generated).

3.3. Maintenance procedures

Unlike the "natural" solitons, which fit exactly the properties of the propagation medium, the "artificial" ones correspond to truncated physical properties, which approximate well (but not exactly) the true (exact) properties. That is why these solitons are subject to some (rather weak, but not null) attenuation processes, for curved trajectories, especially, their Schrödinger non-linear propagation equation being completed with some attenuation terms:

$$i\frac{\partial q}{\partial X}+\frac{1}{2}\cdot\frac{\partial^2 q}{\partial T^2}-|q|^2\cdot q=-i\Gamma\cdot q\,, \tag{4}$$

where Γ is the non-dimensional losses coefficient.

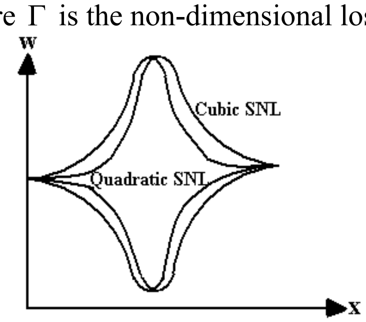

Fig. 4. Comparison of QSNL and CSNL breather standing wave.

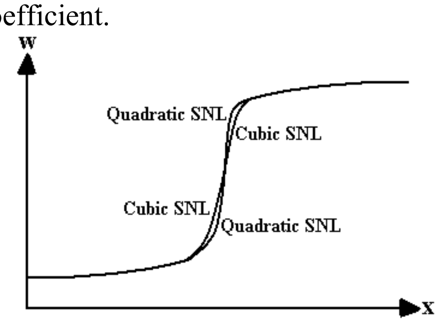

Fig. 5. Comparison of QSNL and CSNL kink standing wave.

In order to keep their amplitudes, the use of some optical amplifiers is necessary (see [8]).

Table 1

Basic features of the studied numerical simulations of the main type of optical and acoustic solitons, respectively

The type of solitary waves pulses	**Equation**	**Programming medium**	**References**
Optical	**Schrödinger with quadratic non-linearity, QNLS** $-2i\omega_\pm \partial_t w_\pm \mp c^2 \partial_z^2 w_\pm + \left(\omega^2 - \omega_\pm^2\right) w_\pm = \alpha_n \left\|w_\pm\right\|^{n-1} \cdot w_\pm$ **[**with $n = 2$, "+" = breather, "-" = kink (inflexion)]	**C**	*B. Denardo,* Non-analytic nonlinear oscillations: Christian Huygens, quadratic Schrödinger equations, and solitary waves, in J. Acoust. Soc. Am., **vol. 104**, no. 3, 1998, pp. 1289-1300.
	Schrödinger with cubic (Kerr) non-linearity, CNLS (the above equation, with $n = 3$) **Schrödinger with saturable non-linearity, SNLS**	**C**	a) *V. E. Zaharov, A. B. Shabat,* in Sov. Phys. JETP, **vol. 34**, 1972, p. 62; b) *S. Cowan et al,* in Can. J. Phys., **vol. 64**, 1986, p. 311; b) *A. W. Snyder, A. P. Sheppard*, in Opt. Lett., **vol. 18**, 1993, p. 482; c) *W. Krolikowski, S. A. Holmstrom*, in Opt. Lett., **vol. 22**, 1997, p. 369; d) *M. H. Jakubowski, K. Steiglitz, R. Squier*, in Phys. Rev. **E, vol. 56,** 1997, p. 7276; e) *W. Krolikowski, B. Luther-Davies, C. Denz*, in IEEE J. Quant. Electron., **vol. 39**, 2003, p. 3.
	Schrödinger with discretized saturable non-linearity, DNLS $i \cdot \dot{u}_n - \beta \dfrac{u_n}{1+\left\|u_n\right\|^2} + \left(u_{n+1} - 2u_n + u_{n+1}\right) = 0$	**C**	*J. Cuevas, J. C. Eilbeck,* Discrete soliton collisions in a wave-guide array with saturable non-linearity, in Physics Letters A, **vol. 358**, no. 1, 2006, pp. 15-20.
Acoustic	**Korteweg – de Vries** $\dot{w} = -V_0 w' - nw \cdot w' - d_1 w''$	**Fortran and C**	*R. H. Landau, M. K. Pàez,* Computational Physics. Problem solving with computers, John Wiley, 1997, pages 453, 459.
		Quick Basic	*D. Iordache, M. Scalerandi, V. Iordache,* et al, FD Simulations of the pulses propagation through non-homogeneous KdV media, in Proc. sci. session Acoustics comm. Romanian Academy, October 1998, pp. 121-26.
		Versions 11 and 12 of Maple (this work)	*A. Petrescu, A. R. Sterian, P. E. Sterian***,** Solitons Propagation in Optical Fibers Computer Experiments for Students Training, Lecture Notes in Computer Science, **vol. 4705**, 2007, pp. 450-461 (Book: Computational Science and its Applications - ICCSA 2007).
	Boussinesq $w_{tt} = c^2 w_{xx} + \dfrac{3c^2}{h^2}\left(w^2\right)_{xx} + \dfrac{h^2 c^2}{3} w_{xxx}$	**C**	*M. Scalerandi, M. Giordano, P. P. Delsanto, C. A. Condat*, private communication, 1995.
	Burgers $u_{tt} + uu_{xx} = \nu u_{xx}$	**C**	*S. de Lillo, P.P. Delsanto, M. Scalerandi*, Analytical and Numerical Results for the Forced Burger Equation, in Proc. Conf. Fluctuation Phenomena: disorder and nonlinearity, ed. by *A. R. Bishop et al.,* World Scientific, Singapore, 1995.
	sine-Gordon $\dfrac{\partial}{\partial x}\left(\dfrac{\partial}{\partial x} u(x,t)\right) - \dfrac{\partial}{\partial t}\left(\dfrac{\partial}{\partial t} u(x,t)\right) - \sin\left(u(x,t)\right) = 0$	**Fortran and C**	*R. H. Landau, M. K. Pàez,* Computational Physics. Problem solving with computers, John Wiley, 1997, pp. 497-498, and 455-456, respectively.

4. Main theoretical models and numerical simulations

4.1. Equations and plots of the main types of solitons

Table 2

Type of SNLE	Breather soliton	Kink soliton
Quadratic	$\frac{A}{2}\mathrm{sech}^2[\xi_+(x-V\cdot t-x_0)]\cdot\exp\left(\frac{i\omega Vx}{c^2}\right)$	$\frac{A}{2}\mathrm{sgn}\,\xi_-(x-V\cdot t-x_0)\cdot\{1+\chi\,\mathrm{sgn}(x-V\cdot t-x_0)-$ $-1.5\,\mathrm{sech}^2[\xi_-(x-V\cdot t-x_0)]\}\cdot\exp\left(-\frac{i\omega Vx}{c^2}\right)$
Cubic (Kerr type)	$\frac{A}{2}\mathrm{sech}[\xi_+(x-V\cdot t-x_0)]\cdot\exp\left(\frac{i\omega Vx}{c^2}\right)$	$\frac{A}{2}\tanh[\xi_-(x-V\cdot t-x_0)]\cdot\exp\left(-\frac{i\omega Vx}{c^2}\right)$

Breather and kink soliton types solutions of the quadratic and cubic SNL equations (SNLE)

4.2. Basic types of computer simulations of optical solitons propagation

The solution of the Schrödinger nonlinear equation of optical solitons is:

$$q(T,X)=\eta\cdot\mathrm{sec}\,h[\eta(T-k\cdot X-T_o)]\cdot\exp\left[-ikT-\frac{i}{2}\left(\eta^2-k^2\right)X+i\cdot\sigma_o\right], \quad (5)$$

where the non-dimensional parameters η, k, T_o and σ_o are determined by the initial and/or the boundary conditions of the optical soliton launching [10b]. We have chosen $\varepsilon = \eta = 1$ and $T_o=\sigma_o=0$:

$$E_{\mathrm{mod}}(t,x)=\sqrt{\frac{\pi}{2g\cdot n_2}}\cdot\mathrm{sech}\left(\frac{V_c t}{\lambda}-\frac{V_c}{V_s}\cdot\frac{x}{\lambda}\right)\cdot\cos\left[k\cdot\frac{V_c t}{\lambda}-\left(\frac{V_c}{V_g}+\frac{k^2-1}{2}\cdot\frac{x}{\lambda}\right)\right], \quad (6)$$

where V_c, V_s and V_g are the characteristic speeds to inside the optical fiber, to the optical soliton and the group-velocity, respectively (see [10b]). For studying the propagation and collisions of some KdV bell-type solitons and of some sine-Gordon kink-antikink solitons we used the powerful PC program Maple 12.

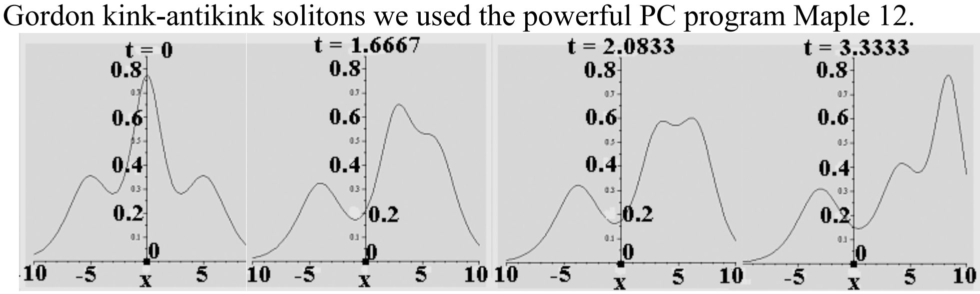

Fig. 6. Succession of snapshots for the propagation and collisions of 3 KDV bell-type solitons

5 Conclusions

The solitary waves fulfil the requirements of an extensive presence in nature (in atmosphere, plasmas, seas, in solid-state physics, proteins, human body, general relativity, high energy physics, etc. stimulating features for theoretical investigations, and many potentially practical applications in technology. Besides their extremely important technical applications in the field of optical communications where they are presently in a financial competition with the trend to reduce very much the energy losses in optical fibres. Starting from the basic features of the theoretical models of solitary pulses, this work obtained a certain classification of the most important types of solitons. The existing numerical simulations were also studied in detail and some comparisons of the "bell" and "kink" type soliton solutions and evolutions were pointed out.

REFERENCES

[1] *A. Hasegawa, F. Tappert,* Transmission of stationary nonlinear optical pulses in dispersive dielectric fibers. I. Anomalous dispersion, in Appl. Phys. Lett., **vol. 23**, no. 3, 1973, pp. 142-144.

[2] *L. F. Mollenauer*, *R. H. Stolen,* Experimental observation of picosecond pulse narrowing and solitons in optical fibers, in Phys. Rev. Lett., **vol. 45,** 1980, pp. 1095-1098.

[3] *R. J. Mears et al.,* in Electron. Lett., **vol. 22**, 1986, p. 160.

[4] *L. F. Mollenauer, K. Smith,* Demonstration of soliton transmission over more than 4000 km in fiber with loss periodically compensated by Raman gain, in Opt. Lett., **vol. 13**, 1988, pp. 675-677.

[5] a) *A. Preda*, Introducere în electronica cuantică, Editura Stiinţifică, Bucureşti, 1995; b) *A. Preda,* Instruments et matériaux opto-électroniques, Univ. "Politehnica" Bucureşti, 1998; c) *S. Şişianu, T. Şişianu, O. Lupan*, Comunicaţii prin fibre optice, Editura Tehnica-Info, Chişinău, 2003.

[6] *E. Bodegom, D. Iordache*, Physics for Engineering Students, vol. 1, 2007, and vol. 2, 2008, Politehnica Publishing House, Bucharest.

[7] a) *J. S. Russell,* Report on waves, 14th Meeting of the British Association for the Advancement of Science, John Murray Publ. House, London, 1844; b) *R. Herman,* Solitary waves, in American Scientist, **vol. 80**, July-August 1992, pp. 350-361.

[8] *G. Keiser,* Optical Fiber Communications, chap. 2, McGraw Hill, 2001.

[9] *D. Iordache, M. Scalerandi, C. Rugină, V. Iordache,* Study of the Stability and Convergence of FD Simulations of Ultrasound Propagation through Nonhomogeneous Classical (Zener's) Attenuative Media, in Romanian Reports on Physics, **vol. 50**, no. 10, 1998, pp. 703-716.

[10] a) *D. A. Iordache,* Contributions to the Study of Numerical Phenomena intervening in the Computer Simulations of some Physical Processes, Credis Printing House, Bucharest, 2004; b) *D. A. Iordache,* De la haos la ordine. Undele solitare (solitonii), in Evrika, **vol. 18**, no. 211, 2008, p. 7-15.

[11] *A. Petrescu, A. R. Sterian, P. E. Sterian***,** Solitons Propagation in Optical Fibers Computer Experiments for Students Training, Lecture Notes in Computer Science, **vol. 4705**, 2007, pp. 450-461 (Book: Computational Science and its Applications - ICCSA 2007).

[12] *H. Gnewuch, E. Smeu, D.A. Jackson, A. Gh. Podoleanu*, Long range extensometer for civil structure monitoring using fibre Bragg gratings, Meas. Sci. Technol., Vol. 16, (2005), pp. 2205-2010.

[13] *D. Mihalache, D. Mazilu, F. Lederer, and Y. S. Kivshar,* Opt. Express 15, 589 2007.

[14] *D. Mihalache, D. Mazilu, F. Lederer, and Y. S. Kivshar,* Opt. Lett. 32, 2091 2007.

[15] *Paul. E. Sterian,* Fotonica, Editura Printech, Bucureşti 2000.

[16] *D. A. Iordache, Şt. Puşcă, C. Toma, V. Păun, A. Sterian, C. Morărescu,* Analysis of the compatibility with the experimental data of fractal descriptions of fracture parameters, Lecture Notes in Computer Science (LNCS) 3980, 804-813(2006), Proceedings of the International Conference on "Computational Science and its Applications" (ICCSA-2006), Glasgow, UK, May 8-11, 2006 (Marina Gavrilova et al., eds.), Springer-Verlag, Berlin-Heidelberg, 2006.