DISCRETE RESISTIVE MODELS FOR MULTI-RATE ANALYSIS OF INTEGRATED ANALOG CIRCUITS

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Circuitele neliniare excitate cu semnale multiton sunt analizate cu metoda variabilelor multiple de timp, deoarece analiza tradițională este greoaie. Această metodă transformă ecuațiile algebrice diferențiale ce descriu circuitele neliniare excitate cu semnale multiton în ecuații cu derivate parțiale. Pentru a rezolva aceste ecuații sunt folosite circuite resistive dsicrete pentru elementele dinamice de circuit. Lucrarea propune noi circuite companion pentru circuitele excitate cu semnale multiton, generate cu algoritmul trapezelor.

For the nonlinear circuits with widely-separated time scales it is used the analysis in multi-time variables because the traditional analysis is very difficult. By applying this approach the differential algebraic equations describing the nonlinear analog circuits driven by multi-tone signals are transformed into multi-time partial differential equations. In order to solve these equations, associated discrete resistive equivalent circuits (companion models) for the dynamic circuit elements are used. This paper propose new companion models for multi-rate analysis of integrated analog circuits, generated by trapezoidal algorithm.

Keywords: companion models, multiple time variables, trapezoidal algorithm, integrated circuits

1. Introduction

The RF-IC applications are, in general, strongly nonlinear and have carrier frequencies into the GHz-range, with modulated signals in the kHz-range. Due to these peculiarities simulating such systems with traditional SPICE-like integration algorithms that work with differential-algebraic equations (DAEs) becomes computationally expensive. The integration time-step must be small enough to accurately capture the fast component and obtaining information about the slowly component needs a large number of time-steps. In these circumstances finding the steady-state by brute force, that starts with an arbitrary initial condition and integrates the system until the transient components vanish, becomes prohibitive [1- 4, 6, 7, 12]. Shooting methods adjust the guess initial condition at the end of the period using a nonlinear solver (usually Newton-Raphson) and integrate by

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transient simulation. When signals with widely separated rates are involved, reaching steady-state needs a large number of time steps. Consequently, shooting cannot handle efficiently circuits driven by multi-tone signals, being definitely a single tone algorithm [5]. The finite-difference time-domain technique (FDTD) discretizes the differential equations over a period yielding a system of algebraic equations, which are solved simultaneously to find solutions in all time points of the discretization network. FDTD method usually converges slower than shooting method [8]. Another approach for the steady state simulation is harmonic balance method (HB) that operates in the frequency domain. It computes the steady state response as the solution of a nonlinear algebraic equation system where the signal is represented by its truncated Fourier series. Because of the multidimensional Fourier transform availability, HB is a natural multi-tone algorithm, but if the circuit is strongly nonlinear, the number of Fourier coefficients needed to describe the unknown waveform accurately is large and therefore the method can be efficiently used for weakly or mildly nonlinearities only [3, 12].

2. The equations for dynamic elements

In this paper are represented the companion models for the dynamic circuit elements, generated by trapezoidal algorithm for the multi-rate analysis. For simplicity, it is consider the two time case.

The characteristics of the nonlinear elements are approximated by piecewise-linear continuous curves and for the numerical integration the implicit trapezoidal algorithm is used.

The characteristic equation for a linear inductor is

$$u_L(t_1,t_2) = L\frac{di_L}{dt} = L\left(\frac{\partial i_L}{\partial t_1} + \frac{\partial i_L}{\partial t_2}\right) \tag{1,a}$$

Using the formula for trapezoidal algorithm the equation (1,a) becomes

$$u_L(i,j) = \frac{2L}{h_1}\left[i_L(i,j) - i_L(i,j-1)\right] - u_L(i,j-1) + \frac{2L}{h_2}\left[i_L(i,j) - i_L(i-1,j)\right] - u_L(i-1,j) \tag{1,b}$$

$$u_L(i,j) = \frac{2L(h_1+h_2)}{h_1 h_2}\, i_L(i,j) - e_{L(i-1,j-1)} \tag{1,c}$$

where

$$e_{L(i-1,j-1)} = \frac{2L}{h_1} i_L(i,j-1) + u_L(i,j-1) + \frac{2L}{h_2} i_L(i-1,j) + u_L(i-1,j) \tag{1,d}$$

The voltage expression for a nonlinear inductor, using the trapezoidal algorithm, is:

$$u_L(t_1,t_2)=\frac{d\varphi_L(t_1,t_2)}{dt}=\frac{\partial\varphi_L(t_1,t_2)}{\partial t_1}+\frac{\partial\varphi_L(t_1,t_2)}{\partial t_2} \tag{2,a}$$

For *(k+1)* iteration, the expression is

$$u_L(i,j)=\frac{2}{h_1}\left(\varphi_{(i,j)}^{(k+1)}-\varphi_L(i,j-1)\right)-u_L(i,j-1)+\frac{2}{h_2}\left(\varphi_{(i,j)}^{(k+1)}-\varphi_L(i-1,j)\right)-$$
$$-u_L(i-1,j) \tag{2,b}$$

$$u_L(i,j)=\frac{2(h_1+h_2)}{h_1h_2}[L_d(s_{(i,j)}^{(k)})i_{L(i,j)}^{(k+1)}+\Phi_L(s_{(i,j)}^{(k)})]=u(i_{(i,j)}^{(k+1)})-e_{L(i-1,j-1)} \tag{2,c}$$

where

$$e_{L(i-1,j-1)}=e_L(i,j-1)+e_L(i-1.j)+\frac{2}{h_1}\varphi_L(i,j-1)+\frac{2}{h_2}\varphi_L(i-1,j) \tag{2,d}$$

is a voltage source with known value from previous time step, L_d is the incremental inductance and $\Phi_L(s_{(i,j)}^{(k)})$ is the flux in origin.

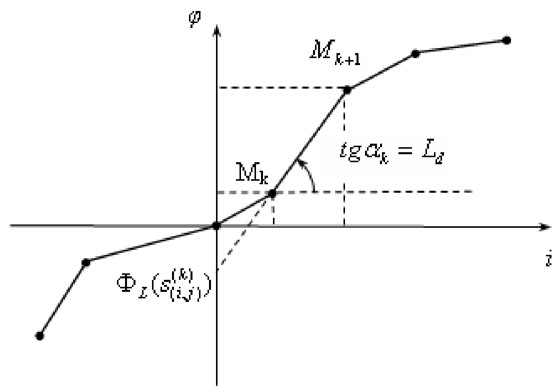

Fig.1. The nonlinear inductor characteristic approximated by piecewise-linear continuous curve

The current expression using the trapezoidal algorithm, for a linear capacitor, is:

$$i_C(t_1,t_2)=C\frac{du_C(t_1,t_2)}{dt}=C\left(\frac{\partial u_C}{\partial t_1}+\frac{\partial u_C}{\partial t_2}\right) \tag{3,a}$$

$$i_C(i,j)=\frac{2C}{h_1}\left[u_C(i,j)-u_C(i,j-1)\right]-i_C(i,j-1)+\frac{2C}{h_2}\left[u_C(i,j)-u_C(i-1,j)\right]-$$
$$-i_C(i-1,j) \tag{3,b}$$

$$i_C(i,j)=G_{ij}u_C(i,j)-j_{C(i-1,j-1)} \tag{3,c}$$

where the current source

$$j_{C(i-1,j-1)} = j_C(i,j-1) + j_C(i-1,j) + \frac{2C}{h_1}u_C(i,j-1) + \frac{2C}{h_2}u_C(i-1,j) \qquad (3,d)$$

has a known value from the previous step.

Using the trapezoidal algorithm for numerical integration, the expression for the current through a voltage-controlled nonlinear capacitor, has the expression:

$$i_C(t_1,t_2) = \frac{dq(t_1,t_2)}{dt} = \frac{\partial q(t_1,t_2)}{\partial t_1} + \frac{\partial q(t_1,t_2)}{\partial t_2} \qquad (4,a)$$

$$i_{C-}(i,j) = \frac{2}{h_1}\left[q_{(i,j)}^{(k+1)} - q(i,j-1)\right] - i_C(i,j-1) + \frac{2}{h_2}\left[q_{(i,j)}^{(k+1)} - q(i-1,j)\right] - \qquad (4,b)$$

$$- i_C(i-1,j)$$

$$i_C(i,j) = \frac{2}{h_1}\left[q_{(i,j)}^{(k+1)} - q(i,j-1)\right] + \frac{1}{h_2}\left[q_{(i,j)}^{(k+1)} - q(i-1,j)\right] = \frac{2(h_1+h_2)}{h_1h_2}[C_d(s_{(i,j)}^{(k+1)})u_{C(i,j)}^{(k+1)} +$$

$$+ Q_C(s_{(i,j)}^{(k)})] - j_{C(i-1,j-1)} \qquad (4,c)$$

$$i_C(i,j) = \frac{2(h_1+h_2)}{h_1h_2}[C_d(s_{(i,j)}^{(k+1)}u_{C(i,j)}^{(k+1)} + Q_C(s_{(i,j)}^{(k)})] - j_{C(i-1,j-1)} = i(u_{(i,j)}^{(k+1)}) - j_{C(i-1,j-1)} \qquad (4,d)$$

where

$$j_{C(i-1,j-1)} = j_C(i,j-1) + j_C(i-1,j) + \frac{2}{h_1}q(i,j-1) + \frac{2}{h_2}q(i-1,j) \qquad (4,e)$$

is a current source with a known value from the previous step and $Q_C(s_{(i,j)}^{(k)})$ is the charge in origin as it shows in Fig. 2.

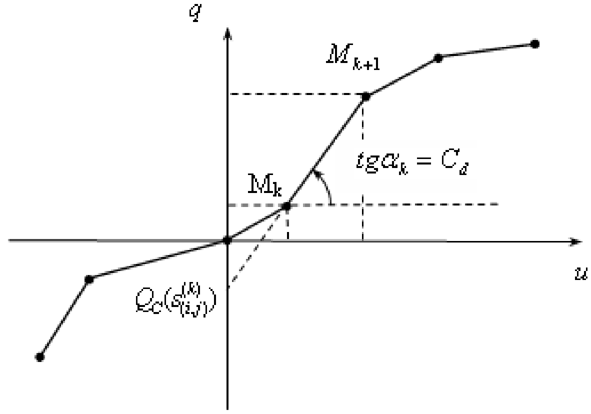

Fig. 2. The nonlinear capacitor characteristic approximated by piecewise-linear continuous curve

3. Discrete resistive models for dynamic circuits elements

The discrete resistive circuits generated by these equations (1)-(4) are represented in Fig. 3 for a linear inductor, in Fig. 4 for a current controlled nonlinear inductor, in Fig. 5 for a linear capacitor and in Fig. 6 for a voltage-controlled nonlinear capacitor.

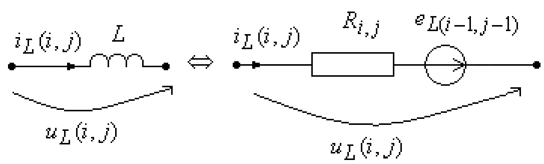

Fig. 3. The discrete resistive model for linear inductor.

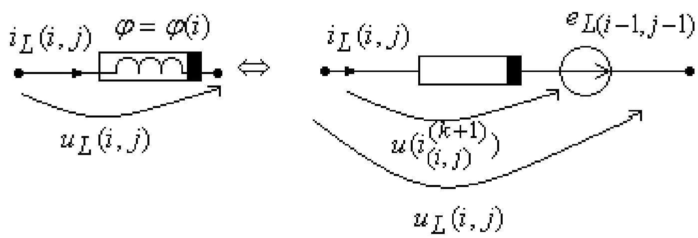

Fig. 4. The discrete resistive model for nonlinear inductor.

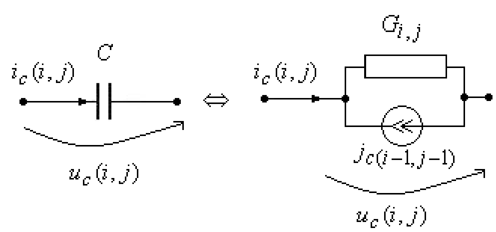

Fig. 5. The discrete resistive model for linear capacitor.

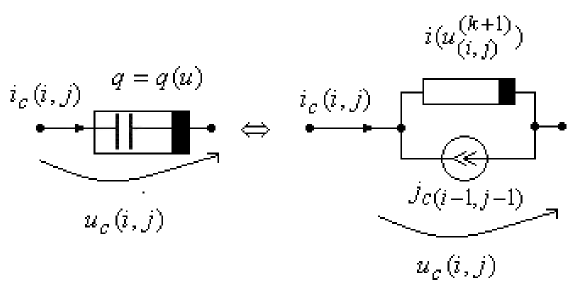

Fig. 6. The discrete resistive model for nonlinear capacitor.

4. Example

Consider the nonlinear circuit shown in Fig. 7:

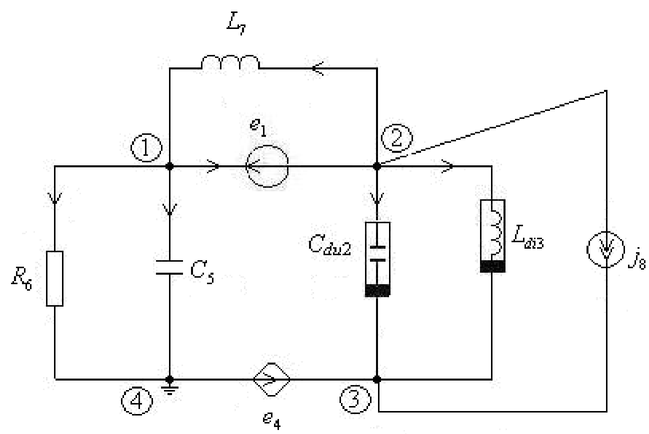

Fig. 7. A nonlinear circuit

Using the equivalent discrete resistive model for each element from the circuit for iteration (k+1), the equivalent circuit is represented in Fig. 8.

$$i_{C2}(i,j)=i(u^{(k+1)}_{C2(i,j)})-j_{C2(i-1,j-1)}$$

$$u_{L3}(i,j)=u(i^{(k+1)}_{(i,j)})-e_{L3(i-1,j-1)}$$

$$i_{C5}(i,j)=G_{C5(i,j)}u_{C5}(i,j)-j_{C5(i-1,j-1)}$$

$$u_{L7}(i,j)=R_{L7(i,j)}i_{L7}(i,j)-e_{L7}(i-1,j-1)$$

If it is used for numerical integration the trapezoidal algorithm, the following data have the expressions:

$$i(u^{(k+1)}_{(i,j)})=\frac{2(h_1+h_2)}{h_1h_2}[C_d(s^{(k+1)}_{(i,j)}u^{(k+1)}_{C(i,j)}+Q_C(s^{(k)}_{(i,j)})]$$

$$j_{C2(i-1,j-1)}=j_{C2}(i,j-1)+j_{C2}(i-1,j)+\frac{2}{h_1}q(i,j-1)+\frac{2}{h_2}q(i-1,j)$$

$$u_{L3}(i^{(k+1)}_{(i,j)})=\frac{2(h_1+h_2)}{h_1h_2}[L_d(s^{(k)}_{(i,j)})i^{(k+1)}_{L(i,j)}+\Phi_L(s^{(k)}_{(i,j)})]$$

$$e_{L3(i-1,j-1)}=e_{L3}(i,j-1)+e_{L3}(i-1.j)+\frac{2}{h_1}\varphi_{L3}(i,j-1)+\frac{2}{h_2}\varphi_{L3}(i-1,j)$$

$$G_{C5(i,j)}=\frac{2C_5(h_1+h_2)}{h_1h_2}$$

$$j_{C5(i-1,j-1)} = j_{C5}(i,j-1) + j_{C5}(i-1,j) + \frac{2C_5}{h_1} u_{C5}(i,j-1) + \frac{2C_5}{h_2} u_{C5}(i-1,j)$$

$$R_{L7(i,j)} = \frac{2L_7(h_1 + h_2)}{h_1 h_2}$$

$$e_{L7(i-1,j-1)} = \frac{2L_7}{h_1} i_{L7}(i,j-1) + u_{L7}(i,j-1) + \frac{2L_7}{h_2} i_L(i-1,j) + u_{L7}(i-1,j)$$

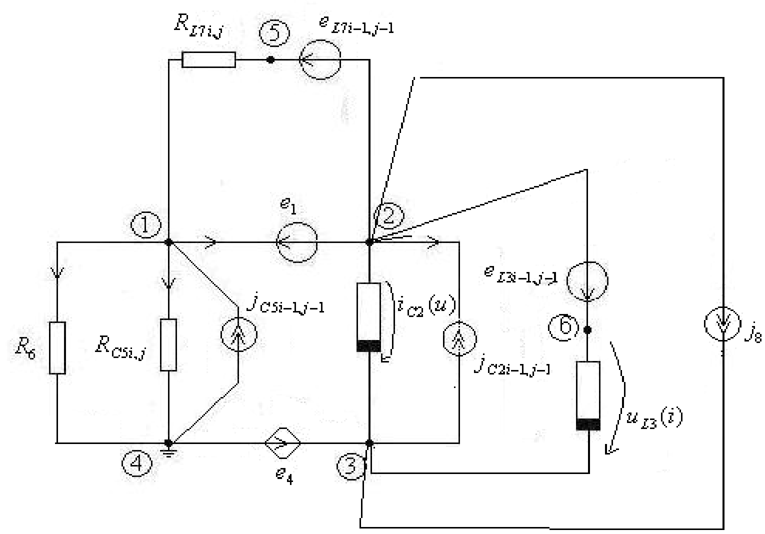

Fig. 8. The equivalent discrete resistive circuit

5. Conclusions

Using the procedure with multiple time variables to describe multirate behavior, the differential algebraic equations describing the nonlinear analog circuits driven by multi-tone signals are transformed into multi-time partial differential equations.

The discrete resistive circuits associated with a numerical implicit integration algorithm like the trapezoidal one or one of Gear algorithms are used to model the dynamic elements. Approximating the characteristics of the nonlinear elements by piecewise-linear continuous curves, in each grid point a new formulation for the modified nodal equations in two time variables can be obtained. Using the companion models for the dynamic circuit elements we can generate an efficient algorithm to solve MPDE.

The main advantage of the multi-time approach consists in improvements in simulation speed compared to Differencial Algebric Equations-based methods, because the simulation result can be numerically represented using far fewer points than in one time variable, while containing all the information to recover the original signal.

Associated discrete resistive equivalent circuits (companion models) for the dynamic circuit elements are used in order to solve the multi-time partial differential equations. The discrete resistive equivalent circuits are depending on the implicit algorithm used for the numerical integration. So, it is important to use a accurate algorithm. The trapezoidal algorithm (the second order Adams-Moulton method) is one of the second order, so it is more accurate than the first order algorithms.

R E F E R E N C E S

[1] R. *Achar, M.S. Nakhla*, Simulation of High-Speed Interconnects, Proceedings of the IEEE, **vol. 89**, no. 5, 2000, pp. 693-728

[2] *V. Borich, J. East, G. Haddad*, A fixed-point harmonic balance approach for circuit simulation under modulated carrier excitation, Proceedings of the ISCAS, **vol. 6**, 1999, pp. 343-349

[3] *H.G. Brachtendorf, G. Welsch, R. Laur*, A novel time-frequency method for the simulation of the steady state of circuits driven by multi-tone signals, Proceedings of the ISCAS,1997, pp. 1508-1511

[4] *Fl. Constantinescu, M. Nitescu*, Computation of the periodic steady-state in RF-IC circuits using fast envelope prediction. Proc. of the International Symposium on Signals, Circuits and Systems, 2001, pp. 217-220

[5*] A. Hodge, R.W. Newcomb*, Semistate Theory and Analog VLSI Design, IEEE Circuit and Systems Magazine, **vol. 2**, no. 2, 2002, pp. 30-49

[5] *M. Iordache, L. Dumitriu, L. Mandache*, Time-Domain Modified Nodal Analysis for Large-Scale Analog Circuits, Revue Roum. Sci. Techn. - Électrotechn. et Énerg., **vol. 48**, 2-3, 2003, pp. 257-268

[6] *M. Iordache, L. Dumitriu*, Efficient Decomposition Techniques for Symbolic Analysis of Large – Scale Analog Circuits by State Variable Method, Analog Circuits and Signal Processing, **vol. 40**, no. 3, 2004 pp. 235-253

[7] *R. Larchevêque, P. Bolcato, E. Ngoya*, New and Efficient Method for the Multitone Steady-State Circuit Simulation, Proceedings of the ISCAS, 1998, pp. 330-333

[8] *D. Topan, L. Mandache*, Chestiuni speciale de analiza circuitelor electrice (Special matters on the electrical circuit analysis), Ed. Universitaria, Craiova, 2007;250 p. B5, ISBN 978-973-742-808-0, pp. 115-151.