POWER AMPLIFIER MODELING FOR MODERN COMMUNICATION SYSTEMS

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Lucrarea propune o nouă metodă de modelare a unui amplificator de putere pentru sistemele de comunicaţii. Metoda realizează o analiză în domeniul timp şi are la bază, spre deosebire de metodele existente până în prezent, excitarea sistemului cu un semnal de tip OFDM. Avantajul metodei constă în modelarea comportamentului neliniar cvasi-fără-memorie pe baza semnalului ce poate fi ulterior utilizat în cadrul transmisiunii. Modelul obţinut nu va depinde de semnal şi poate fi ulterior utilizat pentru a analiza răspunsul sistemului la diverse tipuri de semnale.

The paper introduces a new method to model power amplifier used for communication systems. The algorithm implements a time domain analysis and comparing with existing methods is based on using an OFDM signal as excitation. The method advantages are represented by the ability to model a quasi-memoryless nonlinear behavior based on the signal that will be used within the communication link. The resulted model is independent on the signal therefore can be used to analyze the system response to other signal types.

Keywords: power amplifier modeling

1. Introduction

The purpose of any power amplifier is to bust the radio signal at the output power level that can guarantee the quality of the signal transmitted in the optimum conditions. Unfortunately this is not an easy task and can be achieved only by compromising between efficiency and linearity. In this context, measuring and modeling of the memory and nonlinear phenomena and also of the system that is generating them, represent an important step in designing of any communication system.

The classical two tone test method represents a solution when phenomena like phase distortion, spectral asymmetry and intermodulation behavior need to be

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evaluated [1-3]. On the other hand due to the modern signal complexity, the two tone analysis doesn't provide the necessary information to properly model the nonlinear system [4-7].

The OFDM signal is made by a number of modulated sub-carriers and it has a number of attributes which make it a prime candidate for wireless standards. The biggest disadvantage of the OFDM signal is the big peak-to-average power ratio (PAPR). This ratio generates higher distortions and therefore lower efficiency at the power amplifier (PA) level. The solution of this problem is represented by applying a linearization method. In this context, the analyzing and modeling of the PA nonlinear behavior represent one of the most intensive researches worldwide [1-7].

The present paper proposes a new method of nonlinear PA behavior modeling. The method is based on a time domain analysis. Comparing with the existed methods (as for example the classical two tone tests) the excitation signal is considered to be an OFDM signal. The first advantage of the proposed approach is the possibility to model the nonlinear behavior and partially the memory effects based on the signal that lately will be used within the communication link.

The paper is organized as follows. After introduction, the principle of the PA measuring and modeling is briefly presented in section 2. While section 3 shows a comparison between the classical two tone method and the proposed modeling method. In parallel, in order to prove the general availability of the resulted model, an analysis based on different types of signals is performed. Finally, in section 4 the conclusions of the analyzed method are draw.

2. The principle of nonlinear characterization method

From the beginning the nonlinear model of the PA will be simulated as a polynomial development:

$$y(t) = \sum_{k=0}^{\frac{N-1}{2}} a_k x(t) |x(t)|^{2k} \tag{1}$$

where $y(t)$ is considered to be the system nonlinear response, $x(t)$ the excitation signal, N the maximum polynomial development order and a_k represent the coefficients of the power series. Only the odd terms are considered within the model. The a_k coefficients will be deducted by solving an over-determined linear equation system in the least squares sense, using the QR factorization.

It will be proved that a time domain analysis represent a very good approach when the modeling of a nonlinear system is under discussion. In this respect Fig. 1 shows the measuring setup that allows the characterization of the nonlinear system based on the development (1). The excitation signal at the input of the nonlinear system is considered to be an OFDM signal.
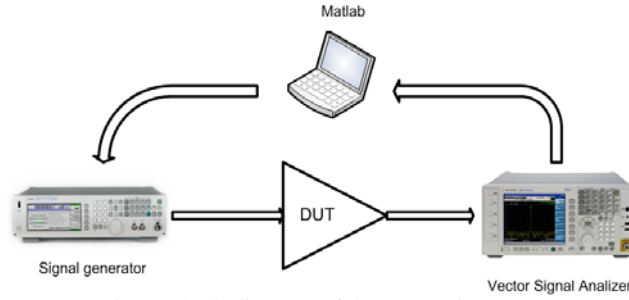
Fig 1. Block diagram of the measuring setup

Any modulated amplitude and/or phase signal can be written as:
$$x(t) = A(t)\cos(2\pi f_0 t + \theta(t)) \tag{2}$$

where f_0 represent the carrier frequency, $A(t)$ and $\theta(t)$ represent the amplitude and phase modulation. The signal expression can be rearranged as:
$$x(t) = \operatorname{Re}\{A(t)e^{j\theta(t)}e^{j2\pi f_0 t}\} = \operatorname{Re}\{s(t)e^{j2\pi f_0 t}\} \tag{3}$$

where $s(t)$ is the base-band signal.

In order to apply the time domain modeling procedure the signals at the nonlinear system input and output have to be known. Those signals have to correspond to the modulated signal extracted at the input ($x(t)$) and output ($y(t)$) of the DUT or to the corresponding base-band signals, $s_1(t)$ and respectively $s_2(t)$. From the practical point of view the base-band analysis is easier to be checked and implemented. The process starts with the generation of the OFDM signal in Matlab. To obtain a good model for a certain application the PAPR of the generated signal has to be high enough so that the resulted model will be valid for other types of signal. To get better performance the model should be obtain for the highest power level at which the system will to be used. The Matlab signal is transferred to a signal generator as shown in Fig. 1. Using the connection Matlab – Signal Generator offers the possibility to generate any type of signal and to adjust the model according to the design needs. From the theoretical point of view applying the signal (2) at the input of system (1) will result a signal with a larger spectrum. Let's note the useful signal as:

$$y(t) \cong A_y(t)\cos[2\pi f_0 t + \theta_y(t)] \tag{4}$$

where $A_y(t)$ and $\theta_y(t)$ represent the amplitude and the phase at the output of the nonlinear system. The extraction of the two mentioned components can be done by using a vector signal analyzer (VSA) as showed in the Fig. 1. The vector analyzer allows the extraction of the signal, which can later be transferred back to Matlab. More exactly the VSA is used for the capture and sampling of the complex waveforms and final for transferring I and Q signals to the Matlab

application. In order to eliminate from the analysis the influence of the measuring system two different steps should be implemented:

- First a calibration process should be done throughout which the signal generator is directly connected to the vector analyzer and excitation signal is captured.
- Second the DUT is connected between the two devices and the system output signal is extracted.

The signal samples captured at the input and output in order to realize the base-band analysis will be:

$$x[n] = s_1(nT_s)$$
$$y[n] = s_2(nT_s)$$

(5)

where T_s is the sampling period.

The model and algorithm accuracy depends on the selected nonlinear polynomial order, on the VSA settings, captured signal level and the Matlab computation accuracy. Analyses will be performed in section 3.

The modeling performance criterion is represented by the comparison of the modeled and measured output signal when both systems are excited by the same signal. The modeling accuracy can be evaluated by using the normalized mean square error (NMSE), defined as:

$$NMSE = \frac{\sum_{k=1}^{M} \left| y_{mes}[k] - y_{mod}[k] \right|^2}{\sum_{k=1}^{M} \left| y_{mes}[k] - \overline{y_{mes}} \right|^2}$$

(6)

where y_{mes} represent the measured response while y_{mod} represent the modeled response and $\overline{y_{mes}}$ is the measured average signal. In the present situation the signals doesn't include a continuous component therefore $\overline{y_{mes}} = 0$. When the error is small enough is preferred to rewrite the expression (6) in dB:

$$NMSE[dB] = 10\log_{10}\left[\frac{\sum_{k=1}^{M} \left| y_{mes}[k] - y_{mod}[k] \right|^2}{\sum_{k=1}^{M} \left| y_{mes}[k] \right|^2} \right]$$

(7)

3. Practical implementation and results

In order to prove the advantages that the proposed modeling method has comparing with the classical two tone test, practical measurements were performed while using a class AB power amplifier. The system was designed for the 2GHz band and it is capable to handle 3% EVM for 34dBm output power at 8dB back-off from OP1dB. For the first tests the main differences between the OFDM and the two tone test modeling methods, were raised. The second set of tests was performed in order to bring up the general availability of the modeling proposed technique.
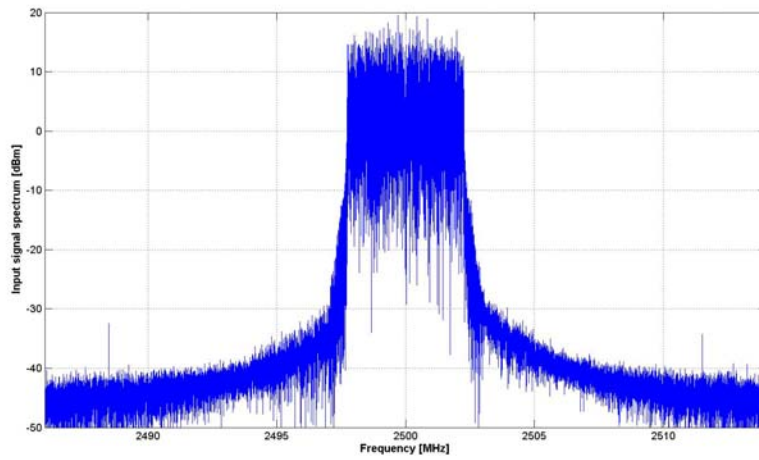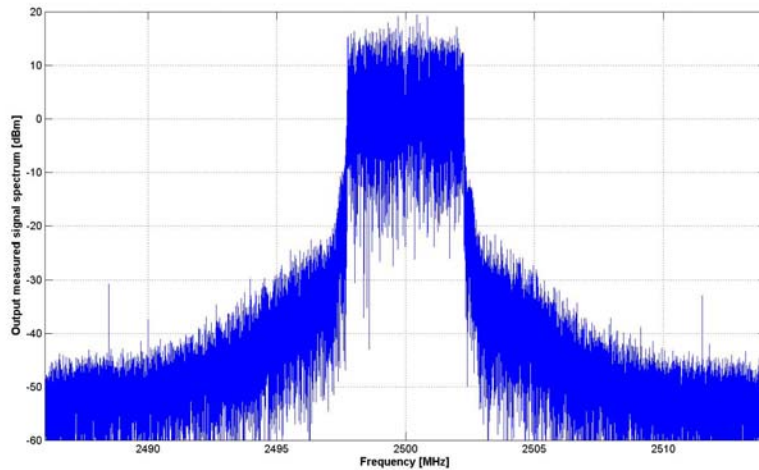

Fig 2. The excitation signal spectrum

Fig 3. The PA measured output spectrum

From beginning the OFDM excitation was considered. Fig. 2 shows the spectrum for the mentioned excitation. The signal was considered to have a 5MHz bandwidth and the output transmitted power was +34dBm. Fig. 3 shows the power amplifier measured output signal. The nonlinear distortion is visible in Fig. 3 comparing with Fig. 2.
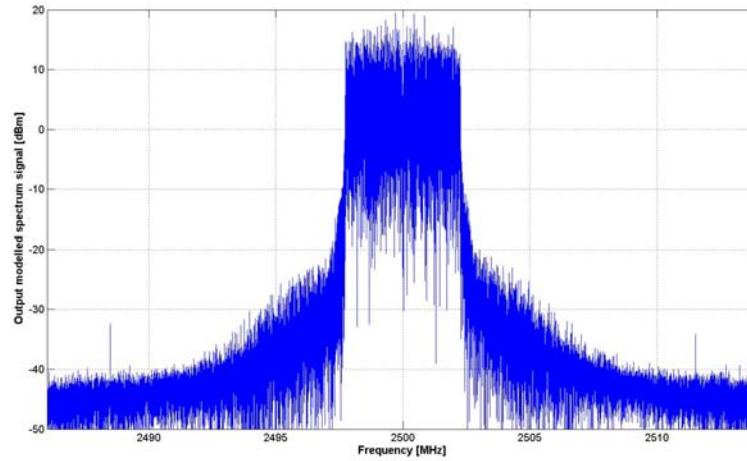

Fig 4. The PA model output spectrum

<div align="right">Tabel 1</div>

Nonlinear coefficients of the PA model

Nonlinear coefficients	OFDM model	Two tone model
a_1	1	1
a_2	0.7028-0.8743i	-0.6661+0.4549i
a_3	-14.8239+2.0803i	24.4544-17.7144i

Based on the input and output measured signal and applying the modeling procedure previous defined a nonlinear polynomial model (as described in equation (1)) was obtained. The model uses a 5th order nonlinear development and the parameters were evaluated in Matlab with the double precision floating-point (10^{-16} precision). The output signal of the resulted model is described within Fig.4. The resulted nonlinear coefficients for an OFDM model are shown in Table1. Comparing the measured and the modeled output spectrum and the time domain response it can be noticed that the PA nonlinear behavior is properly modeled by using the relation (1) and the procedure previously described. In order to prove the disadvantages that a two tone model has comparing with the OFDM model, Fig. 5 shows the nonlinear response of a modeled build with two tone (the

nonlinear coefficients are presented in Table 1) while the OFDM excitation was considered. It is visible how the two tone model presents a stronger nonlinear behavior, much different than the measured spectrum presented in Fig. 3.
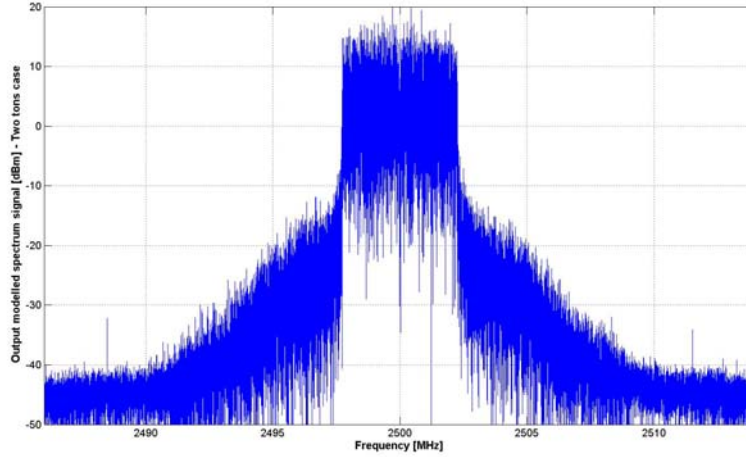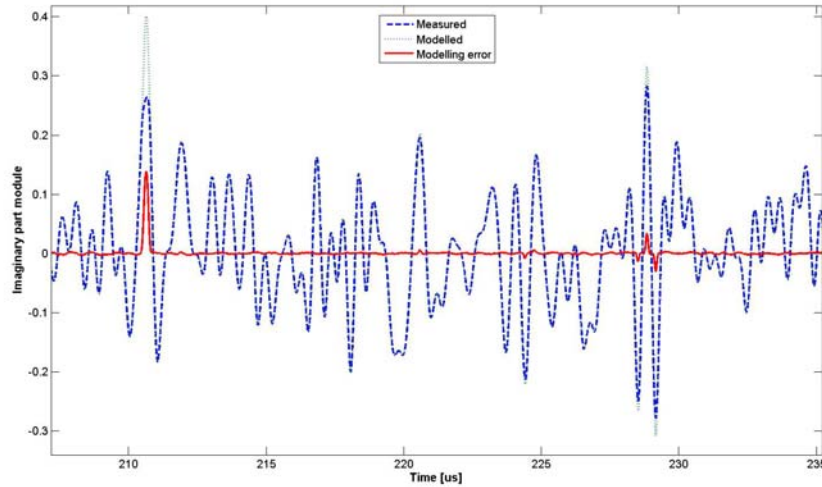

Fig 5. The two tone model output spectrum

Fig 6. The imaginary part of the signal obtained for the two tone model when the OFDM excitation was considered

Comparing Fig. 4 and Fig. 5 from the spectral point of view it results approximately 10dB difference in the adjacent channels behavior between the two models. The explanation of this important difference can be found in the time domain. More exactly, Fig. 6 shows that an important error results in the

estimation of the high peaks of the signal envelope. For easier comparison purposes the system gain was considered of 0dB at a logarithmic scale. In this respect the figure shows the measured and modeled output signal scaled in order to obtain the same average power as the input signal. For illustration purposes the figure presents a randomly chosen area of the signals' imaginary part. As much as the excitation signal presents a high PAPR the resulted two tone model will present a higher error comparing with the real PA but also comparing with the modeled obtained from the OFDM signal. In this respect it can be concluded that the model resulted based on the two tone is not appropriate when the applied excitation signal is a complex one with high PAPR.
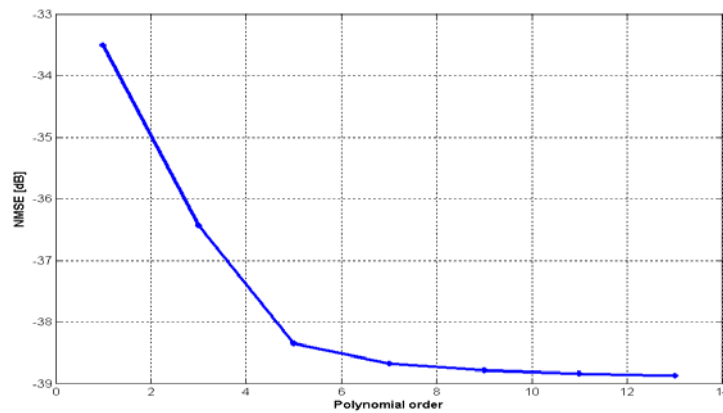

Fig 7. The NMSE variation vs. Polynomial order development

It is not a secret that increasing the nonlinear complexity of a model the time processing will increase substantially. In the same context a polynomial development of lower order will not have the same performances as a higher order one. Therefore Fig. 7 shows the NMSE variation with the order of the polynomial development, while Fig. 8 shows the NMSE variation with the number of samples used within the simulation. From Fig. 7 it can be understood that a linear polynomial function (1st order) will not properly modeled the PA behavior and the NMSE value is about -33.5dB. The value obtained for the error can be justified by the fact that the model was extracted for a class AB power amplifier, situation in which the linear behavior is an important part. In the same time as much as the polynomial order will increase the error between the model and the real system performance will reduce substantially. For example in the case of a polynomial function of 11th order the NMSE is almost of -39dB. In order to make the difference between the two NMSE values previously mentioned it is important to note that on a logarithmic scale, 3dB degradation in NMSE represents increasing to double the modeling system error. In the present case a polynomial

development of 5th order was considered as a compromise between model efficiency and the modeling error. Fig. 7 proves that the NMSE value for a 5th order polynomial function is around -38.4dB which is an acceptable error comparing with the best performance that can be obtained within this type of modeling (NMSE=-38.9dB).

Fig. 8 shows that the two tone model has a constant variation of NMSE versus the samples number variation. The second curve shows the NMSE variation for the OFDM signal which due to the high PAPR value has a completely different behavior. In the present analysis the signal area used in order to extract the model starts at the beginning of the OFDM symbol which consists of 1.6 million samples. It can be seen that while the high amplitude samples of the OFDM signal are not included within the computational algorithm, the modeling error is high. A smaller number of samples comparing with the number presented in the figure can be used while the samples timing is properly chosen. The results in Fig. 8 prove that the two tone model can be obtained faster comparing with the OFDM model. From another point of view, as was proved until now, the two tone model is not so accurate while using OFDM signal as excitation.
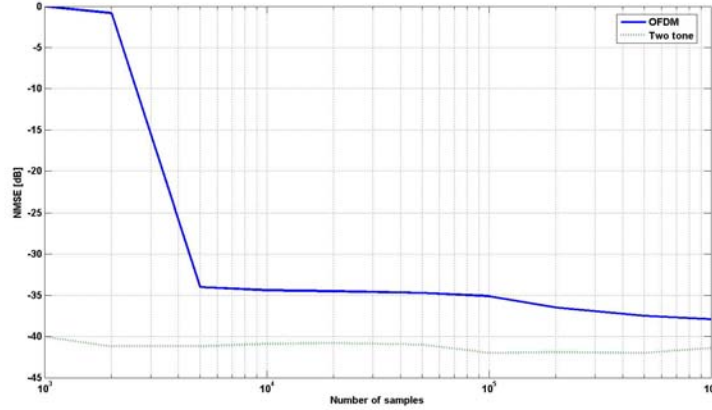

Fig 8. The NMSE variation vs. Number of samples used within the simulation

From the practical point of view it is interesting to know if the resulted model is valid for any other power level and for different other signal types. It was before mentioned that the model is very much dependent on the PAPR of the excitation signal. In order to avoid the influence that this parameter can have over the output power variation of the system it is recommended to build up the model for the highest power level used. To prove, Fig. 9 shows the NMSE results obtained for different power levels. The reference model used was obtained for +34dBm. For each power level one model was build and the corresponding NMSE value ($NMSE_{model}$) was evaluated. This value was compared with the

NMSE values resulted from the reference model ($NMSE_{34dBm}$) used for the corresponding lower power level. The figure presents also a comparison between $NMSE_{model}$ and a linear model NMSE (noted: $NMSE_{lnear}$) corresponding to a certain output power levels. It can be understood from the results presented in the Table 2 and plotted in Fig. 9 that the reference model can represent a solution for modeling the desired dynamic range of the nonlinear system. The resulted error (Δe_1) between the corresponding equivalent power level NMSE and the reference model NMSE is below 0.5dB, while comparing the same NMSE for a certain power with the result obtained for a linear model the error (Δe_2) will increase with the output power. Table 2 also shows a degradation of the NMSE when the captured signal level is decreasing. This shows the importance of the VSA settings and the level of the captured signal. For a good modeling performance, SNR for the signal imported to Matlab should be as high as possible.
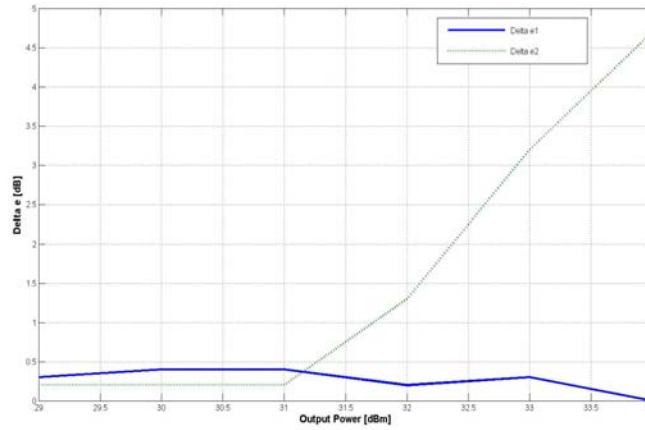

Fig 9. Models comparison based on output power variation

Tabel 2

Models comparison based on output power variation

Output power	34 dBm	33 dBm	32 dBm	31 dBm	30 dBm	29 dBm
$NMSE_{model}$ [dB]	-38.6	-40.1	-38.4	-37.9	-37.2	-34.5
$NMSE_{34dBm}$ [dB]	-38.6	-39.8	-38.2	-37.5	-36.8	-34.2
$NMSE_{linear}$ [dB]	-33.9	-36.9	-37.1	-37.7	-37	-34.3
$\Delta e_1 = NMSE_{34dBm} - NMSE_{model}$	0	0.3	0.2	0.4	0.4	0.3
$\Delta e_2 = NMSE_{linear} - NMSE_{model}$	4.7	3.2	1.3	0.2	0.2	0.2

Such type of approach allows also the system modeling without taking into account the signal type. In this respect Fig. 10 shows the results obtained for different types of signals at the output power level of +34dBm. The same two types of errors, Δe_1 and Δe_2, were considered, just that this time the $NMSE_{model}$ was evaluated in function of the signal not on the power. The reference signal is a Wimax 64-QAM with 5MHz bandwidth, with symbol duration of 0.5ms, FFT size of 256, number of used sub-carriers 200 and a PAPR of 9dB. Applying this signal at the input of an ideal linear model the resulted error (Δe_2) between the NMSE corresponding values is about 2.9dB. On the other side comparing the reference model NMSE with any other NMSE value obtained for different other signal the error (Δe_1) will be below 0.5dB. It can be see that error difference between the NMSE of a linear model and the NMSE of a specific model for a specific type of signal vary in function of the signal type. The highest error value can be obtained for a Wimax QPSK with 50% duty cycle signal and 10MHz bandwidth while the lowest error results for the 16-QAM with 100% duty cycle and bandwidth of 5MHz. Other signal types used within the measurements are: Wimax 64-QAM with 50% duty cycle, bandwidth of 5MHz and a short symbol length (0.5ms); Wimax 64-QAM with 50% duty cycle, bandwidth of 5MHz and a long symbol length (4ms); Wimax 64-QAM with 50% duty cycle, bandwidth of 10MHz; WCDMA with 100% duty cycle and 5MHz bandwidth.
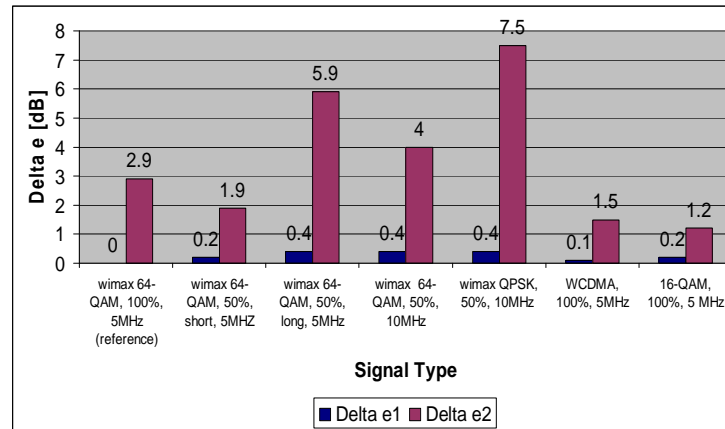

Fig 10. Models comparison based on different signal types

4. Conclusions

It has been proved that a polynomial complex development can represent an alternative for modeling a nonlinear power amplifier while the excitation signal used to extract the model parameters is a complex signal with a high PAPR value.

The proposed modeling method supposes a time domain analysis. The final result includes aspects that cannot be modeled while considering a simple two tone model. The resulted model represents a compromise between efficiency and evaluation time. As long as the model is obtained for the highest power transmitted it can be applied for any other power level in the dynamic range of the nonlinear system. The solution is general valid for different types of signals. All this advantages offer a strong solution for modeling power amplifiers nonlinear behavior. The proposed technique can be very helpful when the designers want to implement different linearization procedures.

List of Acronyms

dB   decibel
DUT   device under test
EVM   error vector magnitude
FFT   fast Fourier transform
NMSE   normalized mean squared error
OFDM   orthogonal frequency division multiplexing
OP1dB   output 1 dB compression point
PA   power amplifier
PAPR   peak-to-average power ratio
QAM   quadrature amplitude modulation
QPSK   quadrature phase shift keying
SNR   signal-to-noise ratio
VSA   vector signal analyzer
WCDMA   wideband code division multiple access
WiMAX   worldwide interoperability for microwave access

R E F E R E N C E S

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