EXPERIMENTAL MODEL OF SHAPE MEMORY ALLOY ACTUATORS USING MODIFIED PRANDTL-ISHLINSKII MODEL

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This article presents the modeling of shape memory alloy actuators. For this purpose, modeling the hysteresis nonlinearity of actuator is performed by using a Modified Prandtl-Ishlinskii (MPI) model. The inverse MPI model is used to reduce the nonlinearity. This part of the article was presented for using it in controller designing in our future work. Generally the shape memory alloy-actuated system can be modeled as a linear model coupled with hysteresis; the linear model is identified with governing heat transfer equation of shape memory alloys. This experimental study attests the derived model to be of good precision.

Keywords: SMA actuator, Hysteresis, MPI model.

1. Introduction

Shape memory alloys (SMA), because of their unique mechanical characteristics and shape memory effect (SME), have been widely used as force and displacement actuators in many fields and applications. SMAs as actuators are well known as they are silent in operation, simple to operate, light in weight, and capable of developing large strains, to mention a few. SMA actuators possess high energy density, in order of 107 J/m3 [1], which is the highest among the presently known actuation principles. Although large force and displacement can be realized with these actuators, the effectiveness of the SMA actuators is typically hampered by two factors: hysteresis and bandwidth limitation.

Hysteresis nonlinearities are the phenomena that are present in the dynamics of many physical systems, and notably in a class of materials such as shape memory alloys, piezoceramics and magnetostrictive materials. This is a complex nonlinearity with memory that may result in multiple outputs for a given input, depending on its time history and is not generally sector-bounded. The hysteresis model in electromagnetic, piezoceramics, magnetostrictives and SMA actuators has been addressed in several researches [2-4]. Two different type of model have been proposed to capture the hysteretic characteristics. The first type

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of models were derived from the physics of hysteresis and combined with empirical factors to explain the models [5, 6]. The second type of the model is based on the phenomenological nature and describes the phenomena mathematically.

Generally speaking, there are two types of approaches to deal with hysteresis nonlinearity:

1. Open Loop compensation: Find a first-principle or phenomenological model, identify the parameters, then invert model, if possible, to remove or at least ameliorate nonlinearity.

2. Closed Loop feedback: Use the output error (measured output subtracting the desired output) to generate the corrective heat input.

In this article we used the first approach to eliminate the hysteresis nonlinearity for controller designing.

Here we list some of the previous work in modeling and control of shape memory alloys:

P. Kloucek and R. Reynolds [7] presented a model describing the thermo-dynamical behavior of shape memory alloy wires, as well as a computational technique to solve the resulting system of partial differential equations. Their model consisted of continuum-level conservation equations based on a new construction of the Helmholtz free energy potential.

G. Song et al [8] proposed a neural network inverse model for tracking control of SMA wire actuator. They used a digital data acquisition and a real-time control system to record experimental data and to implement the control strategy. Based on the training data from the test stand, two neural networks are used to respectively model the forward and inverse hysteresis relation between the applied voltage and the displacement of the SMA wire actuator. To control the SMA actuator without using a position sensor, the neural network inverse model is used as a feedforward controller.

A vast amount of research has been undertaken in modeling SMA's, particularly in capturing their hysteresis properties. Models range from experimentally fitting nonlinear differential equations to the input-output relations, as by Arai et al [9], to atomic interactions of the separate alloys, as by Kafka [10]. Some of the earlier models that were employed for control purpose are: Kuribayashi's model [11] based on experimentally identified relations, the sub-layer model of Ikuta et al. [12], and the constitutive equation of Tanaka [13].

Liang and Rogers extended Tanaka's work to study the three classic cases of SMA actuators, those of free recovery, constrained recovery and controlled recovery [14]. In free recovery the actuator is free to move with no external stress. In constrained recovery the actuator is fixed and controlled recovery involves a motion with an external stress. Their model was complicated however and

required iteration to obtain the stress temperature relations at a specific temperature.

The only reasonably successful mathematical models were developed by Ikuta et al. [15] and Lin et al. [16]. However, Ikuta's model does not adequately account for the presence of minor hysteresis loops. Also, the temperature of the SMA actuators in Ikuta's experimental design was controlled by submerging the actuators in a water bath. Temperature control using electrical current was not considered. The model of Lin offers current-temperature modeling but is geared primarily for simulation rather than control system analysis.

Ahn and Kha [17] investigated a possible application of Preisach model to control SMA actuators using an internal model control strategy. Their developed strategy consisted of Preisach hystresis model and its inverse within the control structure.

Kuhnen and Janocha [18] and Kuhnen [19] proposed a Modified Prandtl-Ishlinskii (MPI) model for the hysteresis nonlinearities. The main advantages of MPI model over the Preisach model are that it is less complex, and its inverse can be computed analytically. Hysteresis, combined with uncertain identification of model parameters, can make design of reliable controller for systems difficult.

2. Basics about shape memory alloys

Shape memory alloys exhibit a shape memory effect which is the result of a phase transformation of martensite crystal structures (which predominates at low temperatures). The martensite phase has low yield strength and is easily deformed. However the original shape of the material is recovered when it is heated, as the deformed martensite transforms into undeformed austenite [21]. During its phase transformation, an SMA generates an extremely large force when encountering resistances or experiences a significant dimension change when being unrestricted.

The shape memory effect is the basis of actuator application for SMA. There are two major types of the shape memory effect.

1. One-way shape memory effect: At the martensite phase, the alloy can be easily stretched using an external force. After removal of the force, the alloy exhibits permanent deformation. It can recover its original shape upon heating. Subsequent cooling does not change the shape unless it is stressed again.

2. Two-way shape memory effect: In addition to the one-way effect, shape change occurs upon cooling and without applying external stress. The SMA usually needs to be trained to exhibit the two-way effect.

The most common shape memory material consists in an alloy of nickel and titanium called Nitinol, or NiTi, which was discovered at the Naval Ordnance Laboratory in the 1960s. This alloy has excellent electrical and mechanical

properties, long fatigue life, and high corrosion resistance. As an actuator it is capable of up to 5% strain recovery or 500 MPa restoration stress with many cycles. For example, a Nitinol wire 0.508 mm in diameter can generate as much as 70 Newton blocked force. Nitinol also has resistance properties which enable it to be actuated electrically by Joule heating. When an electric current is passed directly through the wire, it can generate enough heat to cause the phase transformation.

The unique properties of SMA make it a potentially viable choice for actuators. SMA actuators have over the years been used in a spectrum of applications. When compared to piezoelectric actuators, SMA actuators offer the salient advantage of being able to generate both larger deformations and forces though at a much fabricated into different shapes, including wires and thin films.

The special properties of SMAs result from the reversible phase transformation between their crystal structures. This is shown in Fig. 1. The stronger high temperature is Austenite phase and the weaker low temperature is Martensite phase. When cooling from its high temperature Austenite phase, the alloy undergoes a transformation to a twinned Martensite that can be easily deformed by an external force. This process is often called detwinning. The Martensite phase is then reversed when the twinned structure reverts upon heating to the Austenite phase. Its unique ability of a reversible crystalline phase transformation enables a Nitinol SMA object to either recover its initial heat-treated shape (up to 5% strain) when heated above a critical transformation temperature or alternatively generate high recovery stresses (in excess of 500 MPa).

3. Results and discussion

3.1. Structure of the used SMA actuator

Most of the actuators that are used in researches are wire SMAs, but in this study the SMA is based on the NiTi bar bending and recursive procedure by compression spring. This actuator consists of different parts such as spring, slider, connected wires and SMA bar. The range of actuator's motion is between 0 to 4 mm and the maximum force is about 2 N. Its motion is very smooth. Because of the big diameter of the actuator, it is so resistant against the condition of environment. The schematic and the real actuator that was built are given here.
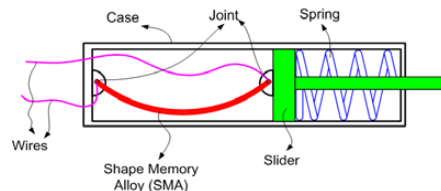

Fig. 1. Schematic of the used SMA actuator

Fig. 2. used actuater

According to Fig. 1, its performance is as follows:
Firstly a deformation is imposed by a compression spring in the actuator. Then with crossing the electrical current within the wire, actuator is heated and phase changing occurrs in SMA bar. So the actuator gets its premier shape that is straight state in here. When electrical current is disconnected and the SMA bar gets cooled under heat transformation with the ambient, it will be compressed by spring force. The role of the spring is so important in this study, because if its tenacity coefficient be large, the actuator's speed is high in recursive motion. So we should design the spring in optimal form, so that the speed is equal in both motions.

3.2. Modeling

As we said before, SMA has hysteresis behavior that makes its modeling a little difficult because of its complex nonlinearity. For achieving model of the proposed experimental work, two blocks were used in series. The first block that is explained below is based on heat transfer equations for thermal cycling. Second is hysteresis block achieved by Prandtle-Ishlinskii model. These two blocks will be described respectively and then the achieved model is shown in some Figs. for comparing to the experimental data.

3.2.1. Heat transfer model for thermal cycling

In order to understand the temperature dependant actuation of SMA, its thermo-elastic response based on heat transfer dynamics needs to be evaluated. It is based on the rate at which heat is added to and removed from the actuator. The simplified one-dimensional heat transfer equation expression [22-24] that describes the SMA actuator for resistive heating and ambient air cooling is expressed as

$$mc_p(dT/dt) = \begin{cases} i^2R - hA(T - T_a) & heating \\ -hA(T - T_a) & cooling \end{cases} \quad (1)$$

where, $m = \rho \pi d^2 / 4$ is a mass of the SMA wire per unit length, ρ is density, d is diameter, c_p is specific heat capacity, T and T_a are maximum

temperature of the wire and ambient temperature respectively. i is the current applied, R is resistance per unit length, h is convective heat transfer coefficient for ambient cooling conditions, $A = \pi dl$ is convective surface area of the wire and l is the length of the wire. The above heat transfer expression is based on the assumption that the effect is one dimensional, the load applied to the actuator is gradual or constant, radiation effect is negligible compared to the conduction mode of heat transfer, volume and changes are significant, T_a, R and h remain constant, wire temperature field is spatially uniform and phase transformation temperature are also considered as constants. The effect of heat radiation is neglected as the range of working temperature is below 200 C [1]. Eq. (1) can be solved for maximum temperature T of the wire separately for heating and cooling process and hence the resulting expression becomes

$$T - T_a = \begin{cases} (T_i - T_a)\exp^{-t/\tau} + i^2\lambda(1 - \exp^{-t/\tau}) & heating \\ (T_i - T_a)\exp^{-t/\tau} & cooling \end{cases} \qquad (2)$$

Where T_i is temperature of the wire at any time t, τ (time constant) $\equiv mc_p/hA$ and λ (current constant) $\equiv R/hA$. From Eq. (2), the current required to heat the actuator to a temperature T is obtained as

$$i_{(heating)} = sqrt\{[(T - T_a) - (T_i - T_a)\exp^{-t/\tau}/[\lambda(1 - \exp^{-t/\tau})]\} \qquad (3)$$

While applying heat to the actuator, the phase transformation and the corresponding shape change starts when the temperature crosses A_s (austenite start) temperature and ends at A_f (austenite finish temperature). In order to maintain the actuator at the actuated position, the temperature has to be maintained just above A_f by the application of optimal power.

3.3. Identification of the hysteresis model
The operators that exist in the MPI model and the related governing equations can be found in the references such as [19-20], so we did not repeat them here. Refer to the governing equations, Unknown parameters of the hysteresis model and its inverse will be finding by measuring the actuator's response experimentally. So the values of r_i and d'_j can be computed by:

$$r_i = \frac{i}{n+1}\max_k\{|x(kT_s)|\} = \frac{i}{n+1}\|x\|_\infty,$$
$$for\ i = 1, 2, ..., n \qquad (20)$$

$$(21)$$

$$d'_0 = 0$$

$$d'_j = \frac{(j - (1/2))}{m} \max_k \{y(kT_s)\}, \quad for \ j = 1, 2, ..., m \tag{22}$$

and

$$d'_j = \frac{(-j - (1/2))}{m} \min_k \{y(kT_s)\}, \quad for \ j = -m, ..., -1 \tag{23}$$

For simplicity:

$$y_{0i} = 0, \quad for \ i = 0, 1, 2, ..., n \tag{24}$$

Then, the constrained quadratic optimization will be used to find the weight parameters w_{hi} and w'_{sj}:

$$\min_w \{w^T \Omega^T \Omega w\} \tag{25}$$

Its constraints are:

$$\begin{bmatrix} U_H & 0 \\ 0 & U_S \end{bmatrix} \begin{bmatrix} w_H \\ w'_S \end{bmatrix} - \begin{bmatrix} u_H \\ u_S \end{bmatrix} \leq \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{26}$$

and

$$\begin{bmatrix} (\|x\|_\infty I - r)^T & 0^T \end{bmatrix} \begin{bmatrix} w_H \\ w'_S \end{bmatrix} - \|x\|_\infty = 0 \tag{27}$$

We have:

$$w^T = \begin{bmatrix} w_H^T & w_S'^T \end{bmatrix} \tag{28}$$

$$w_H^T = \begin{bmatrix} w_{h0} & w_{h1} & \cdots & w_{hn} \end{bmatrix} \tag{29}$$

$$w_S'^T = \begin{bmatrix} w'_{s(-m)} & \cdots & w'_{s0} & \cdots & w'_{sm} \end{bmatrix} \tag{30}$$

$$\Omega = \begin{bmatrix} H_{r_0}[x(0), y_{00}] & \cdots & H_{r_n}[x(0), y_{0n}] & -S_{d'_{-m}}[y(0)] & \cdots & -S_{d'_0}[y(0)] & \cdots & -S_{d'_m}[y(0)] \\ H_{r_0}[x(T_S), y_{00}] & \cdots & H_{r_n}[x(T_S), y_{0n}] & -S_{d'_{-m}}[y(T_S)] & \cdots & -S_{d'_0}[y(T_S)] & \cdots & -S_{d'_m}[y(T_S)] \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ H_{r_0}[x(n_S T_S), y_{00}] & \cdots & H_{r_n}[x(n_S T_S), y_{0n}] & -S_{d'_{-m}}[y(n_S T_S)] & \cdots & -S_{d'_0}[y(n_S T_S)] & \cdots & -S_{d'_m}[y(n_S T_S)] \end{bmatrix} \tag{31}$$

$$U_H = \begin{bmatrix} -1 & 0 & \dots & 0 \\ 0 & -1 & \dots & 0 \\ 0 & 0 & \ddots & \vdots \\ 0 & 0 & \dots & -1 \end{bmatrix} \tag{32}$$

$$U_S = \begin{bmatrix} -1 & \dots & -1 & -1 & 0 & \dots & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & \dots & -1 & -1 & 0 & \dots & 0 \\ 0 & \dots & 0 & -1 & 0 & \dots & 0 \\ 0 & \dots & 0 & -1 & -1 & \dots & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & -1 & -1 & \dots & -1 \end{bmatrix} \tag{33}$$

$$u_H^T = \begin{bmatrix} -\varepsilon & 0 & \dots & 0 \end{bmatrix} \tag{34}$$

$$u_S^T = \begin{bmatrix} -\varepsilon & -\varepsilon & \dots & -\varepsilon \end{bmatrix} \tag{35}$$

$$I^T = \begin{bmatrix} 1 & 1 & \dots & 1 \end{bmatrix} \tag{36}$$

$$r^T = \begin{bmatrix} r_0 & r_1 & \dots & r_n \end{bmatrix} \tag{37}$$

After finding the two parameters that were pointed above, w'_{hj}, z_{0i} and r'_i can be computed by Eqs. (10)- (13), so the inverse model can be obtained. Moreover, w_{sj} and d_j can be solved by Eqs. (14) - (19), and finally the model $H[x]$ can be obtained. In this article we put $\varepsilon = 0.01$, $n = 10$ and $m = 5$. Finally, the output-input error and model errors was optimied with PSO method.

Fig. 3 shows the hysteresis response of the actuator used in this study and Fig. 4 shows its MPI model for $n = 10$ and $m = 5$.
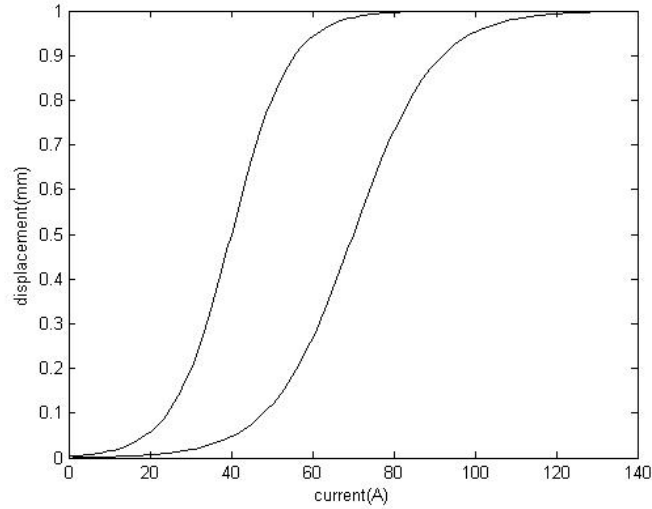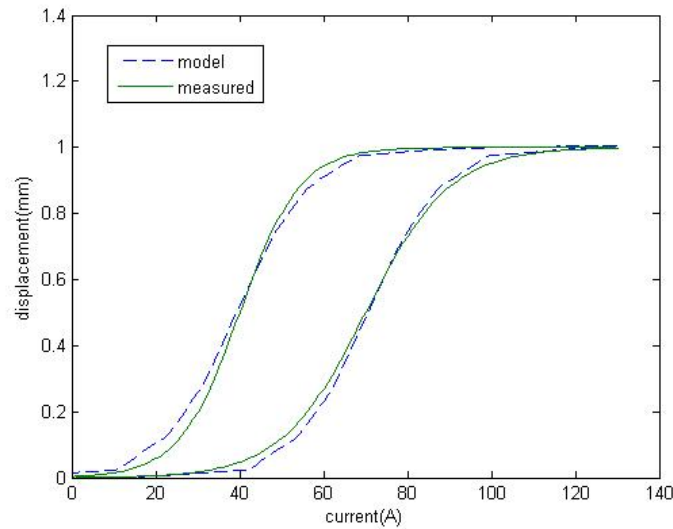
Fig. 3. Measured hysteresis nonlinearity

Fig. 4. Measured hysteresis nonlinearity and its model

When identifying the model, we started with a low model order and then increased the model order until acceptable matching results were obtained.

Fig. (5) shows the experimental output that was achieved from current input that started at zero and took its saturation in 5 A. Fig. (6) shows the achieved model of this experimental result. Despite of nonlinear behavior of the SMA actuator, the achieved model is so precise and has a good fitting.
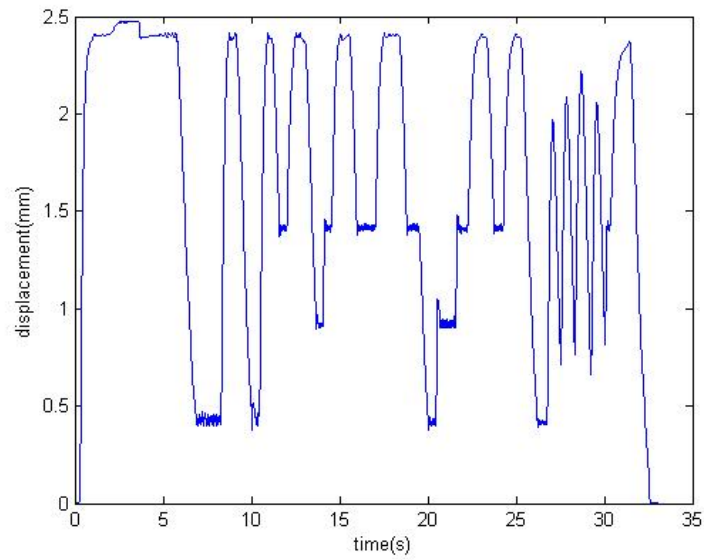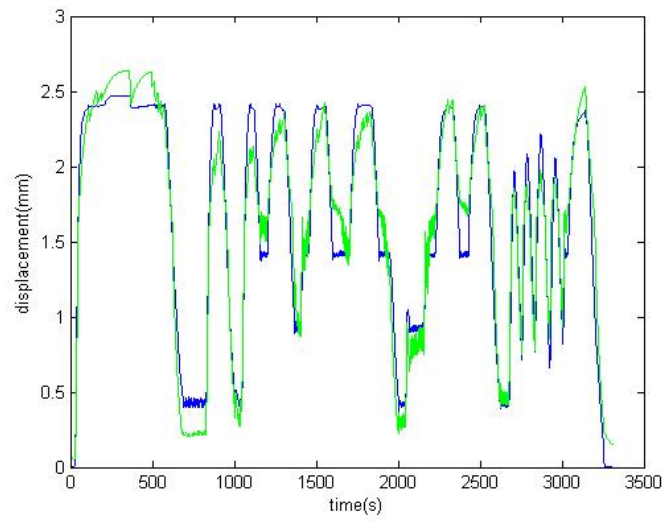
Fig. 5. Output of the actuator

Fig. 6. Achieved model

The parameters of this model and its inverse model are listed in Table 1.

Table 1

Parameters of the hysteresis nonlinearity model

i	r_i	w_{h_i}	j	d_j	w_{s_j}
0	0	0.863	0	0	0.011
1	10.91	-1.06e-17	1	9.13	-0.0018
2	21.82	0	2	30.91	-0.0013
3	32.73	-4.17e-17	3	56.31	0.0007
4	43.63	-6.32e-18	4	79.53	-0.0035
5	54.54	0.250	5	118.9	-0.0005
6	65.45	0	-	-	-
7	76.36	3.84e-18	-	-	-
8	87.27	8.55e-18	-	-	-
9	98.18	1.23e-17	-	-	-
10	109.1	1.66e-18	-	-	-

5. Conclusions

In this article we presented experimental modeling of a shape memory alloy. This model is based on modified Prandtle-Ishlinskii for nonlinear block in series with linear block. Unknown parameters of this model were computed with quadratic optimization method based on experimental results. The achieved model shows that this is a precise model for the nonlinear and complex behavior of the shape memory alloy actuators. As shown, the inverse model of MPI was computed analytically, thus the hysteresis nonlinearity having been reduced, and so the controller designing will be very easy.

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