U.P.B. Sci. Bull. Series D, Vol. 71, Iss. 4, 2009 ISSN 1454-2358

METHOD OF CALCULUS FOR THE POWER INPUT OF THE HELICAL SCREW COMPRESSOR

Dan Codruț PETRILEAN[1]

Lucrarea propune o metodă de determinare a puterii absorbite de compresorul elicoidal. Se cunoaşte metoda clasică, însă metoda grafo-analitică, procedeul grafic este urmat de un model matematic curgeriii gazului prin compressor. Rezultatele obținute se aplică pentru validarea puterii instalate la compresorul elicoidal în timpul funcționării. Valorile obținute prin calcul prin diferite metode sunt apropiate.

The paper proposes a method for power absorption determination of the helical screw compressor. The classic method is known, but at the graphical-analytical method, the graphical procedure is followed by a mathematical model of the gas flow in the compressor. The obtained results are applied for validation of the installed power at a running helical screw compressor. The values obtained by calculus through different methods were very close

Keywords: helical screw compressor, graphical-analytical method, indicated power

1. Introduction

The calculus of a helical screw compressor power input is based on the following parameters:

Piston displacement *V* which is expressed in m^3/m;

The peripheral speed of the main rotor:

$$u = \frac{\pi \cdot D \cdot n}{60}\left[\frac{m}{s}\right] \tag{1}$$

(*D* in meter, and *n* in rot/min).

The theoretical discharge volume:

$$\dot{V}_T = V \cdot u \left[\frac{m^3}{s}\right] \tag{2}$$

The effective discharge which takes in consideration the loss sum by internal and external flowing, dynamic flowing and mechanical flowing is [1]:

[1] Lecturer, PhD, Faculty of Mechanical Engineering and Electrical, University of Petroşani, Romania, petrilean1975@yahoo.com

$$\dot{V}_{ef} = \dot{V}_T - \sum L \left[\frac{m^3}{s} \right] \tag{3}$$

There are four leakage paths for gas to pass from the high pressure end of the compressor to the low pressure end. These are: 1) between the meshing lobes; 2) through the blow hole; 3) across the rotor tips and 4) across the rotor discharge end face.

The external losses refer at the sealing up system of the axes of the two rotors. Through here they produce losses by flowing at input and repression in low pressure spaces.

The volume capacity of the helical screw compressor is:

$$\eta_V = \frac{\dot{V}_{ef}}{\dot{V}_T} \tag{4}$$

These internal and external losses are dependent on the Mach number:

$$Ma = \frac{u}{w_s}; \qquad w_s = (k \cdot R \cdot T_1)^{0,5}$$

The dynamic losses are made by the flowing and air friction in the compressor. They are expressed by the dynamical efficiency δ. Relationship between the two efficiencies is:

$$\eta_i = \eta_v \cdot \delta$$

This efficiency decreases if the peripheral speed increases. The two efficiencies are joining in a point (fig. 1). The optimum value for the internal efficiency η_i is near the intersection of these two curves η_v and δ. This represents the optimum peripheral speed:

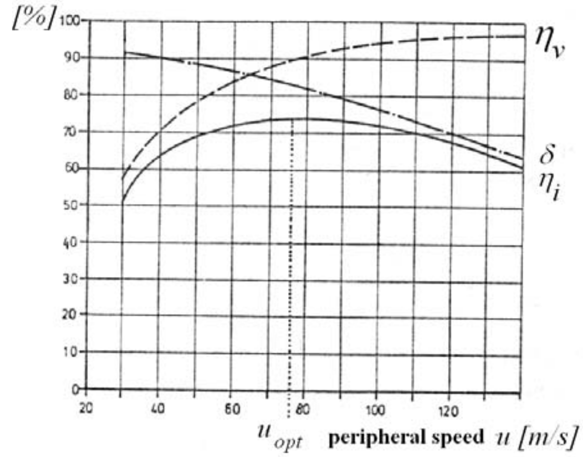

Fig. 1. The efficiency variation in function of the peripheral speed [6]

2. The applied method

2.1. The helical screw compressor power determination by calculus diagram

The powers configuration and energy losses for the compressor are showed by surfaces in Fig. 2:

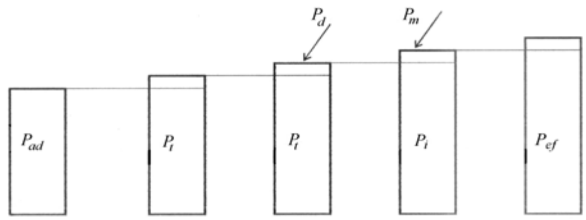

Fig. 2. Helical screw compressor power configuration [6]

If the compression is polytropic, then the isentropic efficiency η_s may be considered as an internal efficiency η_i following the relationship:

$$\eta_s = \eta_i = \frac{\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} - 1}{\left(\frac{p_2}{p_1}\right)^{\frac{1}{\eta_p}\cdot\frac{k-1}{k}} - 1}$$

where η_p is the polytropic efficiency at compression.

In table 1 are presented the values of the polytropic efficiency values as a function of the compression polytropic exponent:

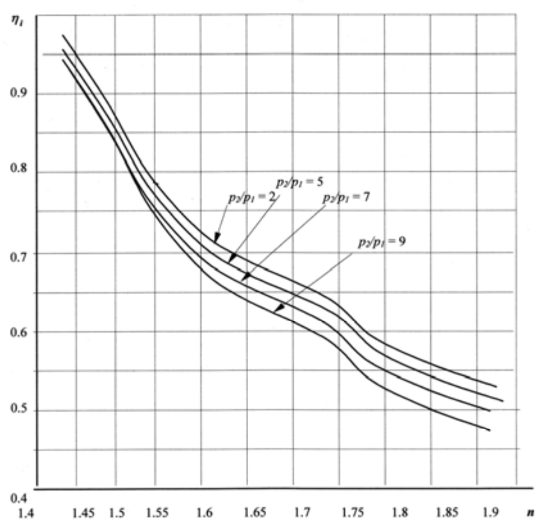

Fig. 3. The η_i value in function of the polytropic exponent and the compressing ratio

Table 1

The polytropic efficiency's values in function of the compressing polytropic exponent

n	1.4	1.5	1.55	1.6	1.65	1.7	1.75	1.8	1.9	2
η_p	1	0.921	0.858	0.806	0.762	0.694	0.667	0.662	0.603	0.57

The internal efficiency $\eta_i = \eta_S$ it is taken from fig. 3:
The compressor efficiency is:

$$\eta = \eta_i \cdot \eta_m \tag{8}$$

The adiabatic input power of the compressor is given by the following relation:

$$P_{ad} = \frac{k}{k-1} \cdot p_0 \cdot \dot{V}_T \cdot \eta_V \left[\left(\frac{p_2}{p_1} \right)^{\frac{k-1}{k}} - 1 \right] \; [W] \tag{9}$$

$$\dot{V}_T = \dot{V}_{max} \cdot z_1 \cdot n_1 \tag{10}$$

$\dot{V}_{max}$ - is the maximum value of the capacity in double quantity;
z_1 – number of lobs of the main rotor;
n_1 – main rotor frequency.

$\dot{V}_{max}$ value that can be determined only from the designed project.
The theoretical power P_t is not considering the volume losses:

$$P_t = \frac{P_{ad}}{\eta_V} \tag{11}$$

The theoretical power may be obtained from figure 4, in which the following parameters are included: the compress ratio $\pi = p_2 / p_1 = 1.5 \div 6$; adiabatic exponent: k = 1.1÷1.7 (for different gases); compressor suction pressure: p_1 = 0.1-10 bar; theoretical volumetric flow rate $\dot{V}_T = 1\, m^3/s$; the inlet air temperature $t_0 = 0\ ^0$C.

If the temperature is different from 0^0C, the coefficient $f_1 = \frac{T_1}{T_0} = \frac{273 + t_1}{273}$ is introduced.

The indicated power may be determined using an algorithm that is further presented.

The method is based on the following system of differential equation[1]:

$$dp = \frac{m \cdot R \cdot dT + R \cdot T \cdot dm - p \cdot dV}{V} \tag{12}$$

$$dT = dT_{ad} + dT_{in} + dT_{cav} + dT_T \tag{13}$$

The first equation expresses the pressure dependence on temperature and volume of the gas. The current value of the volume is a function of the rotation angle of the driver rotor and it is determined in an usual way by a graphical – analytical method.

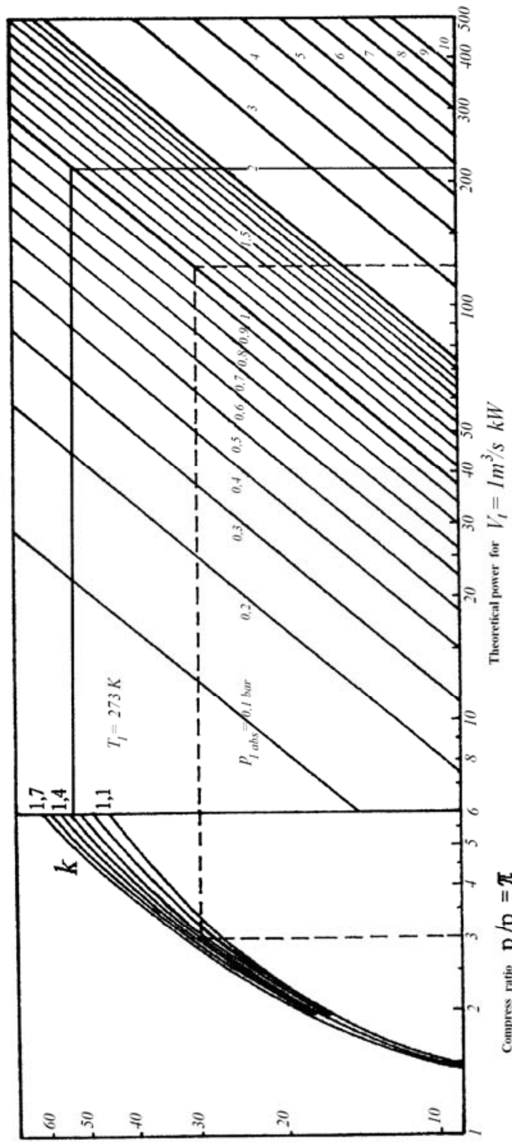

Fig. 4. Nomogram for power calculus

The total differential of the gas modification mass is equal to the algebraically differential of the partial differential of the input and output components:

$$dm = dm_i - dm_e \tag{14}$$

The right part of the equation (13) is developed in the following relations. The modification of the adiabatic temperature is determined by (15):

$$dT_{ad} = \frac{k-1}{k} \cdot \frac{T}{p} dp \tag{15}$$

Equation (16) represents the input result of the gas with another temperature *i*:

$$dT_{in} = \frac{\left[m_i T_i + d\left(m_i \cdot T_{i+1}\right)\right]_{in}}{m_i + dm_i} - T_i \tag{16}$$

The temperature modification based on the gas mass from the cavity is represented as:

$$dT_{cav} = (k-1)\frac{T}{m}\left(dm_{in} - dm_{cav}\right) \tag{17}$$

The temperatures modification due to the heat exchange with the compressor walls is:

$$dT_T = \alpha \cdot F_T \left(T - T_m\right) \frac{d\tau}{c_v \cdot m} \tag{18}$$

where: α is the heat exchange coefficient; F_T – surface area of the cavity from the gas; T_m –average temperature of the gas in the cavity at time $d\tau$; c_v – specific mass heat at constant volume; This term (dT_T) is usually neglected.

The obtained system of equations is solved by numerical methods.

The η_V coefficient is obtained as:

$$\eta_V = \left(\frac{T_{asp}}{T_0}\right)\left(1 - \sum\sum \frac{\Delta m_a}{m_0}\right) \tag{19}$$

where: T_{asp} is the air temperature at the end of the intake process.

$\sum\sum \Delta m_a$ - number of internal drains which reach at the aspiration from all compression and repression cavities during the whole period of the process.

The indicated power is obtained from the diagram in Fig. 5:

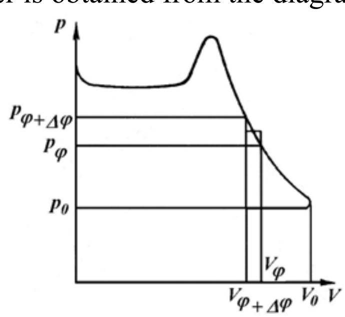

Fig. 5. Indicated diagram by calculus [1]

$$S_i = 0{,}5 \sum \sum \left(p_{\varphi_1} + p_{\varphi_{1+\Delta\varphi_1}} \right) \cdot V_{\varphi 1+\Delta\varphi 1} - p_0 \cdot V_0 \tag{20}$$

where: p_0 is the pressure at the end of the aspiration process;

V_0 – the theoretical volume of the double cavity at the end of the aspiration;

$\Delta\varphi_1$ - drive rotor rotational angle in each limits of cavities, which is divided in several gaps.

Therefore, the indicated power is:

$$P_i = S_i \cdot u \cdot z_1 \tag{21}$$

2.2. The helical screw compressor power determination with the graphical –analytical method

The link between power and variables, which this depends, may be expresses through:

$$P_i = f\left(\pi, n, M, k, c_p, \mu, \eta_V, p_1, T_1\right) \tag{22}$$

Using the criteria equations [3] we obtain:

$$P_i = \frac{k}{k-1}\left(\eta_V \cdot M_a\right) \cdot S_a \cdot w_s \cdot p_1 \cdot \left(\pi^{\frac{k-1}{k}} - 1 \right) \cdot \frac{10^{-3}}{\eta_{ad}} \tag{23}$$

where: $\left(\eta_V \cdot M_a\right)$ represent a dimensionless complex; S_a – conventional aspiration orifice surface.

$$S_a = (8-10)\frac{\dot{V}_1}{w_s} \quad \left[m^2\right] \tag{24}$$

obtained easily.

The efficiencies η_V and η_{ad} are obtained easy from Fig. 6 [3]:

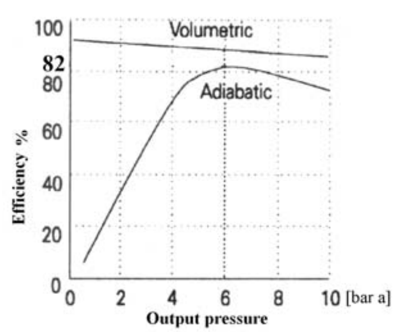

Fig. 6. Adiabatic and volumetric efficiency variation [3]

The effective power is: $P_{ef} = \dfrac{P_i}{\eta_m}$

The mechanical efficiency is considered $\eta_m = 0.98$ for rolling bearings.

The lost dynamical power is $P_d = P_i - P_T$, or one can determine it from Fig. 7:

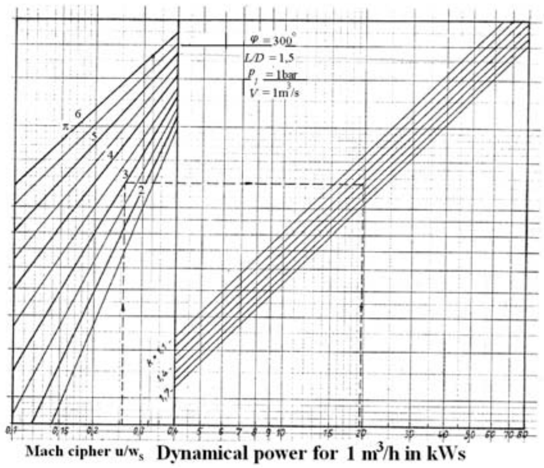

Fig. 7. Nomogram for lost dynamical power [6]

3. Application paragraf

The presented methdology is exemplified on the helical screw compressor GA 250 ATLAS COPCO having the following parameters: nominal flow 43.5 ±4% m^3/min; nominal power 250 kW; air aspiration pressure p_1 =1 bar; aspiration temperature t_1 = 10 ^{0}C; maximum work pressure 10 bar; speed revolution for the asynchronous motor 1490 rot./min.

The energy consumption at an air installation with a helical screw compressor is modified depending on the air parameters at aspiration and pressure variation in the network in which it works.

The absorbed power from the network is calculated through two different methods. The obtained results are approximate.

a) The absorbed power from the network is determined with the classical method. The working parameters of the compressor are: pressure ratio $\pi = p_2 / p_1 = 5$; suction temperature: t_1 =10 ^{0}C; suction pressure: p_1 = 1 bar; the effective flow was measured with the flow - meter. Diaphragm dimensions: d = 45 mm; D = 52 mm; differential pressure Δp = 500 mmHg; $\dot{V} = 0.6943\ m^3/s$ [5].

Polytropic compression was chosen $n = 1.6$:

$$l_{pol} = \frac{n}{n-1} p_1 \left[\pi^{\frac{n-1}{n}} - 1 \right] = \frac{1.6}{0.6} \cdot 1 \cdot 10^5 \left[5^{0.375} - 1 \right] = 2.18 \cdot 10^5 \ \frac{J}{m^3}$$

Indicated power:

$$P_i = \frac{l_{pol} \cdot \dot{V}}{60 \cdot \eta_p} = \frac{2.18 \cdot 10^5 \cdot 41.66}{60 \cdot 0.806} = 187.7 kW$$

Polytropic efficiency was chosen from table 1, $\eta_p = 0.806$.
The effective power:

$$P_{ef} = \frac{P_i}{\eta_m} = \frac{187.7}{0.98} = 191.61 \quad kW$$

In isentropic processes:

$$l_{ad} = 3.5 \cdot 1 \cdot 10^5 \left(5^{0.286} - 1 \right) = 2.044 \cdot 10^5 \quad \frac{J}{m^3}$$

$$P_i = \frac{l_{ad} \cdot \dot{V}}{60 \cdot \eta_S} = \frac{2.044 \cdot 10^5 \cdot 41.66}{60 \cdot 1} = 141.92 \, kW \, ; \; P_{ef} = \frac{P_i}{\eta_m} = 144.8 \, kW$$

b) The graphical – analytical method:

The correctional coefficient for suction temperature is: $f_1 = \frac{273 + 10}{273} = 1.036$. Sound speed at the compressor's input:

$$w_s = \left(1.4 \cdot 284 \cdot 283 \right)^{0,5} = 337.2 \, m/s$$

Choosing the peripheral speed u = 100 m/s, from the nomogram for the lost dynamical power calculus (figure 7), for $Ma = 0.29$ is obtained:

$$P_d = 42 \cdot 0.6943 \cdot 1.036 = 30.21 kW$$

The orifice surface for suction is:

$$S_a = 9 \cdot \frac{0.694}{337.2} = 0.0185 \, m^2 \, ; \eta_V \cdot Ma = \frac{\dot{V}_1}{w_s \cdot S_a} = \frac{0.694}{337.2 \cdot 0.0185} = 0.111$$

From Fig. 6 it results the value $\eta_{ad} = 0.82$
The indicated power is determined with (23):

$$P_i = 3.5 \cdot 0.111 \cdot 0.0185 \cdot 337.2 \cdot 10^5 \left(1.584 - 1 \right) \cdot \frac{10^{-3}}{0.82} = 172.59 \, kW$$

The indicated power for the volumetric flow $0.694 \, m^3/s$, considering the suction temperature:

$$P_i = P_{t10^0} + P_d = (190 + 42)1.036 \cdot 0.6943 = 167.52\ kW$$

The effective power is:

$$P_{ef} = \frac{P_i}{\eta_m} = \frac{172.59}{0.98} = 176.11\,kW$$

4. Conclusions

1. Comparing these two results obtained from different methods, we observe a difference of 5 *kW*.

2. The obtained results for validating the indicated power of the helical screw compressor shows a running limit which is established from design.

3. The calculus algorithm has a theoretical and practical importance, allowing a quick approximation of the input power for different working regimes.

R E F E R E N C E S

[1]. *P.A.Amosov ,* -Vintovie compresornie mashinii (Screw Compression Machines – Handbook), Mashinostroienie, Leningrad, 1977(in Russian)

[2] *M. Marinescu, D. Ştefănescu, I. Ganea*, Termogazodinamica Tehnică Termogazodynamic Technical, Technical Publishing, Bucharest, 1986)(in Romanian)

[3]. *P. O`Neill,* - Industrial compressors, Butterworth-Heinemann London, 1993

[4]. *I. Iulian Irimie, Matei I.* – Gazodinamica rețelelor pneumatice - Metode de calcul (The gas dynamics of the pneumatic network, Calculation methods), Technical Publishing House, Bucharest 1994)(in Romanian)

[5]. *D. C. Petrilean* – " Cercetări privind îmbunătățirea eficienței producerii şi utilizării energiei pneumatice în industria minieră" (Investigation regarding the impruving of efficiency for producing and using the pneumatic energy in the mining industry) In: PhD thesis, University of Petroşani, 2005)(in Romanian)

[6]. *D. C. Petrilean,* Compresoare elicoidale (Helical Screw Compressors) Technical -Info Publishing House, Chişinău, 2006

[7]. *Anca Alexandra Purcarea, Elena Fleaca* - Assessing the quality of labour relationships: an empirical study in energy sector**,** U.P.B. Sci. Bull., Series D, Vol. 69, No. 3, 2007

[8]. *A. Costache, N. Băran* - Computation method for establishing the contour of a new type of profiled rotor 93**,** U.P.B. Sci. Bull., Series D, Vol. 70, No. 3, 2008

[9] *N. Baran, Gh. Baran, Despina Duminica, D. Besnea* - Research regarding the profile of a rotating piston used in the construction of a new type of volumetric pump, U.P.B. Sci. Bull., Series D, Vol. 68, No. 4, 2006

[10]. *N. Stosic, I. Smith, A. Kovacevic* – Screw Compressors- Mathematical modelling and Performance Calculation- Springer Verlag Berlin Heidelberg, 2005

[11]. *D. Stanciu, Al. Dobrovicescu*, Numerical prediction of heat transfer on transonic turbine blades at off design operating conditions**,** U.P.B. Sci. Bull., Series D, Vol. 71, Iss. 1, 2009