THE UTILIZATION OF ELASTICITY MATRIX SPECTRAL DECOMPOSITION IN CALCULATING THE ELASTIC PROPERTIES OF A COMPOSITE BAR HAVING TRANSVERSE ISOTROPY

Dumitru BOLCU[1], Marcela URSACHE[2], Sabin RIZESCU[3], Sonia DEGERATU[4], Nicu George BÎZDOACĂ[5], Marius Marinel STĂNESCU[6]

Lucrarea prezintă descompunerile spectrale ale matricilor coeficienților elastici pentru materiale omogene şi izotrope, precum şi pentru materiale omogene cu izotropie transversă. O bară compozită armată cu fibre de sticlă lungi poate fi considerată ca fiind izotropică transversal, constituenții săi fiind considerați omogeni şi izotropi. Sunt determinate valorile proprii ale matricii coeficienților elastici pentru bara compozită, în funcție de caracteristicile elastice şi de proporțiile volumice ale ambilor constituenți. Este studiat cazul concret al unei bare compozite, armată cu fibre de sticlă lungi, plasate în matrice de răşina epoxidică.

The paper presents the spectral decomposition of the elasticity matrix for a homogeneous and isotropic material and for a material having transversal isotropy. A bar made of a composite material (epoxy resin longitudinally reinforced with long glass fibers), is assumed as being transverse isotropic. Both its constituents are homogeneous and isotropic materials. The paper presents the calculus of the eigenvalues corresponding to the elasticity matrix of the composite bar as a whole. The paper also presents how these eigenvalues do vary, depending on the reinforcing constituent volume ratio.

Keywords: composite materials, spectral decomposition, elastic properties, eigenvalues, matrix constituent, reinforcing constituent

1. Introduction

All theories demonstrate that the mechanic and elastic properties of a certain composite material depend, largely, on many issues, like following:

- the mechanical properties of its constituents;
- the volume ratio of each and every constituent;
- the geometric form of its constituents;

[1] Reader, Faculty of Mechanics, University of Craiova, Romania
[2] Prof., Faculty of Physics, University of Craiova, Romania
[3] Lecturer Faculty of Mechanics, University of Craiova, Romania
[4] Reader, Faculty for Engineering in Electromechanics, Environment and Industrial Informatics, University of Craiova, Romania
[5] Prof., Faculty of Automation, Computation and Electronics, University of Craiova, Romania
[6] Reader, Department of Applied Mathematics, University of Craiova, Romania

- the inner arrangement of its constituents
- the adhesion status between whatever two constituents (phases);
- the technological process to obtain it.

That is why, some big computation-related difficulties do occur during studies concerning the mechanical behavior of composite materials. These studies, basically, consist in building methods to determine the elastic coefficients of a certain composite material.

The first micromechanical model used to evaluate the macroscopic properties of a fiber-reinforced material was the cylinder assemblage model proposed by Haskin [1]. The model was designed to present the analogy between the elastic relaxation modulus and the viscoelastic relaxation modulus in case of heterogeneous materials having the same geometry of phase distribution.

Laws and McLaughlin [2] estimated the viscoelastic creep compliances of several composites by applying the self-consistent method. They used Stieltjes convolution integrals to formulate the problem in the Carson domain and a numerical inversion method to obtain the time domain solution. In the same respect, Yancey and Pindera [3] estimated the creep response of unidirectional linear viscoelastic matrix based composites reinforced with elastic fibers. Also, in case of linear viscoelastic matrix based composite materials reinforced with periodically distributed elastic inclusions, Luciano and Barbero [4] proposed some close-form expressions in the Laplace domain for the elastic coefficients.

In order to determine the elastic coefficients for composites, the finite element-based method was applied, too. Using this method, Huang and Hu [5], analyzed the case of bearing spherical inclusions reinforced composites. Also, Meguid [6], determined the properties of a long fibers reinforced (twice periodically arranged in the transverse section) composite bar.

In this paper, we present the spectral decomposition of the elasticity matrix in case of homogeneous and isotropic materials and in case of materials having transversal isotropy.

The long fibers (periodically arranged in the transverse section) reinforced composite bars, can be considered as having transversal isotropy. Usually, both constituents of such composite material are kind of homogenous and isotropic. Taking into account the existing continuities of deformations and tensions on the boundary surfaces between whatever two constituents (phases) is kind of common sense. Starting from the mathematical expressions of those continuities, we shall establish the dependence between eigenvalues of the matrix of elastic coefficients for the composite bar as a whole and the eigenvalues of the matrix of elastic coefficients of each constituent.

For an epoxy resin based composite bar reinforced with glass fibers, we present the variations of each and every eigenvalue as dependence on the reinforcing constituent volume ratio. For the composite material as a whole, the greatest eigenvalue of the matrix of elastic coefficients depends only on the biggest eigenvalues obtained for each constituent. The following three eigenvalues depend only on the smallest eigenvalues obtained for each constituent.

If the stress tensor and strain tensor are regarded as vectors in an inner product space of suitable dimension, the elasticity tensor can be viewed as a linear transformation on that space. This allows a natural representation of the elasticity tensor in its spectral form and a simple geometrical interpretation of the relationship between stress and strain, regardless the degree of anisotropy. Using the elasticity matrices in their spectral form enhances qualitative comparisons between whatever two different materials within the same elastic symmetry group, and, sometimes, may reveal similarities between materials belonging to different elastic symmetry groups.

In a very comprehensive and substantially founded paper, Mehrabadi and Cowin [7], determined the eigenvalues and eigenvectors in case of homogenous but anisotropic elastic materials. Then, Sutcliffe [8] developed this method and he used it for different types of elastic symmetries.

After pointing out these significant results, the paper continues with original considerations on the spectral decomposition of the elasticity matrix for homogenous materials and for transverse isotropic materials (also homogenous). Next, based on our theoretical results, we present how the elasticity matrix and its corresponding eigenvalues, for a multilayer composite bar, can be obtained. Finally, we give an application designed to validate our theoretically-obtained results in case of epoxy resin matrix, glass fiber reinforced composite bar.

2. Spectral decomposition of the elasticity matrix for homogeneous materials and for transverse isotropic materials

For linear elastic materials, the dependence between the components of deformation tensor and the components of stress tensor is, formally:

$$(\sigma)=[C](\varepsilon), \tag{1}$$

where

$$(\varepsilon)=\left(\varepsilon_{11};\varepsilon_{22};\varepsilon_{33};\sqrt{2}\varepsilon_{23};\sqrt{2}\varepsilon_{13};\sqrt{2}\varepsilon_{12}\right)^t, \tag{2}$$

is the one column matrix of deformations, and

$$(\sigma)=\left(\sigma_{11};\sigma_{22};\sigma_{33};\sqrt{2}\sigma_{23};\sqrt{2}\sigma_{13};\sqrt{2}\sigma_{12}\right)^t, \tag{3}$$

is the one column stress matrix and $[C]$ is the matrix of elastic coefficients.

In order to calculate the elastic coefficients, authors like H.L. Schreyer and Q.H. Zuo [9], S. Sutcliffe [8], use the spectral decomposition of the elasticity tensor. Assuming the same purpose, in this paper, the spectral decomposition of the matrix of elastic coefficients was used. After processing the matrix of elastic coefficients using the spectral decomposition method, the mathematic expressions of obtained eigenvalues are identical to the eigenvalues of elasticity tensor, obtained in [8] and [9]. Using the spectral decomposition method turned out to be more simple than using the tensorial product decomposition.

In case of homogeneous and isotropic materials, $[C]$ has the following form:

$$[C]=\begin{bmatrix} c_{11} & c_{12} & c_{12} & 0 & 0 & 0 \\ c_{21} & c_{11} & c_{12} & 0 & 0 & 0 \\ c_{12} & c_{12} & c_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{11}-c_{12} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{11}-c_{12} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{11}-c_{12} \end{bmatrix}. \tag{4}$$

and $[C]$ has the following spectral decomposition:

$$[C]=\lambda_1[C_1]+\lambda_2[C_2], \tag{5}$$

where: $$\lambda_1=c_{11}+2c_{12}; \quad \lambda_2=c_{11}-c_{12} \tag{6}$$

and:

$$[C_1]=\begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};[C_2]=\begin{bmatrix} \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} & 0 & 0 & 0 \\ -\frac{1}{3} & \frac{2}{3} & -\frac{1}{3} & 0 & 0 & 0 \\ -\frac{1}{3} & -\frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \tag{7}$$

For transverse isotropic materials, the matrix of elastic coefficients has the following form:

$$[C]=\begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{13} & 0 & 0 & 0 \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{11}-c_{12} \end{bmatrix} \tag{8}$$

and its spectral decomposition is:

$$[C] = \lambda_1[C_1] + \lambda_2[C_2] + \lambda_3[C_3] + \lambda_4[C_4], \tag{9}$$

where:

$$\lambda_1 = \frac{1}{2}\left[c_{11} + c_{12} + c_{33} + \sqrt{(c_{11} + c_{12} - c_{33})^2 + 8c_{13}^2}\right],$$
$$\lambda_2 = \frac{1}{2}\left[c_{11} + c_{12} + c_{33} - \sqrt{(c_{11} + c_{12} - c_{33})^2 + 8c_{13}^2}\right], \tag{10}$$
$$\lambda_3 = c_{11} - c_{12}, \quad \lambda_4 = c_{44},$$

and:

$$[C_1] = \begin{bmatrix} \frac{1}{2}\sin^2\alpha & \frac{1}{2}\sin^2\alpha & \frac{1}{\sqrt{2}}\sin\alpha\cos\alpha & 0 & 0 & 0 \\ \frac{1}{2}\sin^2\alpha & \frac{1}{2}\sin^2\alpha & \frac{1}{\sqrt{2}}\sin\alpha\cos\alpha & 0 & 0 & 0 \\ \frac{1}{\sqrt{2}}\sin\alpha\cos\alpha & \frac{1}{\sqrt{2}}\sin\alpha\cos\alpha & \cos^2\alpha & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$[C_2] = \begin{bmatrix} \frac{1}{2}\cos^2\alpha & \frac{1}{2}\cos^2\alpha & -\frac{1}{\sqrt{2}}\sin\alpha\cos\alpha & 0 & 0 & 0 \\ \frac{1}{2}\cos^2\alpha & \frac{1}{2}\cos^2\alpha & -\frac{1}{\sqrt{2}}\sin\alpha\cos\alpha & 0 & 0 & 0 \\ -\frac{1}{\sqrt{2}}\sin\alpha\cos\alpha & -\frac{1}{\sqrt{2}}\sin\alpha\cos\alpha & \sin^2\alpha & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$[C_3] = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & 0 & 0 & 0 & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}; [C_4] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{11}$$

and: $$\sin 2\alpha = \frac{2\sqrt{2}c_{13}}{\lambda_1 - \lambda_2}. \tag{12}$$

In case of isotropic and homogeneous and isotropic materials, two elastic constants, λ_1 and λ_2, are, necessarily, to be known. In case of transverse isotropic materials, four elastic constants, $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ and the parameter α must be known.

3. The calculus of eigenvalues of the elasticity matrix for a multilayer composite bar

A long fibers reinforced composite bar may be assumed as having elastic transverse isotropic elasticity. For this reason, the determination of elastic properties of a composite bar made of two homogeneous and isotropic constituents becomes possible. Assuming a perfect adherence between the two constituents and based on results obtained in [10]. In [11] is presented an algorithm designed to calculate the elastic characteristics of a multilayer composite bar.

In order to estimate the elastic characteristics of a certain composite material, a homogenization theory has to be used. Choosing the appropriate theory is a difficult task and that because those elastic characteristics depend on the specific form of the composite material. Even in simple cases like composite plates and bars, using such kind of theories should be done carefully and taking into account some important issues like the reinforcing constituent distribution and the boundary conditions. In this kind of respect, [12] contains basis of a new theory concerning nonlinear incompressible composites.

Focusing one some authors' recent results and, also, being aware of some new releases [13-14], as well as of a recent paper especially dedicated to an epoxy resin-based matrix carbon fiber reinforced material [15], this paper presents an application of a homogenization theory based on the spectral decomposition of the elasticity matrix in a specific case of a multilayer composite bar.

For such a bar, made from two layers, the matrix of elastic coefficients has the following form:

$$\left[C^*\right] = \begin{bmatrix} c_{11}^* & c_{12}^* & c_{12}^* & 0 & 0 & 0 \\ c_{12}^* & c_{22}^* & c_{23}^* & 0 & 0 & 0 \\ c_{12}^* & c_{23}^* & c_{22}^* & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44}^* & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55}^* & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{55}^* \end{bmatrix}, \tag{13}$$

where: $$c_{11}^{*}=\frac{2}{\Delta}\left(\sum_{i=1}^{2}\frac{V_i}{\lambda_2^{(i)}}\right)\left(\sum_{i=1}^{2}\frac{V_i\left(\lambda_1^{(i)}-\lambda_2^{(i)}\right)}{2\lambda_1^{(i)}+\lambda_2^{(i)}}\right)^2+\sum_{i=1}^{2}\frac{3V_i\lambda_1^{(i)}\lambda_2^{(i)}}{2\lambda_1^{(i)}+\lambda_2^{(i)}};$$

$$c_{12}^{*}=\frac{1}{\Delta}\left(\sum_{i=1}^{2}\frac{V_i}{\lambda_2^{(i)}}\right)\left(\sum_{i=1}^{2}\frac{V_i\left(\lambda_1^{(i)}-\lambda_2^{(i)}\right)}{2\lambda_1^{(i)}+\lambda_2^{(i)}}\right);\ c_{22}^{*}=\frac{1}{\Delta}\sum_{i=1}^{2}\frac{V_i\left(\lambda_1^{(i)}+2\lambda_2^{(i)}\right)}{\lambda_2^{(i)}\left(2\lambda_1^{(i)}+\lambda_2^{(i)}\right)};$$

$$c_{23}^{*}=\frac{1}{\Delta}\sum_{i=1}^{2}\frac{V_i\left(\lambda_1^{(i)}-\lambda_2^{(i)}\right)}{\lambda_2^{(i)}\left(2\lambda_1^{(i)}+\lambda_2^{(i)}\right)};\quad c_{44}^{*}=\frac{1}{\sum_{i=1}^{2}\frac{V_i}{\lambda_2^{(i)}}};\ c_{55}^{*}=\sum_{i=1}^{2}V_i\lambda_2^{(i)};$$

$$\Delta=\left(\sum_{i=1}^{2}\frac{V_i\left(\lambda_1^{(i)}+2\lambda_2^{(i)}\right)}{\lambda_2^{(i)}\left(2\lambda_1^{(i)}+\lambda_2^{(i)}\right)}\right)^2-\left(\sum_{i=1}^{2}\frac{V_i\left(\lambda_2^{(i)}-\lambda_1^{(i)}\right)}{\lambda_2^{(i)}\left(2\lambda_1^{(i)}+\lambda_2^{(i)}\right)}\right)^2. \tag{14}$$

and V_1, V_2 are the corresponding volume ratios of those two components materials; $\lambda_1^{(1)}, \lambda_2^{(1)}$ are the elastic constants for the first homogeneous and isotropic constituent (phase) and $\lambda_1^{(2)}, \lambda_2^{(2)}$ represent the elastic constants for the second one.

Assuming that a multilayer composite bar is obtained by randomly overlapping elementary bars like the bar elastically described by the relations (13) and (14), then, the matrix of elastic coefficients for such a multilayer composite bar can be introduced by:

$$\left[C\right]=\frac{1}{A}\iint_{(S)}\left[S\right]\left[C^{*}\right]\left[S\right]^{t}dS \tag{15}$$

where A is the area of the transverse section S of the bar, and $\left[S\right]$ is the transit matrix from bar's fixed own reference system to the reference system the relation (14) was written with respect to.

The matrix $\left[S\right]$ has the following form:

$$[S]=\begin{bmatrix} \cos^2\varphi & \sin^2\varphi & 0 & 0 & 0 & \sqrt{2}\cos\varphi\sin\varphi \\ \sin^2\varphi & \cos^2\varphi & 0 & 0 & 0 & -\sqrt{2}\cos\varphi\sin\varphi \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\varphi & -\sin\varphi & 0 \\ 0 & 0 & 0 & \sin\varphi & \cos\varphi & 0 \\ \sqrt{2}\cos\varphi\sin\varphi & -\sqrt{2}\cos\varphi\sin\varphi & 0 & 0 & 0 & \cos^2\varphi-\sin^2\varphi \end{bmatrix}, \quad (16)$$

where φ is, in fact, the angle between the x_1 axis and an in-section perpendicular to a separation surface between whatever two constituents of each elementary bar found in the transverse section of the composite bar as a whole [11]. The x_3 axis is the longitudinal axis of the composite bar while the x_1 axis and x_2 axis are describing the planar transverse section of the bar [11].

The elastic coefficients of the multilayer composite bar described by the elasticity matrix (8), are:

$$c_{11}=\frac{3}{8}\overset{*}{c}_{11}+\frac{3}{8}\overset{*}{c}_{22}+\frac{1}{4}\overset{*}{c}_{12}+\frac{1}{4}\overset{*}{c}_{55}, \quad c_{12}=\frac{1}{8}\overset{*}{c}_{11}+\frac{1}{8}\overset{*}{c}_{22}+\frac{3}{4}\overset{*}{c}_{12}-\frac{1}{4}\overset{*}{c}_{55},$$
$$c_{13}=\frac{1}{2}\overset{*}{c}_{12}+\frac{1}{2}\overset{*}{c}_{23}, \quad c_{33}=\overset{*}{c}_{22}, c_{44}=\frac{1}{2}\overset{*}{c}_{44}+\frac{1}{2}\overset{*}{c}_{55}. \quad (17)$$

Considering the restrictions (10) and (12), we obtain for the composite bar, the following characteristics:

$$\lambda_1=\frac{1}{2}\left[\frac{1}{2}\overset{*}{c}_{11}+\frac{3}{2}\overset{*}{c}_{22}+\overset{*}{c}_{12}+\sqrt{\left(\frac{1}{2}\overset{*}{c}_{11}-\frac{1}{2}\overset{*}{c}_{22}+\overset{*}{c}_{12}\right)^2+2\left(\overset{*}{c}_{12}+\overset{*}{c}_{23}\right)^2}\right],$$

$$\lambda_2=\frac{1}{2}\left[\frac{1}{2}\overset{*}{c}_{11}+\frac{3}{2}\overset{*}{c}_{22}+\overset{*}{c}_{12}-\sqrt{\left(\frac{1}{2}\overset{*}{c}_{11}-\frac{1}{2}\overset{*}{c}_{22}+\overset{*}{c}_{12}\right)^2+2\left(\overset{*}{c}_{12}+\overset{*}{c}_{23}\right)^2}\right],$$

$$\lambda_3=\frac{1}{4}\overset{*}{c}_{11}+\frac{1}{4}\overset{*}{c}_{22}-\frac{1}{2}\overset{*}{c}_{12}+\frac{1}{2}\overset{*}{c}_{55},$$

$$\lambda_4=\frac{1}{2}\overset{*}{c}_{44}+\frac{1}{2}\overset{*}{c}_{55}, \sin 2\alpha=\frac{\sqrt{2}\left(\overset{*}{c}_{12}+\overset{*}{c}_{23}\right)}{\sqrt{\left(\frac{1}{2}\overset{*}{c}_{11}-\frac{1}{2}\overset{*}{c}_{22}+\overset{*}{c}_{12}\right)^2+2\left(\overset{*}{c}_{12}+\overset{*}{c}_{23}\right)^2}}. \quad (18)$$

4. Application

We consider the case of a composite bar, which has an epoxy resin-based matrix and is reinforced with glass. The elastic characteristics of both constituents are given in the following table:

Table 1

Elastic characteristics of constituents

Material	$E\left(MPa\right)$	υ	$\lambda_1\left(MPa\right)$	$\lambda_2\left(MPa\right)$
Epoxy Resin	4500	0,4	22500	3210
Glass	74000	0,25	148000	59200

The composite material may be considered as a transverse isotropic material, having the elastic characteristics given by the relations (18).

The graphical representation (figures 1 – 5) of the dependences $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ and $\sin 2\alpha$ given by relations (18) and (12), were obtained by using specific plotting software.

The variation of the characteristic for the multilayer composite bar is presented in figure 1. *V* is the volume ratio of the reinforcing constituent.

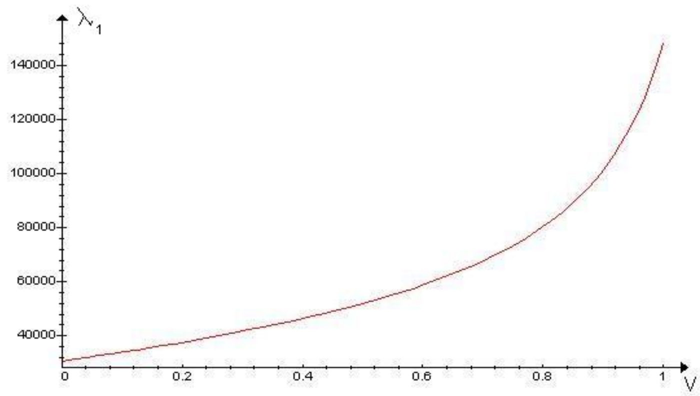

Fig. 1. Variation of λ_1 as dependence on *V*

Is to be noticed that the variation curve starts from the value corresponding to the matrix constituent and arrives at the value corresponding to the reinforcing constituent.

In fig. 2, fig. 3 and fig. 4, the variations of the characteristics and for the multilayer composite bar are presented.

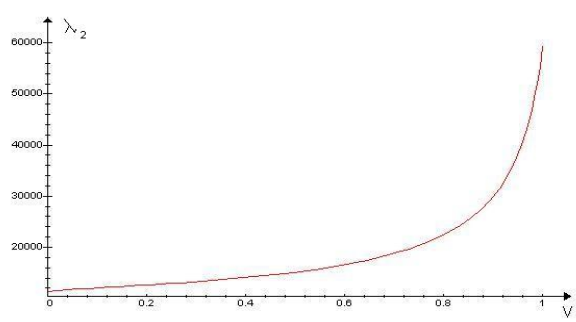

Fig. 2. Variation of λ_2 as dependence on *V*

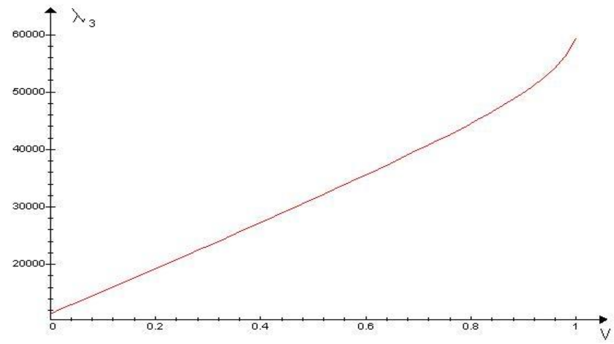

Fig. 3. Variation of λ_3 as dependence on V

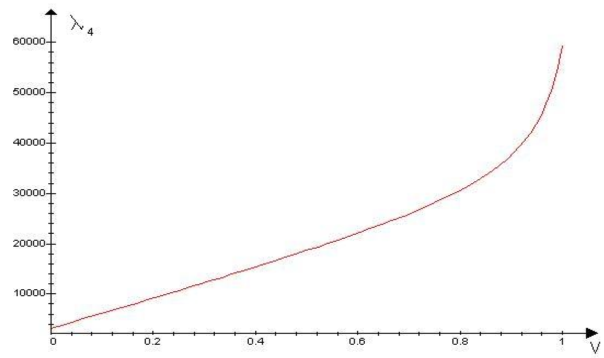

Fig. 4. Variation of λ_4 as dependence on V

In all the last three graphic representations, the variation curves for λ_2, λ_3 and λ_4 start, also, from their values corresponding to the matrix constituent $(V=0)$ and arrive at their values corresponding to the reinforcing constituent $(V=1)$.

Although the extreme values for all the characteristics λ_2, λ_3 and λ_4 are identical, their corresponding variation curves, depending on the volume ratio V of the reinforcing constituent, are quite different.

Examination of the last four graphic representations leads to the following conclusion: if the volume ratio V is smaller than 0,5, then, all the variations curves are pretty much linear.

If the volume ratio V is bigger than 0,5, then, all the variation curves are far from being linear.

In figure 5, the variation of the function $\sin 2\alpha$ is presented. This function characterizes the so-called “non homogeneity degree” of composite material as a whole.

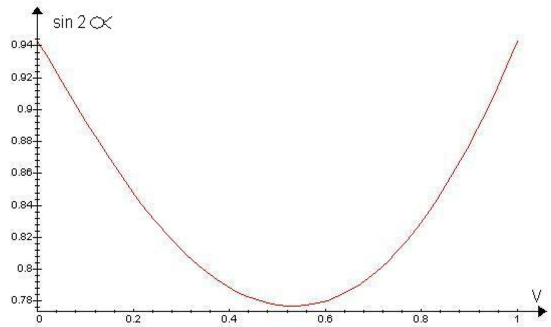

Fig. 5. Variation of sin 2α

At the value $\sin 2\alpha = \frac{2\sqrt{2}}{3}$, the material is considered as "homogeneous". The more the value of the function $\sin 2\alpha$ differs from the specific $\frac{2\sqrt{2}}{3}$ value, the more the composite material is considered less homogeneous.

5. Conclusions

Based mostly on some of their previous works and original results, the authors present a new, original and effective method designed to determine the elastic characteristics of a multilayer composite bar. Bars of this kind are often used in many different technical domains. The originality of this work consists especially in its analytical approach and in its strong mathematic fundamentals. Also, the elasticity matrix spectral decomposition method represents an authors' original, comprehensive and engineering-like method to approach the important and ever actual issue of analytically calculating the elastic coefficients of composite materials. This work paper could be really useful when it comes to make accurate estimations concerning transverse isotropic composite materials – especially multilayer sandwich-shaped composite bars. In this kind of respect, expressing the elastic constants as dependences of each and every constituent volume ratio could turn out to be extremely useful in dynamics (vibrations) of a certain composite structure. In cases like that, the structure mass and the structure mass spatial distribution as well, become very important issues the way that mass distribution and elastic characteristics have work together in order to obtain a desired dynamic response. This paper, by offering an algorithm to estimate each and every elastic constant as dependence of each and every constituent volume ratio, actually offers the possibility to make computation-based choices concerning the nature of constituents as well as concerning their volume ratios and spatial distribution.

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REFERENCES

[1] *Z. Haskin,* "Viscoelastic behavior of heterogeneous media", ASME, J. Appl. Mech., 32, 1965, pp. 630-636.

[2] *N. Laws, J.R. McLaughlin.,* "Self-consistent estimates for the viscoelastic creep compliances of composites materials", Proceedings of the Royal Society, London, **vol. 39**, 1978, pp. 627-649.

[3] *R.N., Yancey, M.J. Pindera,* "Micromechanical analysis of the creep response of unidirectional composites", ASME J. of Engineering Materials and Technology, **vol. 112**, 1990, pp. 157-163.

[4] *R. Luciano, E.J. Barbera*, "Analytical expression for the relaxation moduli of linear viscoelastic composites with periodic microstructure", ASME, J. Appl. Mech., **vol. 62**, 1995, pp. 786-793.

[5] *Y. Huang, K.X. Hu*, "Elastic moduli of microcracked composite with spherical inclusions of cubic anisotropy", Composite Science and Technology, **50(2)**, 1994, pp. 149-156.

[6] *S.A. Meguid, A.L. Kalamkarov*, *A. Zougas*, "Analysis of local and effective elastic properties of fibre-reinforced materials", Compas. Eng., **4(5)**, 1994, pp. 525-536.

[7] *M.M. Mehrabadi*, *S.C. Cowin,* "Eigentensor of linear anisotropic elastic materials", Quartely Journal of Mechanics and Applied Mathematics, **vol. 43**, part. 1, 1990, pp. 15-41.

[8] *S. Sutcliffe*, Spectral decomposition of the elastic tensor, ASME, J. Appl. Mech., **Vol. 59**, 1992, pp. 762-773.

[9] *H.L. Schreyer, Q.H. Zuo*, "Anisotropic yield surfaces based on elastic projection operators", ASME, J. Appl. Mech., **vol. 62, No. 3**, 1995, pp. 780-785.

[10] *D. Bolcu, G. Stanescu, M. Ursache*, "Theoretical and experimental study on the determination of the elastic properties of composite materials", Romanian Reports in Physics, **vol. 56**, No. 1, 2004, pp. 3-13.

[11] *I. Ciucă, D. Bolcu, M.M. Stănescu, G. Marin, S.D. Ionescu*, "Study Concerning Some Elasticity Characteristics Determination of Composite Bars", Materiale Plastice, **vol. 45**, 2008, issue 3, pp. 279-284.

[12] *Z.Y. Guo, X.Q. Peng, B. Moran*, "Mechanical response of neo-Hookean fiber reinforced incompressible nonlinearly elastic solids", International Journal of Solids and Structures, **vol.** 44, issue 6, 2007, pp. 1949-1969.

[13] *J. Yvonnet, D. Gonzalez, Q.C. He*, "Numerically explicit potentials for the homogenization of nonlinear elastic heterogeneous materials", Computer Methods in Applid Mechanics and Engineering, **vol.** 198, issue 33-36, 2009, pp. 2723-2737.

[14] *N. Auffray*, "Analytical expressions for anisotropic tensor dimension", Comptes Rendus Mecanique, **vol.** 338, issue 5, 2010, pp. 260-265.

[15] *A.A. Lukyanov*, "An equation of state of a carbon-fibre epoxy composite under shock loading", European Physical Journal B, **vol.** 74, issue 1, 2010, pp. 35-45.