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# OPTIMIZATION OF SPARK TIMING AND AIR-FUEL RATIO OF AN SI ENGINE WITH VARIABLE VALVE TIMING USING GENETIC ALGORITHM AND STEEPEST DESCEND METHOD

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*In this study, optimization of inlet and exhaust valve timing as well as spark timing and air-fuel ratio in XU7/L3 engine has been conducted in order to reduce fuel consumption and increase engine torque. At first, engine model in GT-POWER software is produced, and then it is optimized using sensitivity analysis. Unknown input data are arranged based on their sensitivity and the most effective of them is selected and optimized. Heat transfer coefficient of the cylinder, temperature and friction coefficient of the inlet port are the most important unknown parameters which affect the model, and optimization of them reduce errors of the model with the experimental results. Since determining the optimal operation points of the engine is not possible just with GT-Power software, it is coupled with MATLAB. The results are in full load case and shows 5% and 5.1% reduction in the fuel consumption as well as enhancement of 5.65% and 6% in torque by using genetic algorithm and steepest descend, respectively.*

**Keywords**: variable valve timing; fuel consumption; optimization; genetic algorithm; engine

## 1. Introduction

Taking into account some important issues such as regulations to reduce air pollutant emissions and less fuel consumption of the automobiles, optimizing the fuel consumption in internal combustion engines would be of high concern for all major engine manufacturers in the world. For this purpose, numerous technologies are being considered and some of them are already used as commercial technologies in new automobiles extensively.

Increasing volumetric efficiency of the engine is one of the methods, which enhances fuel efficiency and efficiency of the engine. Performance of the port and its components including the valves are of great importance in this regard [1]. By raising the amount of inlet air to the cylinder [2] and optimization of the spark timing in a typical engine, one may wish to decrease the fuel consumption and increase the power. The amount of air which enters each cylinder can be adjusted by the valves. Variable valve timing (VVT) system enables the inlet and exhaust valves to be opened and closed at best possible time. Increasing the volumetric efficiency (amount of the inlet air), increasing the number of inlet valves to two or

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three, designing high performance ports or ports with variable length and/or diameter are some other techniques [3].
Brake specific fuel consumption (BSFC) can be saved up to 4.49% just by changing the timing of the inlet and exhaust valve opening [4]. Meanwhile, BSFC can be saved about 6.69% by changing the closing and opening times of these valves [5,6].
The spark timing is also very important in reaching the maximum pressure and fuel consumption of the engine. When the spark is made earlier, the maximum pressure of the cylinder is greater and it occurs sooner. A research work was conducted by Li et al. [7] on a spark ignition engine with direct injection of methanol fuel. The results of this paper which has used a non-uniform mixture of methanol with a classified distribution demonstrated that an optimal value exists for injection and combustion time, in order to reach an adequate combustion with a trivial pollution. In this optimal case, smaller ignition delay and lesser cycle-by-cycle variation will maximize internal pressure of the cylinder, rate of heat transfer and thermal efficiency. Moreover, the studies have revealed that the air-fuel ratio has a significant effect on brake mean efficient pressure (BMEP), thermal efficiency and cylinder temperature. The effect of air-fuel ratio has been investigated in a research by Rahman et al. [2]. The results have showed that BMEP and brake thermal efficiency decrease by increasing the air-fuel ratio, while BSFC increase by raising this ratio. A similar research by Salimi [8] on a four-cylinder engine demonstrated that for reach mixtures, the BMEP decrease almost linearly, followed by a nonlinear decline. Furthermore, it has been shown that the brake thermal efficiency is increased by approaching to the richest fuel conditions and then decreased by increasing the air-fuel condition [9].
Considering the works done so far, it seems that besides using the VVT technology, it is still required to optimize the spark timing and the air-fuel ratio. This is done in engine control unit (ECU) for a typical engine today. For this purpose the model of Kakaee and Pishgooie [5] was optimized first and then, it was employed as the basis.
Some intelligent optimization methods such as genetic algorithm (GA), thermal simulator algorithms, ant colony optimization algorithm and etc. are used in the calculations. An optimization was proposed on valve timing using genetic algorithm in 2005. Genetic algorithm was selected to reduce the time needed to reach the mentioned optimal point. The result of this study is creation of a framework for calibration of an engine and determination of the optimal timing in an engine which is equipped to VVT system.
Cruz-Perago et al. [10] employed genetic algorithm to determine instantaneous pressure inside cylinder of an internal combustion engine. Various operators including selection, mutation, and crossover processes have been applied in this study based on the conventional genetic algorithm. Three engines, one single-

cylinder diesel engine, a three-cylinder spark ignition engine and finally one V-16 diesel engine, were examined and a good pressure profile is estimated in the cylinder. In another research, Kesgin [11] has utilized the genetic algorithm in combination with the neural network to estimate the effect of design and functional elements on efficiency and $NO_X$ pollutant in a gas engine. However, this study has used a computer program to calculate the amount of $NO_X$ based on a kinetic reaction model, the results of which demonstrate enhancement of the efficiency with keeping the $NO_X$ pollutant remaining constant below 250 $mg/Nm^3$. Furthermore, genetic algorithm is used as an optimization method in complicated engineering problems.

Numerous studies have been conducted to predict and reduce emissions of the diesel engine using genetic algorithm and neural network [12]. Togun et al. [13] have applied genetic algorithm in a gasoline engine to predict torque and BSFC in terms of spark timing, throttle position and engine speed.

On the other hand, the steepest descend algorithm is adopted in parallel for the purpose of optimization. The steepest descend method, Newtonian method and quasi-Newtonian method are algorithms based on derivative. Drummond [14] used the steepest descend algorithm to optimize the vectors. Andrus [15] has introduced the steepest descend method as a powerful tool with excellent convergence speed for finding the local minimum of a multivariable function.

In this paper, the effect of using VVT system has been investigated to reduce fuel consumption and increase output torque of the engine. Then, the optimal spark timing, valve timing, and air-fuel ratio have been obtained. GT-Power software is used and coupled with MATLAB software for performing simulation.

## 2. Engine modeling

The model prepared in GT-Power software is a 2D model and the engine geometric specifications are summarized in Table 1. Operating conditions of the engine is also replaced based on experimental results, while the unknown immeasurable conditions have been calculated using sensitivity analysis method and substituted in the model.

*Table1*

**Geometric specifications of the SI engine**

| | |
|---|---|
| Bore (mm) | 83 |
| Stroke (mm) | 81.5 |
| Max. lift of inlet valve (mm) | 9.6997 |
| Max. lift of exhaust valve (mm) | 9.6997 |
| Length of connecting rod (mm) | 150.5 |
| compression ratio | 9.3 |
| Type | Inline four-cylinder |

In the present study, it has been tried to approach the model to the experimental results by optimization of the unknown input parameters. Thermal

conductivity of the cylinder, temperature and friction coefficient of the inlet port are the most important and effective unknown parameters in the model, that optimization them via sensitivity analysis has decreased error of the model with the experimental results.
It can be inferred from Fig. 1 that improving heat transfer of the cylinder will raise BSFC of the engine and reduce its torque. A large amount of the heat generated in the cylinder is transferred to the outside by raising the heat transfer coefficient instead of being transformed to work. As a result, torque and power of the engine is reduced. Consequently, BSFC will be increased, because it is obtained by dividing the rate of inlet fuel by the engine power when the rate of heat transfer is constant.

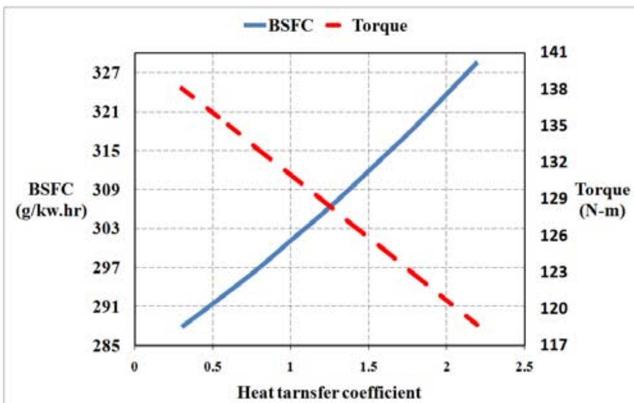

Fig.1. BSFC and torque versus heat transfer coefficient of cylinder

Figs. 2 and 3 depict the effect of temperature and friction coefficient of the inlet port on BSFC and torque of the engine. By increasing temperature of the inlet port, the torque of the engine decreases, while BSFC increases. At higher temperatures, the inlet gas is expanded, so that density and volume efficiency of the engine will decrease. Reduction of the density will let smaller amount of fuel entering the cylinder, and thus, torque and output power of the engine will be decreased. By looking at Fig. 3 it can be observed that BSFC and torque of the engine decrease by increasing the friction coefficient. Thereby, surface roughness will be increased and mixing of the fuel will be done much better. This can reduce BSFC and also the amount of fuel and air which enter the engine such that torque and power of the engine will decrease.

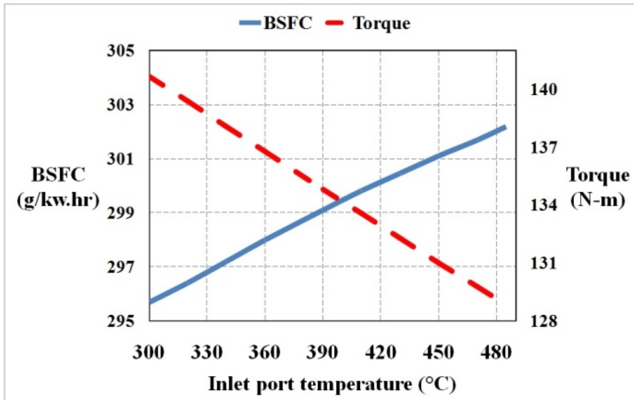


Fig.2. BSFC and torque versus temperature of inlet port

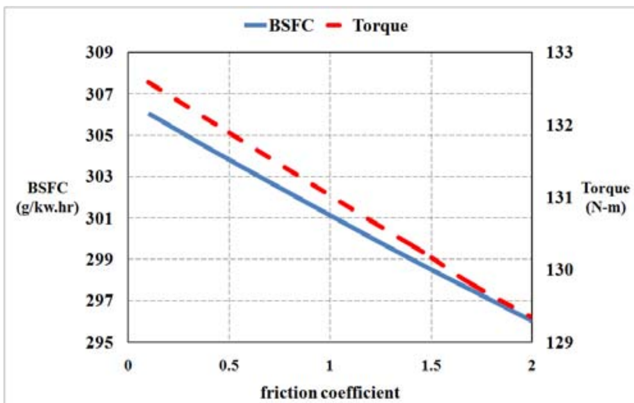


Fig.3. BSFC and torque versus friction coefficient of inlet port

## 3. Coupling GT-Power and MATLAB

General procedure for solving the problem in the coupled softwares is illustrated in Figure 4. By running the code which is written in MATLAB M-File, MATLAB/SIMULINK will be running. At the same time, GT-Power which is coupled with Simulink will be executed and its results will be entered to M-File through Simulink, and the remaining optimization process will be continued.

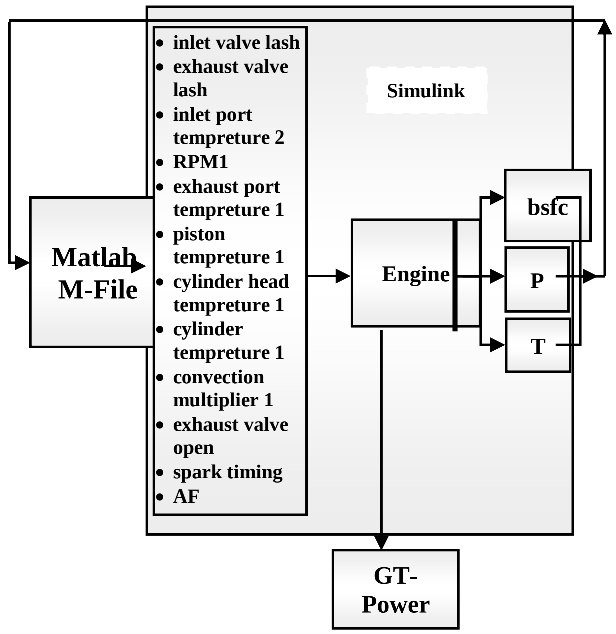


Fig.4. General procedure of problem solving in coupled softwares

## 4. Model validation

Figs. 5 and 6 show behavior of the identified model in comparison with the experimental results. Average error of the BSFC between this model and the experimental sample is equal to 1.71% in all engine speeds. The maximum value of this error is achieved at 6000 rpm speed equal to 3.47% (Fig. 5).

Fig. 6 represents the torque between GT-Power model and the information obtained from the experiment in terms of engine speed. It can be seen that this model is more consistent with the experimental results at lower speeds with the maximum error content (3.47%) being met at 5600 rpm. The average error between the torque from this model and that of experimental results is 0.86 for all engine speeds.

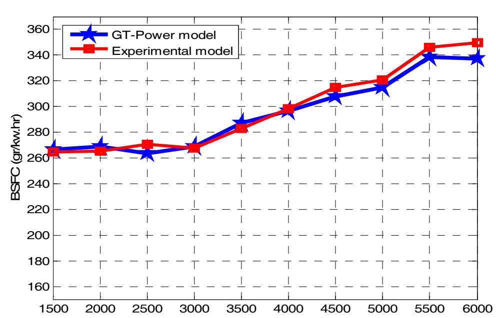


Fig.5. Comparison of BSFC from this model and experimental results

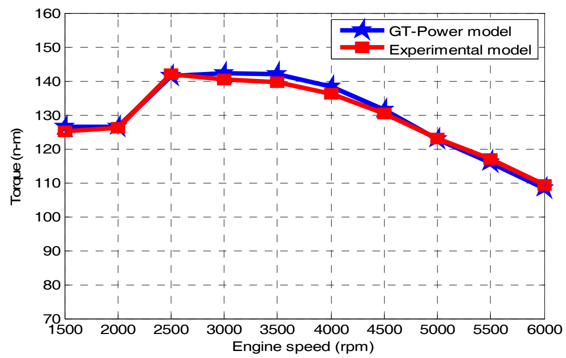


Fig.6. Comparison of torque from this model and experimental results

Fig. 7 depict BMEP of the model in comparison with the experimental results. It can be observed that both of these diagrams properly show the actual behavior of the engine properly.

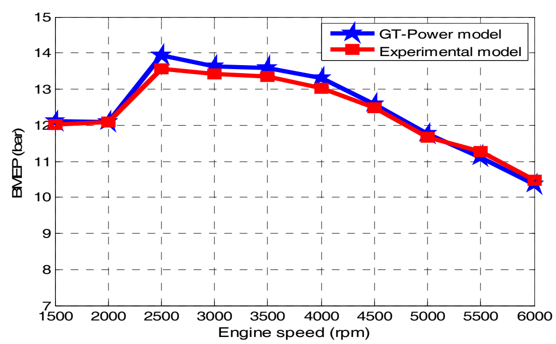


Fig.7. Comparison of effective brake pressure from this model and experimental results

## 5. Optimization methods

### 5.1.Steepest descend method for engine optimization

Steepest descend algorithm is one of the oldest and most famous methods for minimization of a multivariable function which is also known as gradient method. This algorithm is of great importance from theoretical point of view since it is one of the simplest methods with reasonable justification. The main incentive of looking for more advanced algorithms is created by the effort to modify basic form of the steepest descend such that the new algorithm would incorporate superior convergence properties. Therefore, the steepest descend method is not only the first method which is tested for solving a problem, but also a reference to assess other techniques. For a function like $f$ which is uniformly differentiable around $x_k$ point with $g_k = \nabla f(x_k)$:

$$f(x) = \frac{1}{2}\, x^T\, Qx - \; x^T b \tag{1}$$

Where, $Q$ is a definite positive $n \times n$ matrix. Since $Q$ is positive, all of its eigenvalues will be positive. It is assumed that these eigenvalues are sorted in the form of $0 < a = \lambda_1 < \lambda_2 \ldots \leq \lambda_n = A$. From the definite positive value of $Q$ it can be inferred that $f$ is a strictly convex function.
The minimum point of the function $f$ can be directly obtained by putting the gradient vector equal to zero such that:

$$Qx^0 = b \tag{2}$$

Furthermore, by defining function $E(x)$ as below:

$$E(x) = \frac{1}{2}\,(x - x^0)^T\, Q(x - x^0) \tag{3}$$

It is found that $E(x) = f(x) + (1/2){x^0}^T Qx^0$, which indicates that the function $E$ differs with the $f$ function just in a constant value. Thus, it would be much better to minimize the function $E$ rather than the function f.
The gradient (both $f$ and $E$) are explicitly specified as below:

$$g(x) = Qx - b \tag{4}$$

Thus the steepest descend method can be obtained as below:

$$x_{k+1} = x_k - \alpha_k g_k \tag{5}$$

Where, $g_k = Qx_k - b$ and $\alpha_k$ will minimize the function $f(x_k - \alpha g_k)$ but $\alpha_k$ can be directly determined for this special case.

$$f(x_k - \alpha g_k) = \frac{1}{2}(x_k - \alpha g_k)^T Q(x_k - \alpha g_k) - (x_k - \alpha g_k)^T b \tag{6}$$

which is minimized at, $g(x) = Qx - b$

$$\alpha_k = \frac{{g_k}^T g_k}{{g_k}^T Q g_k} \tag{7}$$

as found from differentiating with respect to $a$). Therefore, the steepest descend algorithm is explicitly written as below

$$x_{k+1} = x_k - \frac{{g_k}^T g_k}{{g_k}^T Q g_k} g_k \tag{8}$$

Where, $g_k = Qx_k - b$ [16].

## 5.2.Genetic algorithm (GA) for engine optimization

Some methods like quasi-Newtonian and conjugate gradient need to have an answer which is near to the optimal answer in order to start solving the problem. This will sometimes requires additional techniques to generate an answer that is close enough to the optimal one. These algorithms benefit from a high speed of convergence, though they are simply trapped in local minima or maxima. However, genetic algorithm and methods like random search will adopt to solve the problem from random points in the search space. Thereby, predicting performance of the algorithm would not be that simple. The convergence speed of these algorithms is smaller than those which look for minimum, but they are less prone to be trapped in local optima and they are more promising for finding the global optima.

### *5.2.1. Modified Genetic Algorithm*

Since the genetic algorithm method is based on search, value of the output function must be calculated at each step for each of the input variables of the problem in order to resume the search operation. The function under study here is the simulated engine model which is coupled with MATLAB software. The time required to calculate the outputs each time are about 60 seconds. If the initial population in genetic algorithm is considered 200 and the problem is resumed until 100 generations, a time of 10×100×200×60 will be needed to solve the problem at 10 different engine speeds.

The following recommendations are made in order to reduce the problem solving time and enhance the accuracy of algorithm optimization:

- For reduction of the time, number of each new generation must be smaller than the previous generation in order to accelerate the problem solving.

Assume that $p$ indicates the number of population in each generation and $n$ is a variable. Then the number of ways to select 2 members from $n$ members will be:

$$1 + 2 + 3 + \cdots + (n-1) = \frac{n(n-1)}{2} \quad (9)$$

It is assumed that $0.7p$ members participate in the new generation, as the number of this generation is better to be decreased. If $N_p$ is the number of members in the new generation, then:

$$Np = 0.7p; \qquad \frac{n(n-1)}{2} = 0.7p; \qquad n^2 - n = 1.4\,p$$

$$n = 0.5 + \frac{1}{2}\sqrt{1 + 5.6p} \quad (10)$$

By choosing $n$ from the equation above the value of $N_p$ can be calculated for each step. For example if the first generation has 100 members and 70 members are decided to be in the second generation, then the value of $n$ can be calculated by the Eq. (10) to give number of the 70 selected members from the first generation.

- For improvement of the problem solving accuracy, the input variables must be interpolated in each generation.
- ▪ Since some good answers might be lost during interpolation of the both numbers, 10% of the best answers from the previous generation are also directly introduced to the new generation and participate in this step.

Fig. 8 illustrates variance in the genetic algorithm for optimization with one, two, three and four variables. By increasing the number of the problem variables will raise the amount of variance in the initial generations. The variance is decreased when the algorithm proceeds and it finally reaches to the same value.

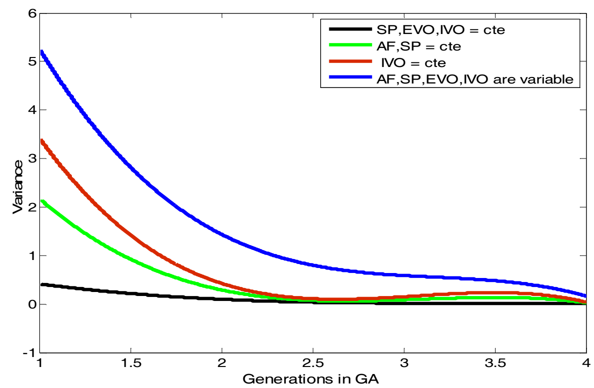


Fig.8. Variance in terms of GA generations at 6000 rpm engine speed

## 6. Result and discussion

### 6.1. Variations of BSFC and Torque

Fig. 9 presents the results of BSFC in terms of engine speed in three optimized states with genetic algorithm, steepest descend and no optimization process. It can be seen that using VVT system and optimization of the spark timing and air-fuel ratio, BSFC will be decreased for all engine speeds. This reduction is especially noticeable at high speeds. Meanwhile, it is seen that the answers obtained from both of the algorithms are similar to each other at lower error content. The maximum BSFC reduction was seen at 6000 rpm which was reported 12.02% and 10.34% for genetic algorithm and steepest descend algorithm, respectively. Generally speaking, using genetic algorithm and steepest descend method for the optimization causes a 5.10% and 5% reduction in the fuel consumption.

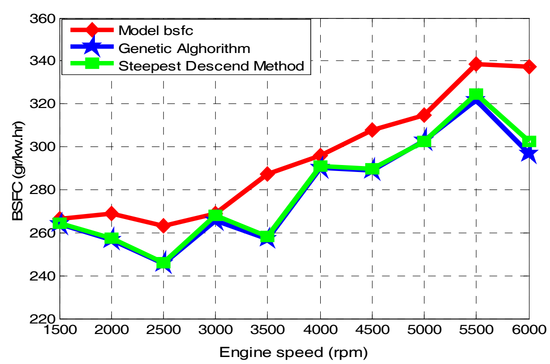


Fig. 9. Variations of BSFC in terms of engine speed in three states: optimization by genetic algorithm, steepest descend method and no optimization

According to Fig. 10, it can be declared that optimization of the spark timing and air-fuel ratio and timing opening of the inlet/exhaust valves will increase the torque almost at all engine speeds. Raising the torque at lower speeds is also evident. At high speeds, the optimized torque diagram is smoother than that of lower speeds. Implementing optimization on the four mentioned parameters will cause a 6.02% and 5.65 growths in the torque by genetic algorithm and steepest descend algorithm, respectively.

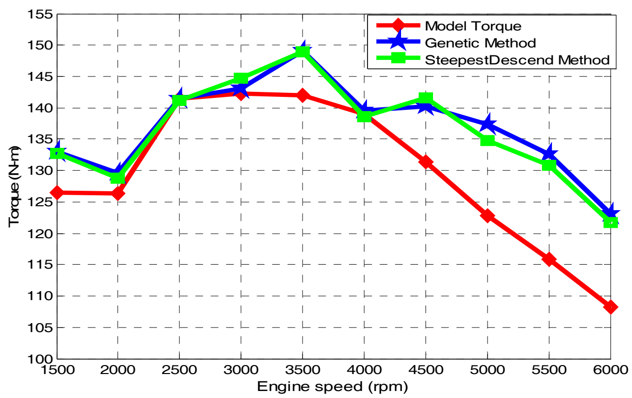


Fig.10. Variations of torque in terms of engine speed in three states: optimization by genetic algorithm, steepest descend algorithm and no optimization

Figs. 9 and 10 suggest different behavior for the engine at different engine speeds especially at 3000 and 4000 rpm. Meanwhile, it can be seen that the optimization at 1500 rpm will improve the torque for 5%, whereas the fuel consumption of the engine is improved just 0.94%. At engine speeds of 2000 and 2500 rpm, BSFC has been improved more versus the torque.

The experimental data of the engine has demonstrated that the air-fuel ratio of the engine was 15.94% at 1500 rpm, whereas at 2000 and 2500 rpm it has decreased to 15.15 and 15.23%, respectively. The air-fuel ratio of 15.94% is indicative of the lean mixture. In other words, at this engine speed, the designer has decided to choose lean mixture in order to reduce the fuel consumption. Since engine torque is reduced due to lean mixture, it can be concluded that at this engine speed the fuel is a limiting factor for the engine. Therefore, the fuel is selected as leaner as possible to reduce the fuel consumption. So it can be argued that since the fuel consumption is rather high at this speed and accounted for a limiting factor, implementing the optimization operation will further improve the torque versus fuel consumption.

## 6.2.Optimal Spark timing

Fig. 11 illustrates the optimum spark timing versus engine speed using genetic algorithm and steepest descend for reduction of the fuel consumption. The variation trend of the spark timing is almost the same for both these algorithms. Generally, it can be declared that the spark must be made sooner when increasing the engine speed for reduction of the fuel consumption and enhancement of torque. This is because by retarding the spark, the maximum cylinder pressure is decreased and the torque is reduced.

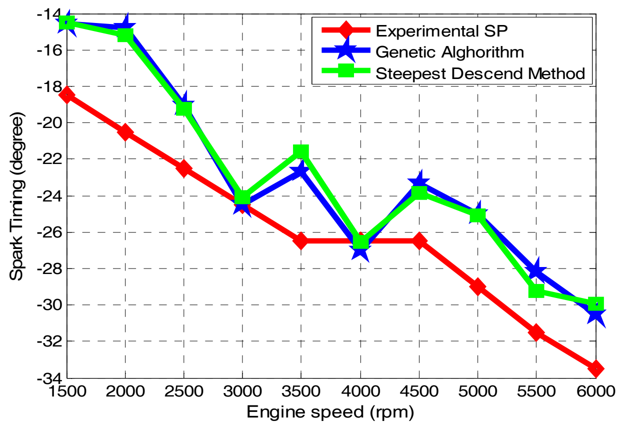


Fig.11. Spark timing in terms of engine speed with genetic algorithm and steepest descend

## 6.3.Optimal Air-Fuel Ratio

Fig. 12 depicts the diagram of air-fuel ratio in terms of engine speed with genetic algorithm and steepest descend for fuel reduction.

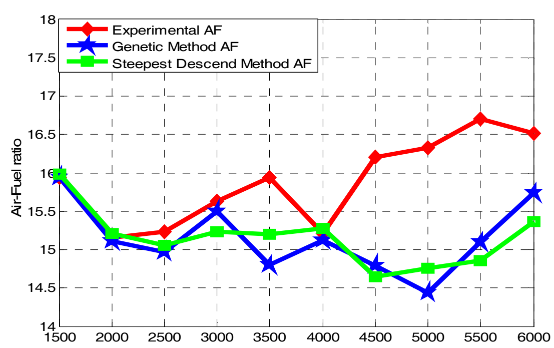


Fig.12. Air-fuel ratio in terms of engine speed with genetic algorithm and steepest descend

## 6.4.Optimal Time for Opening of Inlet and Exhaust Valves

Figs. 13 and 14 depict the optimum IVO and EVO in terms of engine speed using genetic algorithm and steepest descend for reduction of the fuel consumption.

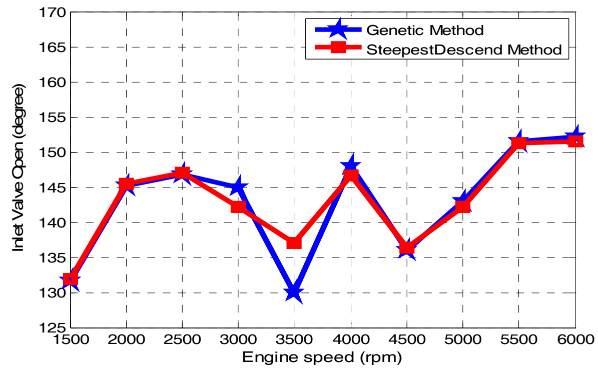


Fig.13 Opening time of Inlet valve in terms of engine speed using genetic algorithm and steepest descend

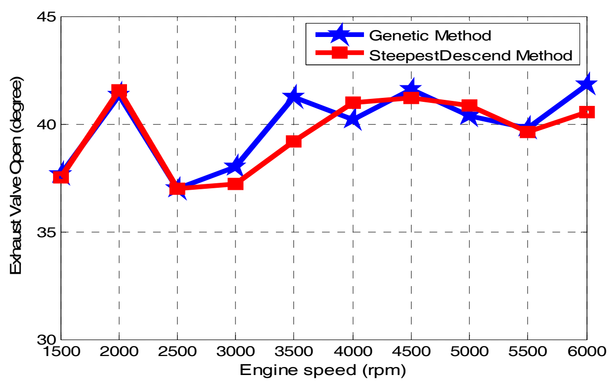


Fig.14. Opening time of exhaust valve in terms of engine speed using genetic algorithm and steepest descend

By looking at the diagram of variations of brake torque and fuel consumption in terms of engine speed, this question may rise that why the fuel consumption of the engine is almost the same for the both algorithms at 3500 rpm, but the output torque is different? To answer this question, torque and fuel consumption diagrams have been drawn versus each of the unknown parameters of the problem.

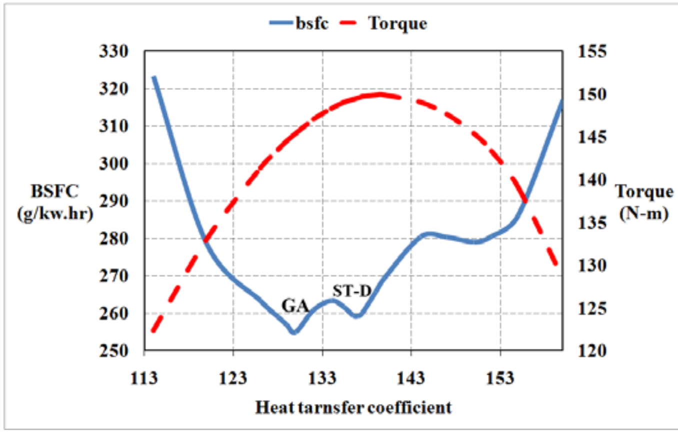


Fig.15. Brake specific fuel consumption and brake torque in terms of intake valve opening (IVO)

It can be observed in Fig. 15 that the diagrams of fuel consumption and torque versus IVO has two minima, one at 137° and the other at 13. Since, there is a relative minimum at 137°, the steepest descend algorithm which moves from right side of the diagram is trapped at this point and takes it as the absolute minimum of the function. But, genetic algorithm which acts based on search is not trapped at relative minimum of 137° and has rather selected the more optimum point of 130°.

## 7. Conclusions

An SI engine has been used in this study for the first time to optimize the spark timing and air-fuel ratio with opening of the inlet and exhaust valves. Thereby, the optimal values have been calculated for these parameters. Critical unknown parameters of the model have been identified first, and then they have been calculated using sensitivity analysis. Heat transfer coefficient of the cylinder, temperature of the inlet port and friction factor of the inlet port were the most significant unknown and effective parameters in the model. Therefore, optimization of them has decreased the error of this model in comparison with the experimental results (average error content between current model and experimental results is smaller than 2%). Creating fundamental changes in the approach of combining genetic algorithm for reduction of the solving time and enhancement of the accuracy is another innovation proposed in this paper.

The results of this research indicate that optimization of the spark timing and the air-fuel ratio, and timing the opening of inlet and exhaust valves in an SI engine equipped with VVT system will lead to a lower BSFC and higher torque at all engine speeds. This improvement is especially more considerable at higher engine speeds. Furthermore, the results demonstrated that the answers obtained from both of these algorithms are very similar with a small error percentage. The maximum

BSFC was seen at 6000 rpm which was 12.02% and 10.34% by genetic algorithm and steepest descend, respectively. Implementing the optimization on the four above mentioned algorithms causes a 6.01% and 5.65% enhancement in the torque by genetic algorithm and steepest descend method, respectively. At the end using genetic algorithm for solving complex problem of engine with many input variable is suggested.

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