FINITE ELEMENT METHOD IN HIGH INTENSITY PLASMA DISCHARGE MODELING

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Se arată cum poate fi modelată plasma descărcărilor electrice la presiune ridicată aflată aproape de echilibrul termodinamic local. Rezolvarea ecuaţiilor diferenţiale cu coeficienţi variabili şi termeni sursă neliniari se face prin metoda elementului finit.

It shows how the high pressure electric gas discharge plasma near to the local thermodynamic equilibrium point can be modeled. Partial differential equations with variable coefficients and non-linear source terms are solved using the finite element method.

Keyword: hot-spot cathode mode, mercury arc discharge, numerical algorithm

1. Introduction

The industrial construction of plasma devices, like high-pressure sodium or mercury lamps, was due to intuition and several successive attempts, and not to a detailed functional analysis. From this point of view, the mathematical and physical modeling is essential.

The early stages of mathematical modeling of electric discharge plasma at high intensity current were based on simplified models that were generally one-dimensional. The models for electrodes created by Tieleman-Oostvoegels [1], Cram [2], and others, were 1D models based on severe approximations. Even the study of the positive column plasma was developed under the assumption of infinite dimensions ignoring boundaries effect. Consequently, the Elenbaas-Heller equation is a second degree ordinary differential equation. Taking into consideration the temperature material coefficients dependencies, the mathematical difficulty of the problem increase, but the problem could not be transferred into a higher-dimensional mathematical topology.

Introduction of the electrode geometry effects and the occurrence of asymmetrical phenomena led to partial differential equations, which requires numerical algorithm based on a finite element method in order to be solved.

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Nowadays, most of the studies are based on 2D models for high symmetries [3-6] or on 3D models [7, 8], as a result of the increased processor power and the development of specialized codes.

2. Theoretical Model

This paper concerns the mathematical modeling of stable lamps that work in dry mercury vapors, in order to obtain the physical parameters that ensure a longer lifetime. The lamp we are talking about (Fig. 1) is considered at the local thermodynamical equilibrium, therefore a number of laws hold true: Planck, Boltzmann, Saha and Guldberg-Waage. The second moment of the Boltzmann equation is the most important. Knowledge of the temperature distribution in every point allows the calculation of all the local parameters.

Thus, for the electrode we must solve the heat transport equation:

$$\rho(T)c_p \frac{\partial T}{\partial t} + \nabla[-k_W(T)\cdot\nabla(T)] = S(r,z) \tag{1}$$

coupled with the conductive media equation:

$$-\nabla\left(\frac{1}{\rho_W(T)}\cdot\nabla V - \vec{j}\right) = Q_j \tag{2}$$
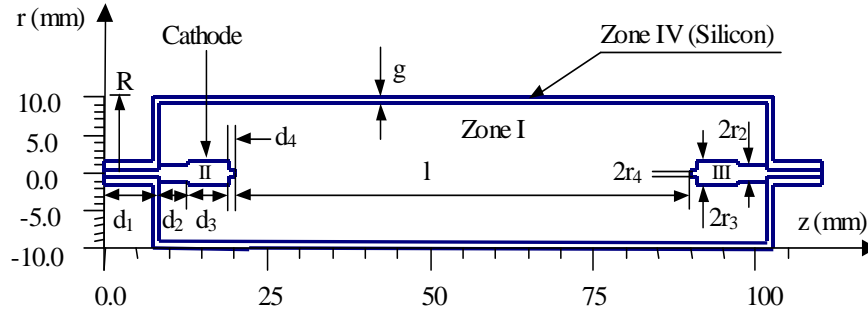
with appropriate Dirichlet or Neumann border conditions.

Fig.1. Lamp geometry. Main characteristics are $d_1 = 8mm$, $d_2 = 4mm$, $d_3 = 6mm$, $d_4 = 1mm$, $r_1 = 0.5mm$, $r_2 = 1mm$, $r_3 = 1.5mm$, $R = 10mm$, $l = 66mm$ and $g = 1mm$.

In the above equations, ρ_W is the electrical resistivity, $\rho = 19300kg/m^3$ is the mass density, $k_W(T)$ is the thermal conductivity, $c_p = 132J/kg/K$ is the specific heat for tungsten, $S(r,z)$ and Q_j are the source terms for the equations.

In general we take for the electrode $Q_j = 0$ and:

$$S(r,z) = \rho_W j^2(r,z) \tag{3}$$

The self-consistency closing condition for a cathode operating in "hot-spot" mode at I discharge current intensity is:

$$I = 2\pi \int\limits_0^{r_{hs}} jr \cdot dr \tag{4}$$

where r_{hs} is the spot radius and j is the current density distribution.

For the positive column plasma we must solve the equation

$$\nabla\{-k_{pl}(T)\nabla T(r,z)\} = S_{pl}(r,z) \tag{5}$$

where k_{pl} and S_{pl} are the thermal conductivity and the source term for the plasma.

The plasma thermal conductivity is given by the formula:

$$k_{pl}(T) = 8.326 \times 10^{-2} \frac{\sqrt{T/m_{Hg}}}{r_m^2 \Omega^{(2,2)}(T^*)} \tag{6}$$

where m_{Hg} is the atom mercury mass, $r_m = 2.898 \overset{\circ}{A}$, k_B is the Boltzmann constant, $\Omega^{(2,2)}$ is the plasma collisions integral at the reduced temperature $T^* = Tk_B/\varepsilon$ with $\varepsilon = 851 k_B$.

The source term is $S_I(r,z) = U_J - U_{rad}$, where U_J is the Joule power contribution and U_{rad}, the radiant term. The last term, related to the plasma net emission, raises many problems, in terms of correct estimation, but also of mathematical convergence after multiple iterations. This is the term most affected by the numerical diffusion. We prefer to use the approximation given by Elenbaas which use the average excited potential $V^* = 7.8V$ and mercury atom distribution n_{Hg}:

$$U_{rad} = 1.08 \times 10^{-10} n_{Hg}(T)\exp\left(-\frac{eV^*}{k_B T}\right) \tag{7}$$

The term describing the heating through Joule effect is:

$$U_J = \sigma_{pl}|\vec{E}|^2 \tag{8}$$

where $\vec{E}$ is the electric field intensity inside the plasma and σ_{pl} is the plasma electrical conductivity.

The intensity of the electrical field $\left|\vec{E}\right|$ is calculated from the local Ohm's law for a given discharge current intensity:

$$\left|\vec{E}\right| = \frac{I}{2\pi \cdot e}\left(\int_0^{R-g} r\sigma_{pl}dr\right)^{-1} \tag{9}$$

where R and g have the meaning from fig. 1. This is the self-consistency relationship.

3. Results

The spot radius was calculated according to the model presented in [7].

For solving the equations (1) and (2) the following boundary conditions were used:

The hot-spot domain and the end part of the electrode were described by the Dirichlet conditions: $T_{hs} = 3683K$ (based on pictures taken with a large focal distance microscope or with CCD cameras [9] which have shown the local melting of tungsten) and $T_{end} = 600K$ (imposed by the designer).

A Neumann condition similar to [10 - 12] was imposed for the rest of the border:

$$\vec{n}k_w\left(\frac{\partial T}{\partial \vec{n}}\right)_b = \vec{n}\left[\varepsilon_w\left(T_b\right)\sigma\left(T_b^4 - T_{amb}^4\right) + \zeta\left(T_b - T_{amb}\right)\right] \tag{10}$$

where T_b is the border temperature, ζ is the convection transport coefficient and $\left(\frac{\partial T}{\partial \vec{n}}\right)_b$ is the normal direction ($\vec{n}$) derivative to the surface.

The ambient temperature was considered $T_{amb} = 300\ K$, and we took $\zeta = 25Wm^{-2}K^{-1}$ for the convection transport coefficient. The thermal conductivity and heat emissivity temperature dependencies were taken from the literature.

We used the Multiphysics module of the FEMLAB software in order to solve the equation (1) coupled with equation (2) under given boundary conditions.

FEMLAB is a powerful, interactive environment for modeling and solving scientific and engineering problems based on partial differential equations (PDEs).

To reduce the runtime, we started with a given temperature distribution and run the equation (2). The result counted as initial data to run equation (1). The

new result counted as new initial data for equation (2) and so on. After several separate run sessions, the equations were coupled and run together, thus reducing the total runtime.

Figure 2 shows several results on the distribution of the temperature in logarithmic scale and also longitudinal cross-section details trough the "hot-spot".
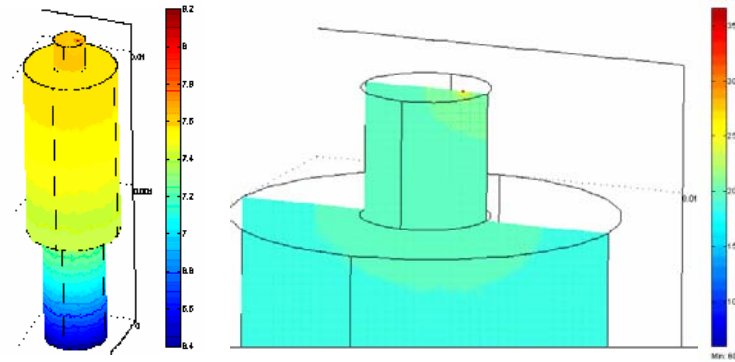

Fig. 2. Temperature distribution inside an electrode during the "hot-spot" mode. Logarithm of the temperature (a), and longitudinal slice detail trough hot-spot (b)

The equation (5) coupled with equation (9), with the boundary conditions were solved using Macsyma Font End (PDE's - Partial Differential Equations). The electrodes-plasma interface borders were taken from the results of the preceding electrode model.

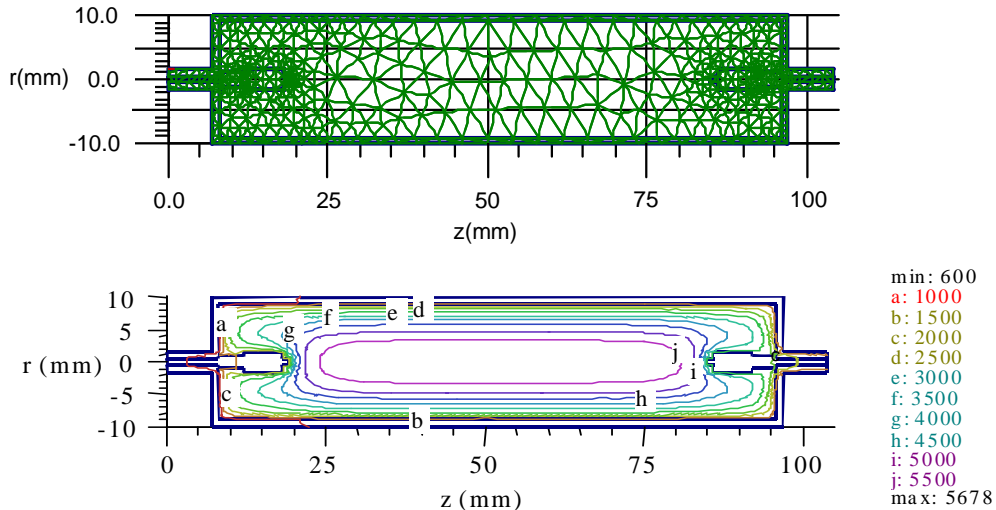Figure 3 show the grid used at a given time and final temperature distribution.

Fig. 3. Discretization at a given time and final temperature distribution .

4. Conclusions

Using specialized algorithms for solving coupled equations of plasma physics is proved to be an efficient and performing tool. Instead of using Fortran language or an algorithm developed in MatLab or Mathematica software, specialized languages developed by IT research groups can be used. So, the FemLab (Finite Element Laboratory) software developed by Comsol - Sweden, Macsyma Font End - PDE's (Partial Differential Equations) - US or very specialized languages created by plasma physics research people, like the PlaSiMo (Plasma Simulation Model) from the Eindhoven University - The Netherlands, represent several powerful instruments to solve complicated problems.

All these software are based on the finite element method (FEM) in order to solve partial differential equations (PDEs) of mathematical physics.

Also, the use of this specialized software, with some self-consistency restrictions, eliminates the errors due to the numerical diffusion and allows us to get results which are subsequently confirmed by experiments.

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