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ECONOPHYSICS APPROACH TO THE DYNAMICS OF THE ROMANIAN EXCHANGE RATE ROL-USD

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Analiza cursului de schimb ROL-USD pe o perioadă de zece ani, de la începutul anului 1991 până la apariţia unei anumite stabilităţi (sfârşitul lui 2000) pune în evidenţă o dimensiune fractală clară a seriei temporale. În această perioadă, considerată perioadă de tranziţie, am constatat prezenţa unor celule caracteristice, care reproduc forma generală a semnalului la diferite scări de analiză, adică am pus în evidenţă o auto-similaritate teoretică. Celulele observate au caracteristici fractale apropiate, sugerând existenţa unui tip de dinamică neliniară dominant monofractală, caracteristică deprecierii puternice a monedei naţionale din perioada de tranziţia spre o economie de piaţă deschisă..

We found a definite fractal dimension of the temporal series describing the daily exchange rate of the Romanian "Leu" (ROL) with respect to the US Dollar (USD) over ten years, from the beginning of 1991 until a certain stabilization occurred (end of 2000); hereafter, we call this the transition period. Besides the statistical self-similarity, unlike other structures reported in the literature, we found repetitive cells that replicate the whole temporal pattern i.e. the evidence of a theoretical self-similarity . The fact that we were able to identify several scaling cells demonstrates the dominant monofractal behaviour of the dynamics during the period of strong depreciation of the national currency that characterizes the transition toward the open market economy.

Keywords: self-similarity, fractal dimension, Hurst exponent, scaling cells

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1. Introduction

Econophysics [1,2] is the newely growing science that applies the well known laws of the Universe to the smaller social system, particularly to the economics dynamics.

Recently, the fractals gained considerable importance because of their universal characteristics. The concept of fractals is most often associated with geometrical objects satisfying two criteria: self-similarity and fractional dimensionality [3]. Self-similarity means that an object is composed of sub-units and sub-sub-units on multiple levels that resemble the structure of the whole object as a result of a non linear underlying dynamics. A fractal object has a distribution of values consisting of a few large values, many medium values, a large number of small and a hugh number of very small values. Mathematically, this property should hold on all scales. However, in the real world, there are necessarily lower and upper bounds over which such self-similar behaviour applies. Thus, the data has a power-law distribution (i.e. the log-log plot of the values versus their relative frequency will be a straight line). If the plot is linear, the data are monofractal. If the plot is nonlinear, the data are multifractal.

The exponential-like growth pattern of a certain quantity usually occurs where things are out of order: it is exactly the case of the economic transitions, when the monetary system is trying to overcome first a collapsing, and then a fragile economy. The net result is a huge inflation. This is masquing all other fractal dynamics and only a dominant monofractal structure is obvious. The Romanian economic system seems to be an appropriate example of the situation and the ROL-USD exchange rate is following an exponential-like trend ([4]).

2. Theoretical self-similarity versus statistical self-similarity

Referring to a certain process, depicted by the lower-bounded quantity $y(t)$, we say it is self-similar if it is possible to identify cells with increasing lengths $(0, T_1)\subset(0, T_2)\subset(0, T_3)\subset\ldots$, with $T_2=\lambda T_1$, $T_3=\lambda^2 T_1,...$, $(\lambda>1)$ and, for any $t_1\in(0, T_1)$ in the smallest cell, one can find a point $t_2\in(T_1, T_2)$ in the immediately greater one, and a point $t_3\in(T_2, T_3)$ in the following one, and so on, so that the size of the samples is scaling according to the formula:

$$y(t_1)=\lambda^{-H}\cdot y(t_2)=\lambda^{-2H}\cdot y(t_3)=\ldots, \qquad (1)$$

with H the Hurst exponent [5].

One has to note that the quantity $y(n)$ is not the measured one. By analogy with the Brownian motion [6], if $x(n)$ is the time series of the measured samples (e.g. the ROL-USD exchange rate), then a suitable choice for $y(n)$ is the fluctuation of the partial sum of x (i.e. the averaged value over all positions n_0 of the root mean square of $s(n)=\sum_{k=1}^{n} x(k)$)

$$y(n)=\left\{ \overline{\left[s(n_0+n)-s(n_0)\right]^2} - \left[\overline{s(n_0+n)-s(n_0)}\right]^2 \right\}^{\frac{1}{2}}. \quad (2)$$

Unlike the usual situations, here no intermediate step of partial summation is necessary because, the huge inflation apparently causes the lack of an upper bond of the series, at least for the transition interval. Thus, instead of $s(n)$ we can directly take $x(n)$.

The theoretical self-similarity is consistent with relation (1) and the supplemental conditions $t_2=\lambda t_1$, $t_3=\lambda^2 t_1$, …, i.e. the scaling samples are in the same relative position in every cell

$$y(t)=\lambda^{-H}\cdot y(\lambda t)=\lambda^{-2H}\cdot y\left(\lambda^2 t\right)=\dots. \quad (3)$$

From a different point of view, we can speak about a statistical self similarity, with the samples being the same as above, but in different positions, according to the causality of the underlying dynamics; however, it’s worth noting that the statistics is the same both for theoretical and statistical analysis. If $y(t)$ is the collection of samples belonging to the cell $(0,T_1)$, and $p(y,t)\big|_{t\in(0,T_1)}$ is its probability distribution function (PDF), then the statistical self-similarity implies the equality of the elementary probabilities

$$\underbrace{p(y,t)\Big|_{t\in(0,T_1)}}_{\text{PDF in the smallest cell}} = \lambda^{-H}\cdot \underbrace{p\left(\lambda^H y,\, \lambda t\right)\Big|_{t\in(T_1,T_2)}}_{\text{PDF in the larger cell}} = \lambda^{-2H}\cdot \underbrace{p\left(\lambda^{2H} y,\, \lambda^2 t\right)\Big|_{t\in(T_2,T_3)}}_{\text{PDF in the following, larger cell}} = .. \quad (4)$$

These relationships indicate that the statistical properties at time scale t are related to the statistical properties at time scale λt.

In practice, however, it is impossible to determine whether two processes are statistically identical, because this strict criterion requires their having identical distribution functions (including not just the mean and variance, but all higher moments as well). Therefore, one usually approximates this equality with a weaker criterion by examining only the means and variances (first and second moments) of the distribution functions. Thus, we say the process is self-similar if any transformation that simultaneously changes the time scale by λ, and the quantity scale by λ^H, i.e. $t \to \lambda t$, $y \to \lambda^H y$, produces a process related to the genuine one by the probabilistic relations (4). It is clear that the statistical self-similarity is not obvious and requires some processing in order to emphasize it. Briefly, the statistical self similarity is weaker than the theoretical one, since it is accounting only for the values, not for the temporal succession. It is far more difficult to prove the existence of the theoretical self similarity (like the Koch curve, the Sierpinski gasket, etc. [3]) than the statistical self-similarity.

Now, for discrete samples $t \to n$, a non trivial solution for (3) is the power function

$$y(n) = \text{constant} \cdot n^H . \tag{5}$$

A value of H=0.5 results from Brownian motion like time series. The Hurst exponent is also related to the fractal dimension D [7]:

$$D=2-H. \tag{6}$$

The fractal dimension D is calculated as the limit

$$D = \lim_{r \to 0} \left| \frac{\log N(r)}{\log r} \right| , \tag{7}$$

where $N(r)$ is number of balls of radius r necessary to cover the curve [3].

It is obvious that the relation between the scaling parameter λ on the horizontal time axis and the scaling parameter η and on the vertical axis is given by

$$\log \eta = H \log \lambda . \tag{8}$$

3. Data acquisition: focusing on the transition epoque

We restrict the time interval subjected to our analysis to January 1991-December 2000 (Fig.1) because of two reasons: *i*/ the entire year 1990 exhibits very peculiar dependence of the characteristic quantities (e.g. in the logarithmic representation exhibits a steeping step by step increase, see Fig.1), so we drop it out; *ii*/ the year 2000 is the beginning of the plateau for the EBRD[7] aggregate index [8], so, at least from the point of view of the deepness of the structural changes, they came to end.

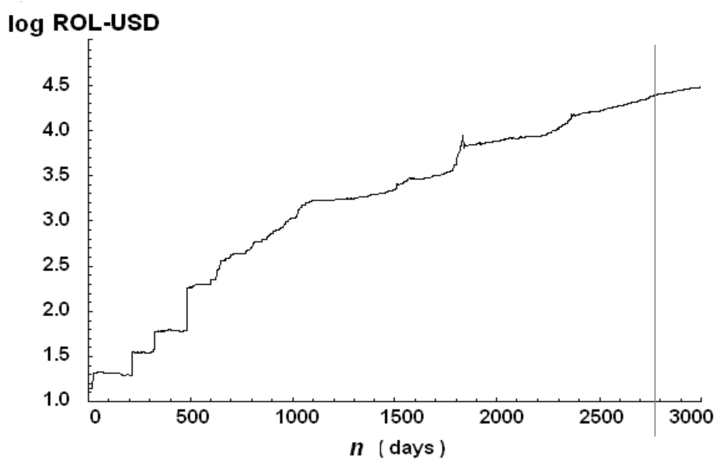

Fig. 1 The exchange rate in the transition époque, logarithmic reprezentation

4. Results

4.1 Fractal dimension

As usually, “holes” from weekends and holidays are ignored and analysis concentrates on trading days. Since 1998, when the Central Bank has announced the daily average parities of ROL with respect to the most important international currencies, we took these numerical values [9], while for the previous interval we took the interbank exchange rate. The absolute values of ROL are expressed in nowadays denominated values.

[7] European Bank for Reconstruction and Development

According to Eq.(7), the fractal dimension was computed as box-counting dimension: the numerical value of D=1.22 is confirmed by FracLab calculus with an error of less than 3%. The log-log plot for 12 different radii is shown in Fig.2.

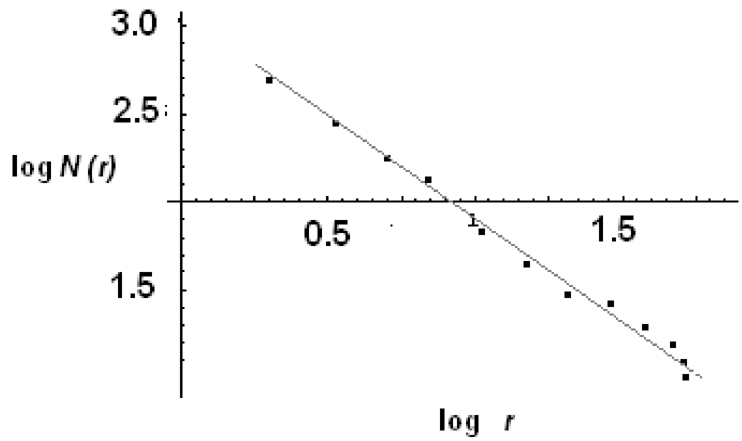

Fig. 2 The fractal dimension computed with the box counting method

Following the well known detrended fluctuation analysis (DFA) test [10] for $y(n)$ we obtain a quite significant deviation from the true Brownian random walk since the Hurst exponent has a value far enough from 0.5 and is confirming the persistence and the long range correlation emerging from the increasing trend of the experimental series. The value H=0.74 is calculated for the interval between samples no.258 and no.2768 corresponding to Jan. 1991-Dec. 2000 (delimited by the dashed vertical lines in Fig.3). The figure confirms the dominant monofractal structure and justifies the dropping out of the year 1990.

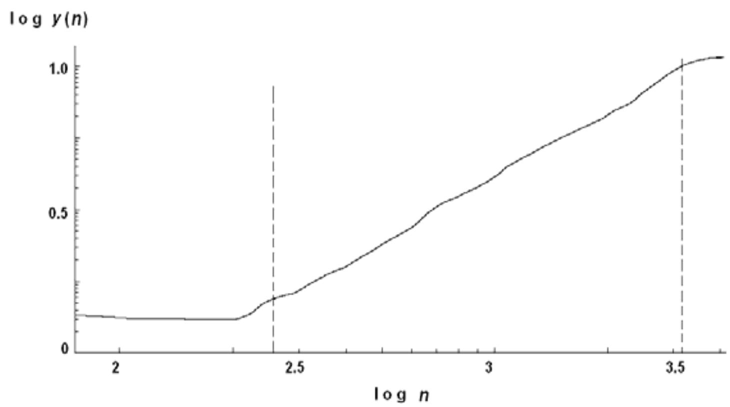

Fig.3 Log-log plot of the fluctuations versus time (n)

4.2 Evidence of self-similar structure

Focusing our attention upon the transition époque, we can isolate some characteristic structure (Fig.4). The most representative shapes are enclosed in the rectangles.

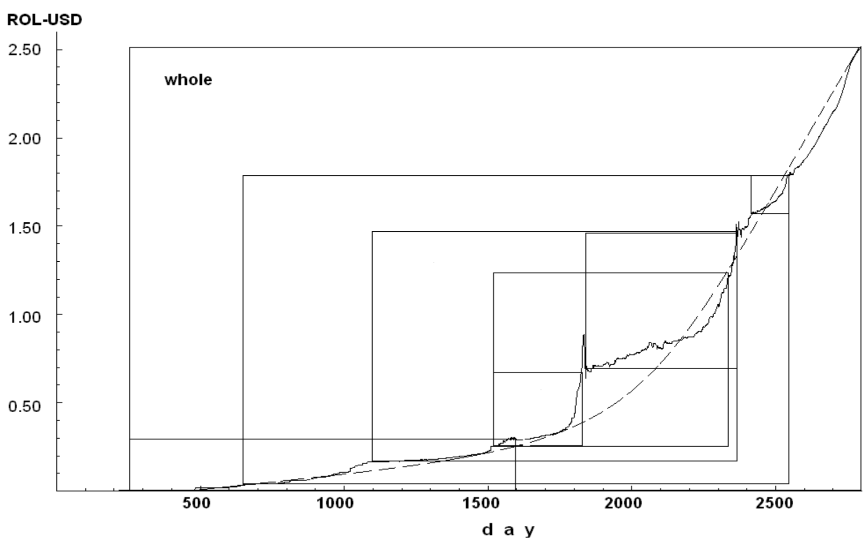

Fig.4 Self-similar structure of the whole time interval with several representative cells

The characteristic exponential-like behaviour of the "whole" interval is also exhibited by the smaller cells but at this scale, they are delimited by sharp angular points.

We make the hypothesis that the real self-similar shape of the exchange rate is the consequence of the economic mechanisms driven by the interest rate. However, a more detailed explanation is necessary, and this will be based on the assumption that in the transition period, the expectations of the economic agents, ranging from the simple human being to large companies, are governed by oversized safety coefficients for the forecasting procedures related to their financial plans.

4.3 Several representatives of the self-similar cells

In Table 1 we summarized the self-similar cells identified in Fig.4.

Table 1

		Window size Δn	**Corresponding size** Δx	**Stand. dev.** ($\times 10^{-3}$)	**Hurst exponent**
Whole		2768–258=2510	2.500–0.010=2.490	2898.26	0.74
Large	1	1100–250= 850	0.170–0.005=0.165	169.57	0.63
	2	1850-1100=750	0.750-0.150=0.600	756.06	0.68
	3	2400–1100=1300	1.450–0.150=1.300	1215.76	0.69
	4	2400–1500= 900	1.450–0.250=1.200	1001.16	0.69
	5	2550– 650=1900	1.800–0.050=1.750	1940.58	0.70
Medium	1	1520–1420= 100	0.260–0.200=0.060	71.23	0.73
	2	1840–1520= 320	0.750–0.250=0.500	430.60	0.80
	3	2350–1850= 500	1.500–0.700=0.800	535.57	0.72
	4	2550–2400= 150	1.800–1.550=0.250	223.65	0.79
Small	1	1943–1926= 17	0.740–0.714=0.026	53.16	0.72
	2	1985–1973= 12	0.770–0.750=0.020	37.31	0.70
	3	2222–2208= 14	0.891–0.871=0.020	31.73	0.81
	4	2271–2252= 19	0.984–0.928=0.056	58.86	0.70

We started by characterizing the structure of the whole curve. Besides the large cells (Δn>500) from Fig.4, we identified several other medium (100≤Δn≤500) and small ones (Δn<100). One can notice that, except for one cell ("large 1"), the Hurst exponent is ranging from 0.68 to 0.81. If we further neglect "medium 2", "medium 4" and "small 3", the Hurst exponent for the remaining cells concentrates in the interval 0.70±0.02. This is a strong indication for a statistical self-similarity. Besides, we have arguments supporting a kind of weak theoretical self-similarity since the mutual correlation between the large cells and the whole series reveals some repetitive structure. The mutual correlation between a core κ and the samples *x* is [1]

$$C(n) = \frac{\frac{1}{N}\sum_{k} \kappa(k)x(k+n) - \bar{x}\,\bar{\kappa}}{\overline{x\kappa} - \bar{x}\,\bar{\kappa}}. \tag{9}$$

For example, the computation of the correlation function between the cell entitled “medium 3” and the whole pattern is given in Fig. 5 that clearly indicates, at least

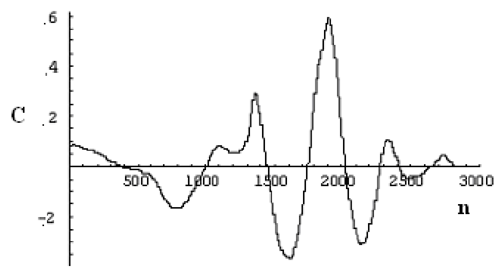

Fig.5 The correlation function

for the large cells, a quasi periodic structure suggested by the five maxima. From the methodological point of view, we have to mention that an intermediate detrending procedure by removing the linear trend was performed in order to have significant results.

5. Conclusion

In this work we present a study of the fractal structure of the temporal series describing the daily exchange rate of the Romanian “Leu” (ROL) with respect to the US Dollar (USD) over ten years, in the framework of econophysics. A self-similar shape of the ROL-USD exchange rate during the transition époque was identified. Unlike other structures reported in the literature, based on the identification of several representative scaling cells that replicate the whole temporal pattern, we conclude that it is not only a statistical self-similarity, but it is close to a theoretical one. Thus, the same underlying non-linear dynamics is characterizing the whole temporal evolution of the Romanian economic system between 1991 and 2000 considered as the transition period from command economy to an open market one.

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