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THE VARIATION OF THE CHARACTERISTICS OF THE IDEAL GAS FLOWS AS A FUNCTION OF MACH NUMBER CONSIDERING TEMPERATURE GAS PROPERTIES VARIATION

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Variația temperaturii, a presiunii şi a densității cu numărul Mach este descrisă de funcțiile gazodinamice. Aceste funcții sunt deduse în mod uzual în ipoteza exponentului adiabatic constant. Se descrie o metodă sistematică, bazată pe ecuația de conservare a energiei şi pe ipoteza de curgere izentropică (evoluție adiabată) care permite calculul funcțiilor gazodinamice în cazul proprietăților termodinamice variabile cu temperatura (capacitate calorică, entalpie, entropie). Metoda ține cont de compoziția amestecului ideal de gaze ideale.

The temperature, pressure and density variation with the March number is described by the gas dynamical functions. These functions are usually deduced from the hypotheses of the constant adiabatical exponent. A systematic method is described, based on the energy conservation equation and the isentropic flow equation (adiabatic evolution) which allows the calculus of the gasdynamic functions in the case of thermodynamic properties varying with temperature (caloric capacity, enthalpy, entropy). The method takes into consideration the composition of the ideal mixture of ideal gases.

Keywords: gasdynamics functions, NASA thermodynamic functions

Notations

h – the specific enthalpy; H – the molar enthalpy;
s – the specific entropy; S – the molar entropy;
c_p, c_v – the specific heat capacity at constant pressure or volume;
C_p, C_v – molar heat capacities;
χ - the adiabatic exponent;
R - the universal gas constant, J/mol/K
$\hat{R}$ - the gas constant, J/kg/K
Ma - the Mach number;
u - the gas speed;
NC – the number of components in the gas mixture;
0 - indices of the stagnation status;

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1. Introduction

Gasdynamics function are used in the calculus of stationary flow parameters through the nozzles, jet engines and rocket engines.

Usually a gas with a constant adiabatic exponent is considered.

The work fluid in engines is a mixture of gases. The thermodynamic properties of the gas mixture depend on the thermodynamic properties of the pure gases and the composition. The heat capacity of the usual pure gases has a pronounced variation with the temperature, which leads to the variation of the adiabatic exponent.

A method of evaluating the gasdynamic functions is presented, using the NASA method for calculus of the thermodynamic properties of pure gases enthalpy and entropy.

The pressure and temperature variation as function of the Mach number is determined by solving a nonlinear algebraic system.

2. Theory

The gas dynamical functions of the flow point out to the dependence of the temperature and pressure fraction on the Mach number or the Ceaplaghin number. In the following, we will use the Mach number.

These functions are deduced from the energy conservation equation, considering an isentropic transformation:

$$\begin{aligned} h_0 &= h(T) + 0.5u^2 \\ s(T_0, p_0) &= s(T, p) \end{aligned} \tag{1}$$

In the above hypotheses and with the usual notations in the gas dynamics, we have:

$$c_p = \frac{\chi \hat{R}}{\chi - 1}\ ;\ \chi = \frac{c_p}{c_v}; h_0 = c_p T_0\, ; h = c_p T\, ; c_p \ln\frac{T}{T_0} = \hat{R}\ln\frac{p}{p_0}\ \ , \tag{2}$$

and from equations (1) one obtains:

$$c_p T_0 = c_p T + 0.5u^2\, ; \frac{p}{p_0} = \left(\frac{T}{T_0}\right)^{\frac{\chi}{\chi+1}}\ . \tag{3}$$

By introducing the Mach number, *Ma*:

$$Ma^2 = \frac{u^2}{\chi R T}\ , \tag{4}$$

from equations (3), (1), (2) one yields:

$$\frac{T}{T_0}=\left(1+\frac{\chi-1}{2}Ma^2\right)^{-1};\frac{p}{p_0}=\left(1+\frac{\chi-1}{2}Ma^2\right)^{\frac{-\chi}{\chi-1}}. \tag{5}$$

The above equations have been obtained in the hypotheses that the adiabatic exponent is constant and the constitutive equations (2) are valid for ideal gas mixtures, the air for example. Also, the speed in state 0 is considered very small, that is:

$$h_0 >> 0.5u_0^2 \tag{6}$$

The working fluid in aerodynamics is almost always a gas mixture, and in the usual interval of the state variables, the gases can be considered almost perfect. It results that perfect gas mixtures represent the most important thermodynamical system for aeronautical applications [3].

Gas mixtures are not simple thermodynamic systems because the properties of the mixture depend on two state parameters (usually temperature and volume or temperature and pressure) and on the composition.

If the component gases are inert, there are no chemical reactions and the composition remains constant, independent of temperature and pressure. In this case the mixture will act as a single perfect gas (a simple thermodynamic system). Air for example, is a mixture of oxygen, azoth, traces of carbon dioxide and argon. On a considerably large interval of temperatures and pressures the component gases are inert and air can be considered a singular perfect gas (after mentioning the composition).

In the following, we leave aside the hypotheses that the adiabatic coefficient is constant and we will calculate the molar enthalpy and molar entropy of the pure ideal gas with the thermodynamic formulas [4], [5]:

$$\begin{aligned}
&C_{pj}(T)=R\left[\frac{a_{1j}}{T^2}+\frac{a_{2j}}{T}+a_{3j}+a_{4j}T+a_{5j}T^2+a_{6j}T^3+a_{7j}T^4\right]\\
&C_{vj}(T)=C_{pj}(T)-R\ ;\\
&H_j(T)=H_j^0+\int_{T_{ref}}^{T}C_{pj}(T)dT\ ;\\
&S_j(T,p)=S_j^0+\int_{T_{ref}}^{T}\frac{C_{pj}(T)}{T}dT-R\ln\frac{p}{p_{ref}}\ .
\end{aligned} \tag{7}$$

The integration constants and the constants from the molar caloric capacity expression are given in the NASA [4] data base, or in the TPIS [5] data base. The reference thermodynamic state is T_{ref}=298.15 K, p_{ref}= 1bar.

The effective relations for calculus of nondimenssional thermodynamic functions of pure gas j are, Appendix C, NASA TP/2002-211556:

$$\frac{C_{pj}(T)}{T}=\frac{a_{1j}}{T^2}+\frac{a_{2j}}{T}+a_{3j}+a_{4j}T+a_{5j}T^2+a_{6j}T^3+a_{7j}T^4$$
$$\frac{H_j(T)}{RT}=-\frac{a_{1j}}{T^2}+\frac{a_{2j}\ln(T)}{T}+a_{3j}+a_{4j}\frac{T}{2}+a_{5j}\frac{T^2}{3}+a_{6j}\frac{T^3}{4}+a_{7j}\frac{T^4}{5}+\frac{b_{1j}}{T}$$
$$\frac{S_j(T,p)}{R}=-\frac{a_{1j}}{2T^2}-\frac{a_{2j}}{T}+a_{3j}\ln(T)+a_{4j}T+a_{5j}\frac{T^2}{2}+a_{6j}\frac{T^3}{3}+a_{7j}\frac{T^4}{4}+b_{2j}-\ln\frac{p}{p_{ref}} \tag{8}$$

The ideal gas mixture is composed of NC pure chemical substances with a composition characterized by the molar fractions x_j, j=1,2,…; NC, by mass fractions ω_j or by the mass concentrations, mole per unit mass , σ_j :

$$\omega_j=\frac{x_jM_j}{\sum_{j=1}^{NC}x_jM_j} \qquad j=1,\ 2,\dots,\ \mathrm{NC} \qquad \sigma_j=\frac{\omega_j}{M_j} \tag{9}$$

The specific enthalpy and the specific entropy of the ideal gas mixture are calculated with formulas [6], [7], [8]:

$$h(T)=\sum_{j=1}^{NC}\sigma_jH_j(T) \qquad s(T,p)=\sum_{j=1}^{NC}\sigma_j\left(S_j(T,p)-R\ln x_j\right) \tag{10}$$

By substituting equations (4) and (10) in equations (1) it is obtained a nonlinear algebraic system of two equations with two unknowns T and p:

$$h(T_0)=h(T)+\frac{\chi(T)\hat{R}T}{2}Ma^2 \tag{11.1}$$

$$s(T_0,p_0)=s(T,p) \tag{11.2}$$

For solving the nonlinear equation system (11) the composition, the zero state and the Mach number are given. The temperature *T* and pressure *p* are determined. It is possible to solve first the equation (11.1) for the unknown T and then the nonlinear equation (11.2) for the unknown p.

The gas constant and the adiabatic exponent are calculated using the following formulas:

$$\hat{R}=R\sum_{j=1}^{NC}\sigma_j;c_p(T)=\sum_{j=1}^{NC}\sigma_jC_{pj}(T);c_v(T)=\sum_{j=1}^{NC}\sigma_jC_{vl}(T). \tag{12}$$

3. Example

We consider a mixture of nitrogen 79%, oxygen 21% in molar percentages. State zero is T_0=2500 K, p_0=50 bar. The gasdynamic functions are presented in Figs. 1, 2, for case χ=1.4 and by solving the nonlinear system (10). The variation of the adiabatic exponent with the Mach number is presented in Fig. 3.

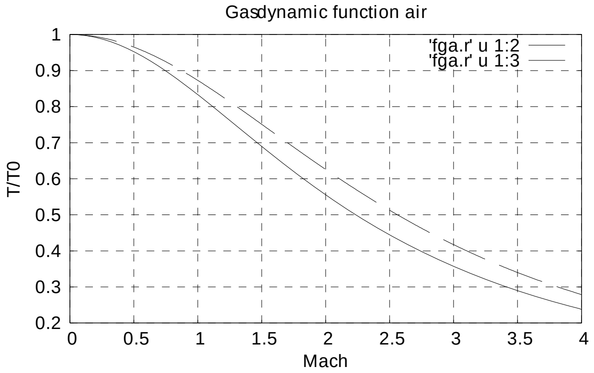

Fig. 1. Gasdynamic function T/T_0. 1:2 C_p constant; 1:3 C_p variable

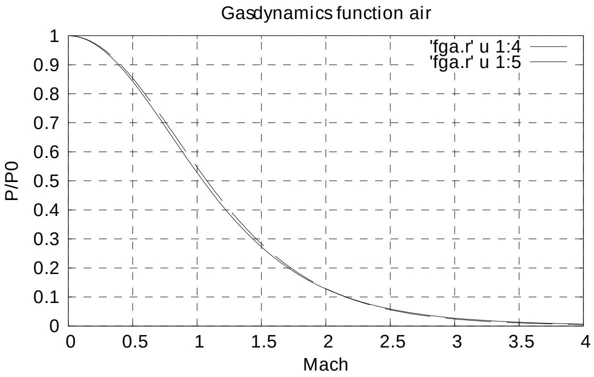

Fig. 2. Gasdynamic function p/p_0. 1:2 C_p constant; 1:3 C_p variable

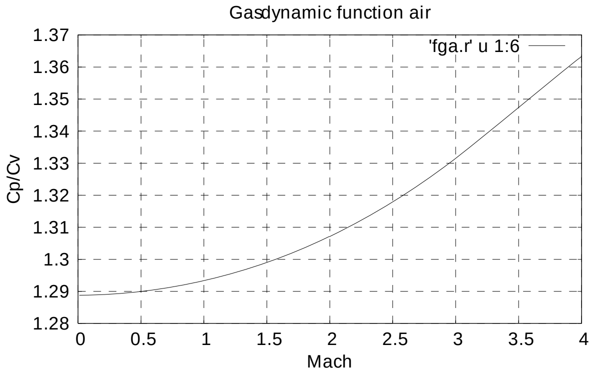

Fig. 3. Variation of adiabatic exponent with Mach number

3. Conclusions

The values of the gas dynamical functions calculated in the constant specific heat hypotheses have a qualitative character. The exact values are obtained by considering the variation of gas mixture properties with temperature, pressure and composition. The calculus requires solving a nonlinear algebraic system of two equations with two unknowns, using the Newton method, having as an initial approximation the values calculated in the constant adiabatic exponent hypothesis. The differences are larger in the case of the temperatures fraction. In the case of the pressures, compensation is produced due to the growth of the adiabatic exponent, at the decrease of temperature during expansion.

For each composition of the gas mixture the molar fractions are calculated, the mass composition variables, moles per mass unit, and then the thermodynamic properties of the mixture. In the case of combustion gases (rocket motors, jet engines) the composition is determined according to the stoechiometrics of the chemical reactions of fuel combustion.

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